REDUCED ORDER SHUNT NONLINEAR ADAPTIVE CONTROLLER

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Abstract

The method of parallel feedforward compensator ("shunt") developed earlier for adaptive control of linear plants is extended to a class of nonlinear minimum phase plants. The new design of shunt output feedback adaptive controllers achieves global stability of the system solving the regulation and tracking problems. The total order of the auxillary filters is reduced by half compared to existing results. The performance of the proposed adaptive controller is illustrated by example of controlling chaotic trajectories of Duffing equation to the periodic orbits.

1 Introduction

The problem of adaptive control using plant output measurements attached attention of researchers for more than two decades (see [2, 18, 19]). Among different existing approaches it is necessary to mention "augmented error signal" [17, 5], high-gain or variable structure observers ([21, 15]). Recently a new approach based on parallel feedforward compensators (shunts) drew much attention because it provided relatively simple adaptive control laws for high order plants. Shunt is an auxillary linear system which looks like a by-pass augmenting the plant and is implemented as a feedback in the controller. A special session was devoted to this approach at 13th IFAC Congress (see Preprints of 13th IFAC World Congress, Volume K, session 3b-11).

The method of shunt was extended to nonlinear plants in [3, 14], where the inverse of some stabilizing feedback was suggested to use as a shunt. However to apply the method of [3] it is necessary to find some stabilizing controller which itself is not an easy task. An alternative version of shunt for nonlinear plants was proposed in [8]. It was shown that it provided asymptotic output feedback stabilization and tracking for arbitrary minimum phase plants in normal form with known gain under some mild technical restrictions. However the adaptive controller in [8] includes a number of linear filters of total order $l \cdot (2r-2)$, where r is relative degree of the plant, l is number of unknown parameters.

The purpose of the present paper is to propose the shunt adaptive contoller of reduced order. It is shown in section 2 that the wide class of nonlinear minimum-phase plants having relative degree r can be passified and therefore stabilized by shunt of order $r-1$. The total order of filters is reduced by half compared to [8]. This result gives possibility of simple adaptive controllers design for regulation and tracking problems considered in the following sections.

2 Stabilization of SISO minimum-phase plants

Consider nonlinear affine in control plant model

$$\dot{x} = f(x) + g(x)u, \; y = h(x) \; , \tag{2.1}$$

where $x \in \mathcal{R}^n$ is state vector, $u \in \mathcal{R}^1$ is input, $y \in \mathcal{R}^1$ is output, f, g, h are smooth functions such that $f(0) = 0$, $h(0) = 0$, i.e. origin $x = 0$ is equilibrium of free system $\dot{x} = f(x)$.

The problem is to stabilize the system (2.1) by means of dynamic output feedback

$$u = U(y, x_u), \; \dot{x}_u = \chi(y, x_u) \; , \tag{2.2}$$

ensuring boundedness of all the trajectories of (2.1), (2.2) and achievement of the goal

$$x(t) \to 0, \quad x_u(t) \to 0, \text{ when } t \to \infty. \tag{2.3}$$

where $x_u \in \mathcal{R}^{n_u}$ is controller state vector.

Recall that the plant (2.1) is said to have relative degree r at the open set $\mathcal{D} \subset \mathcal{R}^n$, if for all $x \in \mathcal{D}$ the following conditions are satisfied

$$\begin{aligned} &L_g L_f^k h(x) = 0, \quad k = 0, 1, \ldots, r-2 \; , \\ &L_g L_f^{r-1} h(x) \neq 0, \end{aligned} \tag{2.4}$$

where $L_\varphi \psi(x) = \sum_{i=1}^n \frac{\partial \psi}{\partial x_i} \varphi_i(x)$ stands for the derivative of function $\psi(x)$ along vector field $\varphi(x)$ (Lie derivative, see [13]). If system (2.1) has relative degree r in the open set $\mathcal{D}$, then there exists smooth nonsingular coordinate change $z = \Phi(x)$, $x \in \mathcal{D}$, such that system (2.1) model has in new coordinates so called Isidori's normal form

$$\begin{aligned} \dot{z}_i &= z_{i+1} \ , \quad i = 1, \ldots, r-1 \ , \\ \dot{z}_r &= a(z) + b(z)u \ , \\ \dot{\overline{z}} &= q(z) \ , \quad y = z_1 \ , \end{aligned} \tag{2.5}$$

where $a(z) = L_f^r h(\Phi^{-1}(z))$, $b(z) = L_g L_f^{r-1} h(\Phi^{-1}(z))$, $\overline{z} = (z_{r+1}, \ldots, z_n) \in \mathcal{R}^{n-r}$. The subsystem

$$\dot{\overline{z}} = q_0(\overline{z}) \ , \tag{2.6}$$

where $q_0(\overline{z}) = q(0, \ldots, 0, \overline{z})$ describes the motions of (2.1) consistent with $y(t) \equiv 0$, it is called *zero dynamics* of (2.1). System (2.1) is called *weakly minimum phase* (resp.*minimum phase, exponentially minimum phase*), if zero dynamics (2.6) are Lyapunov stable (resp. asymptotically stable, exponentially stable).

Let us consider functions $a(z)$ which include not only directly measured quantities but also any linear functions of $y^{(i)}, i = 1 \ldots r-1$, i.e.

$$a(z) = a_0(y) + A(p)y, \quad a(0) = 0 \tag{2.7}$$

where A(p) is polynomial of degree $r-1$, $p = d/dt$ is operator of time differentiation.

Introduce *shunt system* (parallel feedforward compensator) as follows

$$(p+1)^{r-1}\eta = \kappa\epsilon(p\epsilon+1)^{r-2}(b(z)u + a_0(y)) \ , \tag{2.8}$$

where η is auxillary variable, $\kappa > 0$, $\epsilon > 0$ and consider augmented plant model described by equations (2.1), (2.8) and output equation

$$y_a = y + \eta \ , \tag{2.9}$$

It is to be noted that after change of control variable $\overline{u} = b(z)u$ equations of augmented plant (2.5),(2.8) may be rewritten in form

$$\begin{aligned} & p^r y = a_0(y) + A(p)y + \overline{u} \ , \\ & \dot{\overline{z}} = q(z) \ , \\ & (p+1)^{r-1}\eta = \kappa\epsilon(\epsilon p+1)^{r-2}(\overline{u} + a_0(y)) \ . \end{aligned} \tag{2.10}$$

The first equation of (2.10) after adding to both sides the term

$$H(p)y = (p+1)^r y - p^r y, \quad deg H(p) = r-1$$

reads as follows

$$(p+1)^r y = a_0(y) + \tilde{A}(p)y + \overline{u} \ , \tag{2.11}$$

where $\tilde{A}(p) = A(p) + H(p)$, $deg\tilde{A}(p) = r-1$.

Therefore the augmented output y_a satisfies equation

$$(p+1)^r y_a = \tilde{A}(p)y + G(p)(a_0(y) + \overline{u}) \ , \tag{2.12}$$

where $G(p) = 1 + \kappa\epsilon(\epsilon p+1)^{r-2}(p+1)$ is polynomial of degree $r-1$ with leading coefficient $g_0 = \kappa\epsilon^{r-1}$. Since $(p+1)(\epsilon p+1)^{r-2}$ is Hurwitz polynomial, it can be shown [10], that there exist number $\kappa_0 > 0$ and function $\epsilon_0(\kappa) > 0$ such that $G(p)$ is Hurwitz polynomial for $\kappa > \kappa_0$, $0 < \epsilon < \epsilon_0(\kappa)$. Pick up such κ and ϵ and introduce function $\overline{a}(t)$ satisfying differential equation

$$G(p)\overline{a}(t) = \tilde{A}(p)y, \quad \overline{a}(0) = 0 \tag{2.13}$$

Now we are in position to formulate the main result of this section.

Theorem 1 *Let the system (2.1) have relative degree r in any bounded set and its normal form (2.5) satisfies the following assumptions:*

A1. Function $a(z)$ has the form (2.7) and is locally Lipschitz.

A2. Function $b(z)$ is available for measurement, i.e. $b(z) \equiv b(y)$, and $b(y) \neq 0$ for all $y \in \mathcal{R}^1$.

A3. Function $q(z)$ is locally Lipschitz and plant (2.1) is exponentially minimum phase.

A4. Function $q(z)$ can be represented in the form

$$q(z) = q(\tilde{z}, \overline{z}) = q_0(\overline{z}) + q_1(\tilde{z}, \overline{z})\tilde{z} \ ,$$

where $||q_1(\tilde{z}, \overline{z})|| \leq C_\alpha(1 + ||\overline{z}||)$ for $||\tilde{z}|| \leq \alpha$, $C_\alpha > 0$.

Then there exist numbers $\kappa > 0$, $\epsilon_0 > 0$ such that for any bounded set $\mathcal{D}_0$ of initial conditions there exists positive nonincreasing function $K_0(\epsilon)$ such that for any $\epsilon : 0 < \epsilon < \epsilon_0$ system (2.1), (2.8), (2.9) closed by feedback

$$u(t) = -\frac{1}{b(y)}\left[K y_a + a_0(y) + \overline{a}(t)\right] \tag{2.14}$$

where $\overline{a}(t)$ is the output of the filter (2.13), is asymptotically stable for $K > K_0(\epsilon)$, its trajectories are bounded and the goal (2.3) is achieved.

It follows from Theorem 1 that the plant (2.1) with relative degree r satisfying A1–A4 can be asymptotically stabilized by dynamic output feedback (2.8),(2.9),(2.13),(2.14).

Proof: Suppose with no loss of generality that system (2.1) already has normal form (2.5). Fix open bounded set $\mathcal{D}_0 \subset \mathcal{R}^{n+r-1}$ of initial conditions of system (2.5), (2.8), (2.9), (2.14).

It is clear that y_a is output of linear system

$$(p+1)^r y_a = G(p)(a_0(y) + \overline{a} + \overline{u}) \tag{2.15}$$

with new input $a_0(y) + \overline{a} + \overline{u}$. This system is minimum-phase and has relative degree 1. Therefore it can be represented in special coordinate basis with state vector $\overline{w} = (y_a, \xi)$, $\xi \in \mathcal{R}^{r-1}$ in normal form, similar to (2.5):

$$\begin{aligned} \dot{y}_a &= d_1 y_a + \overline{d}\xi + g_0(a_0 + \overline{a} + \overline{u}) \ , \\ \dot{\xi} &= G\xi + \overline{g}y_a \ , \end{aligned} \tag{2.16}$$

where G is $(r-1) \times (r-1)$ matrix, $\overline{d}$,$\overline{g} \in \mathcal{R}^{r-1}$ and $\det(\lambda I - G) = G(\lambda)/g_0$ [20, 21]. Finally, represent the second equation of plant (2.10) in form

$$\dot{\overline{z}} = q_0(\overline{z}) + q_1(\tilde{z}, \overline{z})\tilde{z} \ , \tag{2.17}$$

where $\tilde{z} = (z_1, \dots, z_r) = S\overline{w}$, S is $r \times r$ constant matrix; $q_1(\tilde{z}, \overline{z})$ is smooth function, continuous in $\tilde{z} = 0$, $\overline{w} = (y_a, \xi)$. Introduce also state vector of the whole system $w = (\overline{w}, \overline{z})$.

Pick up initial conditions $w_0 = (y_a(0), \xi(0), \overline{z}(0))$ in system (2.16), (2.17) from the compact set $\mathcal{D}$ corresponding to above defined set $\mathcal{D}_0$. To prove asymptotic stability of system (2.14), (2.16), (2.17) use Lyapunov function of form

$$V_1(w) = \mu \ln(1 + V_0(\overline{z})) + \xi^T P \xi + y_a^2 , \qquad (2.18)$$

where $\mu > 0$, $V_0(\overline{z})$ is Lyapunov function establishing exponential stability of zero dynamics (2.6); $P = P^T > 0$ is $(r-1) \times (r-1)$ positive definite matrix satisfying $PG + G^T P = -2I_{r-1}$, I_{r-1} is identity matrix. Function $V_0(\overline{z})$ satisfies quadratic type inequalities [16, 11]

$$\rho_1 ||\overline{z}||^2 \le V_0(\overline{z}) \le \rho_2 ||\overline{z}||^2 , \quad ||\nabla V_0(\overline{z})|| \le \rho_3 ||\overline{z}|| , \qquad \nabla V_0(\overline{z})^T q_0(\overline{z}) \le -\rho_0 ||\overline{z}||^2 \qquad (2.19)$$

with some positive ρ_0 , ..., ρ_3.

Apparently function (2.18) is positive definite and proper, i.e. set $\overline{\mathcal{D}} = \{w : V_1(w) \le V_0\}$ is compact for all $V_0 \ge 0$. Choose V_0 such that $\overline{\mathcal{D}} \supset \mathcal{D}$ and calculate derivative of function (2.18) along trajectories of (2.16),(2.17) using assumption A4 written in form $||q_1(\tilde{z}, \overline{z})||^2 \le C_0(1 + V_0(\overline{z}))$ for some $C_0 > 0$ (see [8]). Then $\dot{V}_1(w) \le A + B + C$, where

$$\begin{aligned} A &= \mu \frac{-\rho_0 ||\overline{z}||^2}{1 + V_0(\overline{z})} + \mu \frac{\rho_3 C_0 ||S|| \cdot ||\overline{z}|| \cdot ||\overline{w}||}{\sqrt{1 + V_0(\overline{z})}} - \\ &\quad -||\xi||^2 - |y_a|^2 , \\ B &= -||\xi||^2 + 2||\xi|| \cdot |y_a|(||P\overline{g}|| + ||\overline{d}||) - \\ &\quad -(||P\overline{g}|| + ||\overline{d}||)^2 |y_a|^2 , \\ C &= y_a^2 \big[2d_1 + 1 + (||P\overline{g}|| + ||\overline{d}||)^2 - 2g_0 K \big]. \end{aligned}$$

The quantity A is a quadratic form of variables $||\overline{z}||/\sqrt{(1 + V_0(\overline{z}))}$ and $||w||$. Therefore it is negative definite if $4\mu\rho_0 > (\mu\rho_3 C_0 ||S||)^2$, i.e.

$$\mu < 4\rho_0 / \left(\rho_3 C_0 ||S||\right)^2 . \qquad (2.20)$$

The quantity B is already nonpositive, and C becomes nonpositive for $K > K_0$, where $K_0 = \big[2d_1 + 1 + (||P\overline{g}|| + ||\overline{d}||)^2\big]/2g_0$. Obviously, K_0 depend on ϵ and function $K_0(\epsilon)$ can be made nonincreasing.

We have proved that $\dot{V}_1(w) \le 0$ for $w \in \overline{\mathcal{D}}$ and, therefore all the trajectories of system are bounded. Decreasing μ, if necessary, one may ensure that $\dot{V}_1(w) \le -\delta ||w||^2$ for some $\delta > 0$. Therefore function $||w(t)||^2$ is integrable and in view of boundedness of trajectories, using Barbalat lemma, we have $y_a(t) \to 0$, $\xi(t) \to 0$, $\overline{z}(t) \to 0$, when $t \to \infty$. This in turn yields $y(t) \to 0$, $y^{(k)}(t) \to 0$, $k = 0, 1, \dots, r-1$, that proves theorem. ∎

Remark 1. Assumption A4 overbounds quadratically the rate of growth in $\overline{z}$ of right hand side of the last equation (2.5). However the knowledge of this bound is not required for control algorithm. Note also that stabilization is global in case when functions $q_1(z)$ and $a(z)$ are bounded since in this case parameter μ of Lyapunov function (2.18) can be chosen from (2.20) independently of initial conditions.

Remark 2. The proposed controller is not adaptive and contains design parameters: gain K and parameters of shunt κ, ϵ. Note that shunt parameters do not depend on parameters of the plant. Moreover for $r = 2$, 3 one may take $\kappa = 1$ as it seen from Hurwitz criterion for $G(p)$.

Remark 3. Extension of Theorem 1 to MIMO plants is straightforward when the plant (2.1) has uniform relative degree $(r, r, \dots, r)$.

Remark 4. It follows from the proof of Theorem 1 that the control law

$$u = -\big[b(y)\big]^{-1} \left[K y_a + \overline{a}(t) + a_0(y) + v(t)\right] \qquad (2.21)$$

with new input $v(t)$ ensures semiglobal output strict passivity [9] of the system (2.5),(2.8),(2.13),(2.21) with storage function (2.18).

3 Adaptive stabilization of minimum-phase plants

Although the proposed controller does not need much apriori information about controlled plant, the required information may be further reduced by means of tuning gain K. Adaptation algorithm can be derived by speed-gradient method [7, 6], taking Lyapunov function (2.18) of nonadaptive system as the goal function:

$$\dot{K} = \gamma_0 y_a^2 , \qquad (3.1)$$

where $\gamma_0 > 0$. Standard arguments based on Lyapunov function

$$V_2(w, K) = V_1(w) + g_0 \gamma_0^{-1} \left(K - K_0\right)^2$$

show that all the trajectories of system (2.1), (2.8), (2.13), (2.14), (3.1) are bounded and the control objective (2.3) is achieved.

If additional structural information about plant nonlinearity $a(z)$ is available then it may be taken into account when choosing the adequate structure of controller. Suppose for example that $a(z)$ has the following form

$$a(z) = a_0(y) + A(p)y + \sum_{i=1}^{l} \theta_i a_i(y) , \qquad (3.2)$$

where $a_i(y)$ are known (measured) functions, θ_i are unknown constant coefficients. Then the structure of adaptive controller can be taken as follows:

$$u = -\frac{1}{b(y)} \left[K y_a + a_0(y) + \overline{a}(t) + \sum_{i=1}^{l} \hat{\theta}_i \overline{a}_i(t) \right] , \qquad (3.3)$$

where $\hat{\theta}_i$ are estimates of unknown parameters θ_i and functions $\overline{a}(t), \overline{a}_i(t)$ are the outputs of $l+1$ identical filters:

$$\begin{aligned} &G(p)\overline{a}(t) = \tilde{A}(p)y, \\ &G(p)\overline{a}_i(t) = a\big(y(t)\big) \quad i = 1, \dots, l . \end{aligned} \qquad (3.4)$$

After applying speed-gradient method with objective function (2.18) the following adaptation algorithms are obtained

$$\begin{aligned} \dot{K} &= \gamma_0 y_a^2 \ , \ \gamma_0 > 0 \ , \\ \dot{\hat{\theta}}_i &= \gamma_i y_a \overline{a}_i(t) \ , \ \gamma_i > 0, \ i = 1, \dots, l \ . \end{aligned} \tag{3.5}$$

Theorem 2 . *Let the conditions of Theorem 1 be valid and plant model (2.5) have structure (3.2). Then there exist values of parameters κ , ϵ such that algorithm (2.8), (2.9), (3.3)–(3.5) ensures boundedness of the trajectories and achievement of the goal (2.3).*

Proof of the theorem is standard. It is based on Lyapunov function

$$V_3(w, K, \hat{\theta}) = V_2(w, K) + g_0 \sum_{i=1}^{l} \gamma_i^{-1} (\hat{\theta}_i - \theta_i)^2 \tag{3.6}$$

Remark 1. Functions $a_i(\cdot)$ in (3.2) may depend also on time and include not only directly measured quantities but also their derivatives up to the order of relative degree of filters (3.4), i.e. up to $r-1$. Therefore any linear function of $y^{(i)}$, $i = 1, \dots, r-1$, multiplied by unknown parameters may be added to (3.2).

4 Time-varying plants and tracking

All the above designs apply to the time-variant plant if the plant model can be transformed to the normal form, similar to (2.5):

$$\begin{aligned} &\dot{z}_i = z_{i+1} \ , \quad i = 1, \dots, r-1 \ , \\ &\dot{z}_r = a(z,t) + b(z,t)u \ , \\ &\dot{\overline{z}} = q(z,t) \ . \end{aligned} \tag{4.1}$$

Assumptions A1–A4 should be replaced by the following ones:

A1. Function $a(z,t)$ is smooth and locally bounded together with first partial derivatives uniformly in $t \geq 0$; and $a(0, \dots, 0, \overline{z}, t) \equiv 0$.

A2. $b(z,t) \equiv b(y,t)$ and there exists $\delta > 0$ such that $|b(y,t)| \geq \delta$ for all y, t .

A3. Function $q(z,t)$ is smooth, $q(0,0) \equiv 0$ and system $\dot{\overline{z}} = q(0, \dots, 0, \overline{z}, t)$ is exponentially stable, i.e. there exists Lyapunov function $V_0(\overline{z}, t)$, satisfying standard quadratic-type inequalities [16, 11].

A4. Function $q(z,t)$ can be represented in the form

$$q(\tilde{z}, \overline{z}, t) = q_0(\overline{z}, t) + q_1(\tilde{z}, \overline{z}, t)\tilde{z} \ ,$$

where $||q_1(\tilde{z}, \overline{z}, t)|| \leq C_\alpha(1 + ||\overline{z}||)$ for $||\tilde{z}|| \leq \alpha$, $t \geq 0$.

Theorems 1,2 hold true for plant (4.1) under assumptions *A1–A4*. It allows to solve problem of tracking where the goal (2.3) is replaced by the goal

$$e(t) \to 0 \quad \text{as} \quad t \to \infty \ , \tag{4.2}$$

where $e(t) = y(t) - y_d(t)$, $y_d(t)$ is desired trajectory of the plant output. To reduce this problem to the previous one just take $z_1 = e(t)$ in (4.1). Then the highest derivative of command signal $y_d^r(t)$ will appear in the second equation of (4.1), while augmented error will be $e_a(t) = y(t) - y_d(t) + \eta(t)$.

All proposed algorithms apply with obvious changes and Theorems 1,2 hold true (i.e. the goal (4.2) is achieved) if the command signal $y_d(t)$ is bounded for $t \geq 0$ together with its derivatives $y_d^{(k)}(t)$, $k = 1, \dots, r$.

5 Example: Adaptive control of Duffing equation

To illustrate the proposed design consider the controlled forced Duffing equation

$$\begin{aligned} &\ddot{y}(t) + a_1\dot{y}(t) + a_2 y(t) - a_3 y^3(t) = \\ &= a_4 \cos \ \omega t + bu(t) \end{aligned} \tag{5.1}$$

that has become a traditional example of oscillating system with complex dynamics. In (5.1) $u(t) \in \mathcal{R}^1$ is control action, $y(t) \in \mathcal{R}^1$ is plant output, a_1, a_2, a_3, a_4 are unknown parameters.

Denoting $x_1(t) = y(t)$, $x_2(t) = \dot{y}(t)$, rewrite (5.1) in normal form (2.5):

$$\begin{aligned} &\dot{x}_1(t) = x_2(t), \\ &\dot{x}_2(t) = a(x_1, x_2, t) + bu(t). \end{aligned}$$

where

$$a(x_1, x_2, t) = -a_1 x_2(t) - a_2 x_1(t) + a_3 x_1^3(t) + a_4 \cos \ \omega t$$

Obviously in this case $r = 2$ and there is no zero-dynamics. Introduce the shunt filter

$$(p+1)\eta(t) = \epsilon b \cdot u(t) \ . \tag{5.2}$$

Assuming $b = 1$ the augmented output $y_a(t) = y(t) + \eta(t)$ satisfies the following equation

$$(p+1)y_a(t) = a(y, \dot{y}, t) + G(p)u(t) + H(p)y \ ,$$

where $H(p) = 2p+1$ is polynomial of degree 1 and $G(p) = 1 + \epsilon(p+1)$ is Hurwitz polynomial for all $\epsilon > 0$.

To obtain the adaptive control law recall that $a(y, \dot{y}, t)$ has the form (3.2)

$$a(y, \dot{y}, t) = \theta_1 \dot{y}(t) + \sum_{i=2}^{4} \theta_i a_i(y, t).$$

where θ_i are unknown parameters.

Choose control action as

$$u(t) = -K \cdot y_a(t) - \overline{a}(t) - \sum_{i=1}^{4} \widehat{\theta}_i \overline{a}_i(t) \ , \tag{5.3}$$

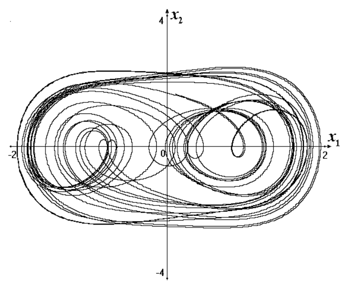

Figure 1: Plant phase plot

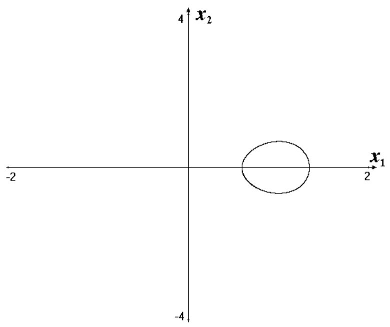

Figure 2: Reference model phase plot

and adaptation algorithm as

$$\begin{aligned} \dot{K}(t) &= \gamma_0 y_a^2(t) \ , \\ \dot{\widehat{\theta}}_i(t) &= \gamma y_a(t) \overline{a}_i(t), \ i = 1,2,3,4, \end{aligned} \tag{5.4}$$

where $\overline{a}(t)$, $\overline{a}_i(t)$ are outputs of the following filters

$$\begin{aligned} &(\epsilon p + \epsilon + 1)\overline{a}(t) = (2p+1)y, \\ &(\epsilon p + \epsilon + 1)\overline{a}_1(t) = -py(t), \\ &(\epsilon p + \epsilon + 1)\overline{a}_2(t) = -y(t), \\ &(\epsilon p + \epsilon + 1)\overline{a}_3(t) = y^3(t), \\ &(\epsilon p + \epsilon + 1)\overline{a}_4(t) = \cos\ \omega t. \end{aligned} \tag{5.5}$$

The convergence of the error y_a to zero follows from the Theorem 2. To solve the tracking problem output $y(t)$ should be replaced by the error $e(t) = y(t) - y_d(t)$ and $y_a(t)$ should be replaced by $e_a = e + \eta$. Also additional filter for the derivative of reference signal should be introduced as follows

$$(\epsilon p + \epsilon + 1)\overline{y}_d(t) = (p+1)^2 y_d(t), \tag{5.6}$$

and its output $\overline{y}_d(t)$ should be subtracted from the right-hand side of control law (5.3).

Simulation results are shown at Figures 1–5. Uncontrolled plant has parameters $a_1 = 0.4$, $a_2 = -1.1$, $a_3 = 1.0$, $a_4 = 1.8$, $b = 1$, $\omega = 1.8$, corresponding to chaotic behavior (Fig.1). Command signal is generated by the reference model also described by equation (5.1). It has stable limit cycle (Fig.2) for parameter values $a_{1m} = 0.6$, $a_{2m} = -1.3$, $a_{3m} = 1.1$, $a_{4m} = 0.6$, $b_m = 1$, $\omega = 1.8$. Parameters of filters are: $\epsilon = 0.02$ and adaptation gains are $\gamma_0 = \gamma = 5.0$. Transients of plant and model are shown at

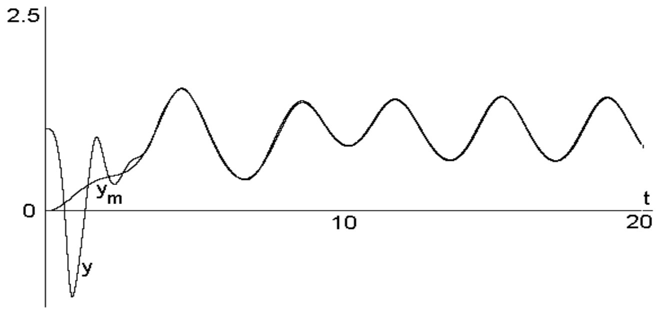

Figure 3: Outputs of the plant and reference model vs time

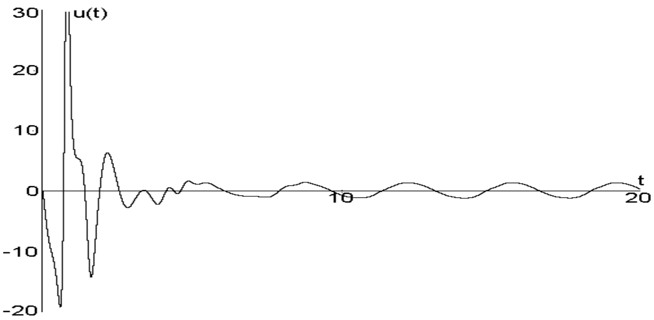

Figure 4: Control action

Fig.3; control action is drawn at Fig.4, typical time history of adjustable parameters can be seen at Fig.5. One may see from the pictures that tracking error approaches zero after a few seconds, while the limit values of adjustable parameters may differ from the true plant parameter values. Such kind of behavior is typical for adaptive systems with implicit reference models [1].

6 Conclusion

The proposed controllers can be used in various application problems (control of robots, oscillatory and chaotic systems, *etc.*). The results of the paper extend passification approach [9] to nonlinear systems with arbitrary relative degree. Compared to results of [8] our new algorithm not only reduces filters order but also avoids discontinuities of control law. Note that the algorithm (3.1) is just special case of the so called Byrnes-Willems controller [4] or universal stabilizers [12] which was studied still earlier

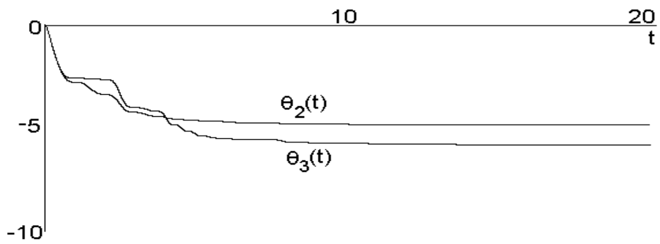

Figure 5: Adjustable parameters vs time

(see [6]).

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