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NONLINEAR ADAPTIVE CONTROL: REGULATION-TRACKING-OSCILLATIONS

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ABSTRACT. An overview of nonlinear continuous-time adaptive control is given focusing on evolution of control objectives from conventional regulation and tracking towards synchronizing and modifying complex system dynamics: periodic or chaotic oscillations. The unified framework for such presentation employs the concept of the speed-gradient. The schemes of control and adaptation algorithms are given as well as stability condition.

KEY WORDS. Nonlinear adaptive control; oscillations; chaos.

1. INTRODUCTION

The importance of taking into account the plant nonlinearity when adaptive control design has been demonstrated recently in robotics (Ortega, Spong, 1989), in power systems (Vesely, Mudroncik, 1991), in chemical and biotechnology (Bastin, 1992). Some attempts were also taken to use the concept of adaptive control for investigating the systems with complex nonlinear (e.g. chaotic) dynamics (Hübler, 1989; Vassiliadis, 1994), having more applications in physics, biology, communications, etc.

The development of nonlinear adaptive control (NAC) theory is also stimulated by recent progress in general nonlinear control theory (see, e.g. Isidori, 1989). However the existing NAC theory is still far from completeness. Therefore it worth trying to track main tendencies of its development. The evolution of such concepts and approaches in NAC as feedback linearization, compensation, Lyapunov and passivity approaches was briefly exposed in Fradkov, Hill (1993) (see also surveys by Kokotovic, Kanellakopoulos, Morse, 1991; Praly *et al.*,1991; Fradkov, 1990).

The present paper is focused on the role and evolution of the control goal which is (together with the plant model) the main component of the control problem. Both the conventional goals (regulation, tracking) are considered, but also some new ones (swinging, synchronizing) which are typical for control of oscillating systems. It is shown that the general "speed-gradient" framework that has been widely used for treating the conventional problems of nonlinear adaptive control (see Fradkov, 1991; Stotsky, Fradkov, 1992) is also applicable to the control of nonlinear oscillations. Examples of speed-gradient control of periodic and chaotic oscillations are given extending some previous results obtained by other methods. The main results are formulated for continuous-time case.

The paper is organized as follows. In Section 2 the general adaptive control problem statement is given. The speed-gradient approach is briefly described in Section 3. Section 4 is devoted to the adaptive synchronization of periodic and chaotic motions. In Section 4 the problem of swinging control is considered.

2. THE FORMULATION OF THE ADAPTIVE CONTROL PROBLEM

Consider the continuous-time version of general adaptive control problem statement suggested by V.A.Yakubovich (1968) (see also Fomin, Fradkov, Yakubovich,1981). Given the plant state equations

$$\dot{x} = F(x,u,t,\xi), \quad y = G(x,u,t,\xi), \ t\ge 0, \qquad (2.1)$$

where $x\in R^n$, $u\in R^m$, $y\in R^l$ are vectors of the plant state , input and output , correspondingly; $\xi\in\Xi\subset R^n$ is vector of unknown parameters which is from a priori known set Ξ . The control objective is given as

$$Q_t \le \Delta \quad \text{for} \quad t\ge t_* \qquad (2.2)$$

where $Q_t = Q_t[x(s)\ ,\ u(s)\ ;\ 0\le s\le t] \ge 0$ is the objective functional.

The problem is to find the two level control algorithm

$$u(t) = U_t[y(s),u(s),\theta(s);\ 0\le s\le t]\ , \qquad (2.3)$$

$$\theta(t) = \Theta_t[y(s),u(s),\theta(s);\ 0\le s\le t]\ , \qquad (2.4)$$

ensuring the aim (2.2) in system (2.1),(2.3),(2.4) for any $\xi\in\Xi$. Here $\theta(t)$ is a vector of adjustable parameters, Q_t , U_t , Θ_t are nonaticipative operators. The first level of the algorithm (2.3) is called the main loop control law. Algorithm (2.4) is called adaptation law.

The above general problem encompasses different more specific problems, arising for special choices of plant equations (2.1) and the objective functional in (2.2). For example choice

$$Q_t = \|x-x_*\|_P^2 \quad (\text{ or } Q_t = \|y-y_*\|_P^2\ ) \qquad (2.5)$$

where $P=P^T>0$ is positive definite $n\times n$- (or $l\times l$) matrix, corresponds to state (output) regulation problem. More generally, the choice

$$Q_t = \|x-x_*(t)\|_P^2 \quad (\text{or } Q_t = \|y-y_*(t)\|_P^2 \;) \qquad (2.6)$$

corresponds to the state (output)tracking for signal $x_*(t)$ (or $y_*(t)$).

Note that the vector-function $x_*(t)$(or $y_*(t)$) can either be given explicitly or appear as a solution of some auxillary equations. Those equations are often reffered to as reference equations or reference model because it specifies the behavior of the adaptive system. For example, the reference model can be of the form

$$\dot{x}_*=F_*(x_*,t), \quad y_*=G_*(x_*,t) \qquad (2.7)$$

The reference model can also appear implicitly as a set of parameters, e.g. when the objective functional is given in the form

$$Q_t = \int_0^t \|\dot{x}-F_*(x,t)\|_P^2 \qquad (2.8)$$

3. SPEED GRADIENT ALGORITHMS

A lot of existing NAC algorithms can be analyzed and designed in the following framework. Suppose the main loop (2.3) is already chosen where $U_t[...]$ is memoryless operator and substitute (2.3) into (2.1). The obtained dynamic equations

$$\dot{x}=F(x,\theta,t), \quad t\ge 0 \qquad (3.1)$$

(dependence on ξ is omitted for simplicity) represent some controlled plant with new input vector θ. Now the achieving the control goal

$$Q_t \to 0 \text{ for } t\to\infty \qquad (3.2)$$

for the plant (3.1) can be regarded as an independent control problem.

To solve this problem introduce function $\omega(x,\theta,t)$ as a speed of changing Q_t along trajectories of (3.1). Particularily for memoryless functional $Q_t=Q(x(t),t)$ we have $\omega(x,\theta,t)=(\nabla_x Q)^T F(x,\theta,t)$. Then we can build the speed-gradient algorithm in its most general, so called combined form looking as follows (Andrievsky *et al.*,1988):

$$\frac{d}{dt}[\theta+\psi(x,\theta,t)]=-\Gamma\nabla_u\omega(x,\theta,t) \qquad (3.3)$$

where $\psi(\ )$ satisfies pseudogradient condition $\psi^T\nabla_u\omega\ge 0$ and $\Gamma=\Gamma^T>0$ is $m\times m$ gain matrix. The equation (3.3) can be rewritten in integral form

$$\theta=-\psi(x,\theta,t)-\Gamma\int_0^t \nabla_u Q_s ds$$

The main special cases of (3.3) are SG-algorithm in differential form

$$\frac{d}{dt}\theta = -\Gamma\nabla_u\omega(x,\theta,t) \qquad (3.4)$$

and SG-algorithm in the finite form

$$\theta=-\psi(x,\theta,t) \qquad (3.5)$$

The typical forms of algorithm (3.5) are linear and relay ones:

$$\theta = - \Gamma\nabla_u\omega(x,\theta,t), \qquad (3.5a)$$

$$\theta = - \Gamma\text{sign}\{\nabla_u\omega(x,\theta,t)\} \qquad (3.5b)$$

where components of vector sign{z} are signs of the corresponding components of vector z. The following stability theorems for SG-system (3.1),(3.3) can be proved. (see e.g. (Fradkov, 1990).

Theorem 1 *(combined form).* Assume that the right hand sides of the system (3.1),(3.3) are smooth functions in x,θ which are bounded together with derivatives in any region where the function $Q(x,t)$ is bounded. Assume also that $\omega(x,\theta,t)$ is convex in θ and the following *stabilizability condition* is valid:

there exists $\theta_*\in R^m$ such that

$$\omega(x,\theta_*,t)\le 0. \qquad (3.6)$$

for all $x\in R^n$. Then $Q(x,t)$ is bounded along each trajectory of (3.1),(3.3).

Besides, if the *asymptotic stabilizability* condition is valid

$$\omega(x,\theta_*,t)\le -\rho(Q(x,t)) \qquad (3.7)$$

where $\rho(Q)>0$ for $Q>0$, then the goal (3.2) is achieved for all trajectories of (3.1), (3.4).

The proof of the theorem is based on Lyapunov function

$$V(x,\theta,t)=Q(x,t)+(2\gamma)^{-1}\|\theta-\theta_*\|^2 \qquad (3.8)$$

In the case when it is difficult to find constant "ideal" control θ_*, satisfying (3.6) or (3.7), SG-algorithms in finite form may be applied. Their applicability conditions are as follows.

Theorem 2 *(finite form).* Assume that function $Q(x,t)$ is smooth and the right hand side of the system (3.1) is smooth function in x,θ , bounded together with its derivatives in any region where function $Q(x,t)$ is bounded. Assume that equation (3.5) is solvable for θ for any $x\in R^n$ and a solution of system (3.1),(3.5) (e.g. Filippov solution) exists locally for any initial $x(0)\in R^n$. Assume also that $\omega(x,\theta,t)$ is convex in θ and satisfies *stabilizability condition* (3.6). Then $Q(x,t)$ is bounded along each trajectory of (3.1),(3.5).

Besides, if the *asymptotic stabilizability* condition (3.7) and the following *strong pseudogradient condition*

$$\psi^T\nabla_u\omega(x,\theta)\ge\beta\|\nabla_u\omega(x,\theta)\|^\delta \qquad (3.9)$$

are valid for some $\beta>0$, $\delta>0$, then the goal (3.2) is achieved for all trajectories of (3.1), (3.5).

Remark. It can be shown that the goal (3.2) is still achieved under weakened stabilizability conditions: for some bounded $\theta_*(x,t)$ the inequality (3.6) is valid as well as the following *stabilizability condition in integral form:*

there exist the sequence of time instances $t_k \to \infty$, $k=1,2,\ldots$ and the sequences of nonnegative numbers α_k, ρ_k, such that

$$Q_{k+1} - Q_k \le -\rho_k Q_k + \alpha_k, \quad \sum_{k=1}^{\infty} \rho_k = \infty, \quad \alpha_k/\rho_k \to 0 \qquad (3.10)$$

where $Q_k = Q(x(t_k), t_k)$.

Note also that Lyapunov function proving the theorem 2 is just the objective function $Q(x,t)$. More statements of the SG-algorithm properties can be found in (Fradkov, 1990). Choosing various types of plant equations, input and output vectors various objective functionals and vector-funtions $\psi(x,\theta,t)$, one can obtain different structures of control algorithms in specific problems. For system stability analysis the theorems 1,2 can be used (as well as the other statements, see Fradkov (1990) for studying the properties of SG algorithms in the presence of disturbances). This is the essence of speed gradient method.

In adaptive control problems the SG-method can be used both for the main loop and for adaptation algorithm design. In the latter problem the equation (3.1) represents the generalized controlled plant , obtained by substitution the main loop control law into the plant equation. In this case the inputs of the plant are adjustable parameters (controller parameters , the parameters of adjustable model etc.). Different applications of SG-method to regulation and tracking algorithms design can be found in (Andrievsky *et al,* 1988; Stotsky, Fradkov, 1992). Consider some applications of SG-method to adaptive control of oscillations.

4. MODEL REFERENCE CONTROL AND SYNCHRONIZING OSCILLATIONS

The problem of (two or more) oscillating processes synchronization has a variety of applications in mechanics, biology, electronics, etc. (Lindsey, 1972; Blechman, 1988). It is formulated as follows (Blechman, 1971):

Given equations of N interacting subsystems

$$\dot{x}_i = \overline{F}_i(x_1,\ldots x_N, u, t), \quad x \in R^{n_i}, \quad i=1\ldots r \qquad (4.1)$$

and connection system equation

$$\dot{u} = U(x_1,\ldots x_N, u, t), \quad u \in R^m, \qquad (4.2)$$

find conditions of existence and stability of T-periodic solutions $\overline{x}_i(t)$, $\overline{u}(t)$ of (4.1), (4.2).

The adaptive synchronization problem can be posed as follows. Given equations

$$\dot{x}_i = \overline{F}_i(x_1,\ldots x_N, u, t, \xi), \quad i=1\ldots r \qquad (4.3)$$

find the equation of the control algorithm

$$u = U(x_1,\ldots x_N, \theta, t)) \qquad (4.4)$$

and adaptation algorithm

$$\dot{\theta} = \Theta(x_1,\ldots x_N, \theta, t) \qquad (4.5)$$

ensuring the control goal

$$\|x_i(t) - \overline{x}_i(t)\| \le \Delta \text{ for } t > t_* \qquad (4.6)$$

The problem (4.3)-(4.6) looks very much like the general adaptive control problem of Section 2, if the notation is introduced $x=(x_1,\ldots x_N) \in R^n$, $n=\Sigma n_i$ and the objective $Q_t = \|x(t) - \overline{x}(t)\|_\infty$. The difference is that $\overline{x}(t)^T$, and Q_t in (4.3)-(4.6) are not given a priori. Moreover, the periodicity of $\overline{x}(t)$ also is not necessary. The complete reduction to the problem of Section 2 can be performed for special case: adaptive synchronization with reference model. In this case $\overline{x}(t)$ is a solution of conventional system (4.1), (4.2) playing a role of reference model. However the existing MRAC methods cannot be applied directly because they usually require asymptotic stability of reference model, while oscillating systems correspond to the boundary of stability region and chaotic systems are locally unstable (see Wiggins, 1988).

For special case of linear controlled plant the high-gain adaptive stabilizer was suggested by Helmke et al (1991). Kazakov, Shirokov (1992) studied the identification-based adaptive phase-locked loop design. The synchronization algorithm for two chaotic systems (drive and response) was suggested by Carrol, Pecora (1991). Their idea is to replace the part of the response system state variables by the corresponding state variables of the drive system. The synchronization model is as follows:

$$\dot{v} = f_1(v,w), \quad \dot{w} = f_2(v,w) \quad \text{(drive)} \qquad (4.7)$$

$$\dot{v}' = f_1(v',w'), \quad \dot{w}' = f_2(v,w') \quad \text{(response)} \qquad (4.8)$$

Kosarev et al.(1993) gave the general convergence (synchronization) conditions for scheme with linear synchronizing signal:

$$\dot{\bar{x}} = \bar{f}(\bar{x}) \quad \text{(drive)} \qquad (4.9)$$

$$\dot{x} = f(x) + K(\bar{x} - x) \quad \text{(response)} \qquad (4.10)$$

Kosarev et al. showed that the distance between solutions of the systems (4.9) and (4.10) is small provided $K>0$ is sufficiently large.

In fact both the scheme (4.7),(4.8) and the scheme (4.9),(4.10) are special cases of the system

$$\dot{\bar{x}} = \bar{f}(\bar{x}), \quad \dot{x} = f(x) + g(x)u, \qquad (4.11)$$

where $u=U(x,\bar{x})$ is the control/synchronization signal to be determined (In case of (4.7),(4.8) we may assign $x=(v,w)$, $g(x)=1$, $u=f_2(v,w')-f_2(v',w')$).

Note that the known applications of Pecora-Carrol's method (Rössler system (Pecora-Carrol, 1991), Chua's cirquit (Murali, Lakshmanan, 1992), Lorenz equation (Cuomo *et al.*,1993)) have function $f_2(v,w)$ linear in v. Hence the convergence of the synchronized trajectories in this scheme (as well as in the scheme (4.9),(4.10)) can be established by means of speed-gradient method (Theorem 2 with $Q_t = \|v-v'\|^2$ or $Q_t = \|x-\bar{x}\|^2$). Using other objectives leads to the extended convergence conditions.

Note also that if the right-hand sides of (4.7),(4.8) (or functions $f(.)$, $\bar{f}(.)$ in (4.9),(4.10)) differ in finite number of parameters then the adaptive control scheme of section 2 applies (see also example in section 5).

5. CONTROL OF CHAOTIC OSCILLATIONS.

After the first publications in control of chaos the class of available control goals extended significantly. The following types of the control goals for oscillating systems can be mentioned:

- swinging oscillations (transition from rest position to periodic or almost periodic oscillations);
- transition from periodic to chaotic motions;
- transition from chaotic to periodic motions;
- suppressing oscillations (transition from oscillations to the rest position.

The simple kind of control was suggested by Hübler(1989), Hübler, Luscher(1989). It corresponds in (4.11) to the control law

$$u=K[\bar{f}(\bar{x})-f(\bar{x},0,\xi,t)]=K[\dot{\bar{x}}-f(\bar{x},0,\xi,t)] \qquad (5.1)$$

for K=1. It is just the control counterpart of synchronization law (4.7), (4.8) by Pecora, Carrol (1991). For appropriate value of K the feedback disconnects and the law (5.1) determines the open loop system. It means that the signal (5.1) can be obtained without any measurement, by means of numerical integration, (Hübler, 1993).

Another kind of open loop control is excitation by external periodic force. It is known that such an excitation is able to both stabilize (synchronize) the system and "chaotize" it (Leonov,1986; Neimark, Landa, 1987; Wiggins, 1988). However the feedback laws, e.g. SG-laws, are more robust to the disturbances and hence allow for achievement of more complicated aims.

To demonstrate application of SG-algorithms consider the simple 2nd order system: Duffing equation:

$$\ddot{y}_*+p\dot{y}_*+p_1y_*+y_*^3= \xi \cos \omega t \qquad (5.2)$$

As it is known (see, e.g. Chen,Dong,1993) the dynamic behavior of solutions of (5.2) is different for different ξ (e.g., for p=0.4, p_1=-1.1, ω=1.8, ξ=0.62 - periodic; ξ=1.8 and ξ=2.1 - chaotic, ξ=2.3 - periodic). Introduce the controlled plant

$$\ddot{y} +p\dot{y} +p_1y +y^3= \xi_0\cos \omega t + u \qquad (5.3)$$

where ξ_0 is unknown parameter and the main loop control law

$$u = \theta \cos \omega t + \bar{u} \qquad (5.4)$$

where θ is adjustable parameter, $\bar{u}$ is the bias signal.
The control goal $y(t)-y_*(t)\to 0$ when $t\to\infty$ can be reformulated in form (3.2) where $x=(y,\dot{y})$ and $Q_t(x) = (x-x_*)^TP(x-x_*)$, $P=P^T>0$ - 2x2-matrix.

Calculating the speed-gradient $\nabla_\theta \dot{Q}_t$ gives

$$\nabla_\theta\dot{Q}_t = \begin{bmatrix}0 & \cos \omega t\end{bmatrix} P\ e, \quad e=x-x_* .$$

All the conditions of the theorem 1 are valid except for the stabilizability condition (3.7). To satisfy (3.7), choose $\bar{u}=-K(y-y_*)+y^3-y_*^3$ and take $P=P^T>0$ as a solution of Lyapunov equation $PA+AP^T=-R$, where $R=R^T>0$, $A=\begin{bmatrix}0 & 1\\ -(K+p_1) & -p\end{bmatrix}$

It follows from the theorem 1 that the control goal (3.2) is achieved, if the adaptation law is taken in PI form

$$\theta(t)=-\gamma_0\delta(t)\cos\omega t-\gamma_1\int_0^t \delta(s)(\cos\omega s)ds, \qquad (5.5)$$

where $\delta(t)=e^TPb$, $b=[0,1]^T$.

More details can be found in (Fradkov, Pogromsky, 1994). Note that as in the case of synchronization there are some results on modifying dynamic behavior of the chaotic systems using linear proportional feedback (see (Chen,Dong,1993), (Hartley, Mossayebi 1992), (Murali, Lakshmanan, 1993). These results also can be derived and extended using speed-gradient approach.

Other existing approaches to control of chaos are based upon harmonic balance (Genesio et all,1993) and bifurcation analysis (Wang, Abed, 1993).

6. SPEED-GRADIENT ALGORITHMS FOR HAMILTONIAN SYSTEMS

The convenient mathematical description for oscillating system is Hamiltonian form. It allows for explicit describing surfaces of constant energy where unforced motions stay. The Hamiltonian form of controlled plant equations is as follows:

$$\dot{p} = -\frac{\partial H}{\partial q} + bu , \quad \dot{q} = \frac{\partial H}{\partial p} , \qquad (6.1)$$

where $p,q\in R^n$ are generalized coordinates and impulses; $H=H(p,q)$ is Hamiltonian function (total energy of the system); $u=u(t)\in R^m$ is input (generalized force); b is $n \times m$ input matrix. Introduce the control goal as approaching the given energy surface:

$$S = \{ (p,q) : H(p,q) = H_* \}, \qquad (6.2)$$

The control goal can be formulated as

$$H(p(t),q(t)) \to H_* \quad \text{when } t \to\infty , \qquad (6.3)$$

or in the form (3.2), where x=(p,q), and

$$Q(x) = \tfrac{1}{2} [H(p,q) - H_*]^2 \qquad (6.4)$$

To build SG-algorithm calculate $\dot{Q}$:

$$\dot{Q} = (H-H_*) \frac{\partial H}{\partial p} bu \qquad (6.5)$$

The differential SG-algorithm (3.4) can be represented in the form:

$$u = - \gamma (H-H_*) b^T \dot{q} \qquad (6.6)$$

where γ >0 - gain coefficient.
The finite forms (3.5a), (3.5b) look as follows :

$$u = - \gamma (H-H_*) b^T \dot{q} \qquad (6.7)$$

$$u = - \gamma \operatorname{sign} [(H-H_*) b^T \dot{q}] \qquad (6.8)$$

To analyze the behavior of systems with algorithms (6.6)-(6.8) theorems 1, 2 can be used. It can be easily seen that the differential algorithm (6.6) satisfies conditions of the theorem 1 with stabilizability condition in form (3.6) for the constant $u_* = 0$. It follows from theorem 1 that $H(p,q)$ is bounded along the trajectories of the system (6.1), (6.6) together with $Q(x)$. However the theorem does not ensure achievement of the initial goal (6.3). As a matter of fact the goal (6.3) is not achieved by the system (6.1),(6.6), demonstrating complex behavior (see Fradkov, Guzenko 1994).

The better convergence give algorithms (6.7), (6.8). Taking, e.g. $u_* = -(H-H_*)\, b^T \dot{q}$ we obtain from (6.5) $\dot{Q} = -2\gamma Q(\dot{q}^T bb^T \dot{q})^2$. It means that condition (3.7) is not valid, because Q may vanish in some instants $t_k>0$, $k=1,2,\ldots$. However the stabilizability condition is valid in the averaged (integral) sense (3.10) and the goal (6.3) is achieved according to the remark to theorem 2 , if rank b=m.

The important feature of control algorithm is its ability to achieve arbitrarily large level of objective function with arbitrarily small level of control action. It looks like the ability of the seesaw to be swung up by small pushes and can be called *swinging* property. The remark to the theorem 2 implies that any Hamiltonian system admits swinging control. The simulation results for pendulum (Fradkov, Guzenko, 1994) confirm swinging properties of the algorithm (6.7).

The above approach applies for the case of more complicated desired system behavior. For systems with several degrees of freedom composed from several subsystems the objective function can be taken in the form :

$$Q = \alpha_1(H_1 - H_1^*)^2 + \ldots\ldots + \alpha_p(H_p - H_p^*)^2$$

where H_i is an energy of i-th subsystem; $\alpha_i \geq 0$ is weighting coefficient.

7. CONCLUSIONS

The speed-gradient scheme of nonlinear adaptive control can provide systems with various properties, including synchronized and chaotic behavior. To extend the applicability of the approach some more sophisticated objectives may be used, e.g. energy, enthropy, Hausdorff dimension, etc. The further extension may consist in weakening matching conditions utilising, e.g. the results of Kanellokopoulos *et al.*(1992) or Druzhinina, Fradkov (1994).

An important property of "good" control of oscillations is one of "swinging", that means achievability of arbirtary value of objective function by means of arbitrary small (law gain) control. It was shown that SG-algorithms provide swinging in Hamiltonian systems.

Notice, finally, that for discrete-time system the speed-gradient should be replaced by the usual gradient. The general convergence results for algorithms of Ott *et al.* (1990), Nitsche, Dressler, (1992), Chen, Dong (1992) can be obtained on this waym that will be demonstrated elsewhere.

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