SPEED-GRADIENT CONTROL FOR PASSAGE OF UNBALANCED ROTOR THROUGH RESONANCE IN PLANE MOTION

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Abstract: New algorithm for passing through resonance zone of an unbalanced rotor in plane motion is proposed and analyzed by computer simulation. The algorithm is based on speed-gradient method and allows to significantly reduce the required level of the controlling torque. The algorithm has only one design parameter. Compared with the known Malinin-Pervozvansky algorithm it is more simple for design and exhibits stronger robustness properties.

Keywords: oscillations, nonlinear control, speed-gradient method.

1. INTRODUCTION

Vibrational units with unbalanced (eccentric) rotors are widely used in the industry. It is well-known that the maximum power of driving motor is required during the spin-up mode (Blekhman, 2000). The decrease of the spin-up power leads to decrease of nominal power and, therefore to decrease of the weight and the size of the motor. Another problem is that in order to obtain the desired mode of vibration it is necessary to control the rotor speed in a broad range including both pre-resonance and post-resonance regions. It means that the problem of passage through resonance arises naturally. It is important for development of new generation of vibrational equipment with improved technological characteristics.

The key idea to reduce the power of the unbalanced rotor is to swing the rotor during the spin-up period by feedback control. The control algorithms implementing this idea were proposed in (Kinsey *et. al.*, 1992; Kel'zon and Malinin, 1992; Malinin and Pervozvanskii 1993; Tomchina and Nechaev, 1999). In (Kel'zon and Malinin, 1992) and (Malinin and Pervozvanskii, 1993) the optimal control method was used leading to complicated and not sufficiently robust controller. Kinsey *et. al.* (1992) proposed the algorithm based on derivation of the averaged controlled plant equation which is labor-consuming. The algorithm of (Tomchina and Nechaev, 1999) is based on the speed-gradient method (Fradkov, 1990; Fradkov *et. al.*, 1999) and energy-based goal functions. As it was shown in (Fradkov, 1996) the speed-gradient algorithms for energy control of conservative systems allow to achieve an arbitrary energy level by means of arbitrarily small level of control power (so called swingability property). Using this approach for systems with losses allows to spend energy only to compensate the losses, and to reduce the power of driving motor significantly. However, reduction of the motor power for systems with several degrees of freedom may increase the influence of resonance and lead to appearance of Sommerfeld phenomenon and capture (Blekhman, 1971; Blekhman, 2000). Sommerfeld phenomenon is caused by a limited power of motors. It may prevent the system from passing through resonance region and achieving the desired post-resonance value of rotor speed.

Therefore it is important to develop control algorithms allowing to decrease the power of motor at the stage of passing through resonance. An additional requirement of achieving fast passage through the resonance zone by electrical correction means is also important (Tomchina and Nechaev, 1999). In the paper by (Tomchina and Nechaev, 1999) only the case of one-dimensional motion of the rotor axis was considered. The case of plane motion was studied by Malinin and Pervozvanskii (1993), who designed the controller using optimal

Supported by Russian Foundation for Basic Research (grant RFBR 02-01-00765) and Complex Program of the Presidium of RAS #19 "Control of mechanical systems", project 1.4.

control technique, see also (Kel'zon and Malinin, 1992).

In this paper the problem of controlling flexural-and-torsional oscillations of a rotating shaft with an unbalanced rotor in the middle (Blekhman, 1971) is solved by means of speed-gradient method. The proposed algorithm allows to significantly reduce the required level of the controlling torque. The efficiency and robustness of the algorithm are investigated by means of computer simulation for different values of plant and algorithm parameters.

2. PROBLEM STATEMENT

Consider the following system of differential equations describing the flexural-and-torsional oscillations of a rotating shaft with an unbalanced rotor in the middle (Blekhman, 1971):

$$J\ddot{\varphi} = m\varepsilon(\ddot{x}\cos\varphi + \ddot{y}\sin\varphi) + u(t) - k_\varphi\dot{\varphi}$$
$$m\ddot{x} + cx = m\varepsilon(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi) - k_x\dot{x} \qquad (1)$$
$$m\ddot{y} + cy = m\varepsilon(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi) - k_y\dot{y},$$

where φ – rotor angle, x,y – coordinates of the rotor center of mass, $u(t)$ – control action (rotating torque of a motor), J - moment of inertia of an unbalanced rotor (disk), m - mass of a rotor (disk), ε - eccentricity of the rotor center of mass, c - shaft torsional stiffness, k_φ, k_x, k_y - damping factors.

It is well-known (Blekhman, 1971; Kononenko, 1964), that the "capture" of angular velocity of a rotor (Sommerfeld phenomenon) sometimes takes place in the near-resonance zone. The capture phenomenon happens when the level of constant control action $u(t) \equiv M_0$ is small. If the level of constant control action $u(t) \equiv M_0$ is higher, the system passes the resonance zone. Simulation results for system (1) are shown in Fig. 1.1, 1.2. for the parameters: J = 0.014 [kg·m^2], m = 1.5 [kg], ε = 0.04 [m], k_φ = 0.02 [J·sec], $c = 130$ [N/m], $k_x = k_y = 1$ [kg/sec] and the constant control action M_0 = 0.6 [N·m] (Fig. 1.1) and M_0 = 0.7 [N·m] in Fig. 1.2.

The problem is to design the control algorithm $u = \boldsymbol{U}(z)$, providing the spin-up of unbalanced rotor until the system passes through resonance zone, where $z = [x, \dot{x}, y, \dot{y}, \varphi, \dot{\varphi}]^{\mathrm{T}}$ - state vector of the control plant. It is assumed that the level of control signal is restricted and does not allow the passage through resonance when the control signal is constant. Passage through resonance is understood as significant decrease of the rotor center of mass oscillations. Detecting of the exit from resonance zone is a separate problem which will be considered in the next section as a part of control algorithm design.

3. SYNTHESIS OF CONTROL ALGORITHM

To describe the proposed control algorithm first describe the way to define the time of passing through resonance zone. It is easy to see that the capture of rotor speed is equivalent to the increase of average sum of coordinate squares $x^2 + y^2$. Also, the sum of coordinate squares decrease when system is passing through the resonance zone. This fact is confirmed by simulation: the sum $x^2 + y^2$ increases when the level of constant control action is small and does not allow system to pass through the resonance zone (see Fig. 2.1 for $u(t) \equiv M_0$, M_0 = 0.6 [N·m]). In case of higher control torque, the average sum of coordinate squares $x^2 + y^2$ increase in the pre-resonance zone and decrease in the post-resonance zone (see Fig. 2.2, M_0 = 0.7 [N·m]).

In order to smooth the variable $x^2 + y^2$ we introduce the additional low pass filter:

$$T_\theta\dot{\theta}(x,y,t) = -\theta + x^2 + y^2, \quad \theta(0) = \dot{\theta}(0) = 0, \qquad (2)$$

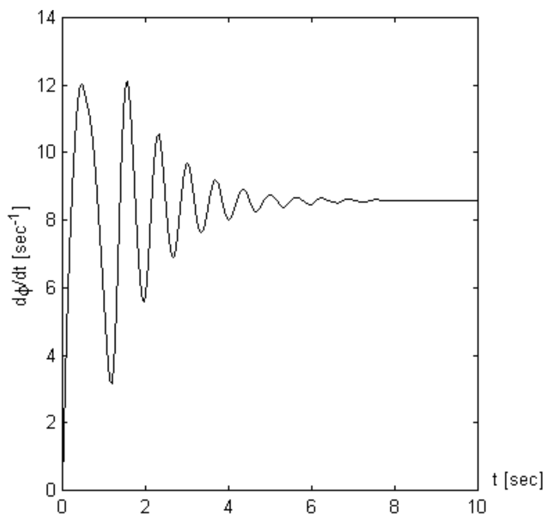
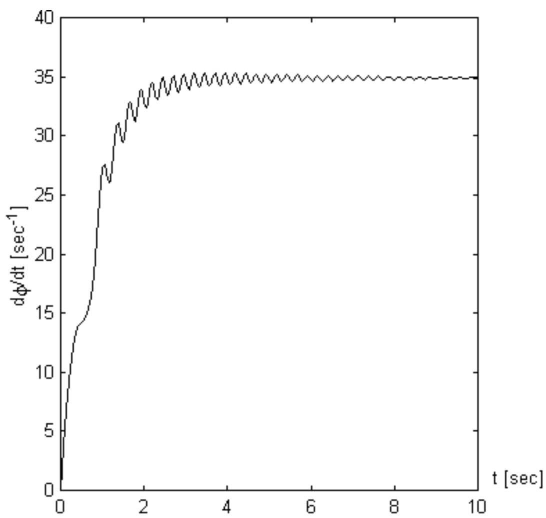


Fig. 1.1.



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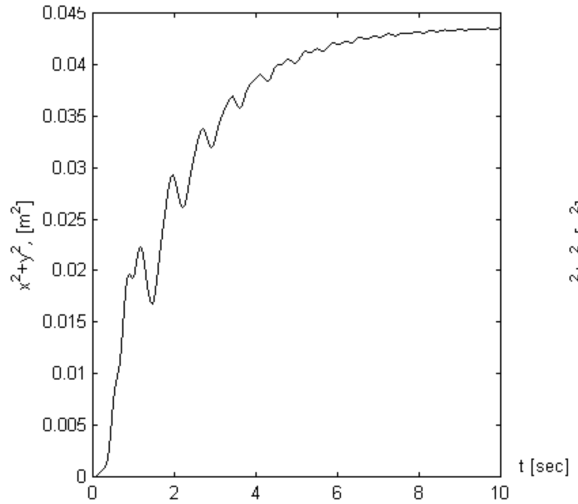

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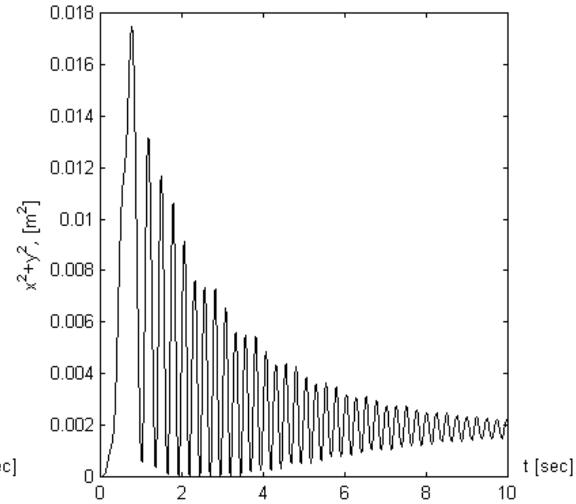

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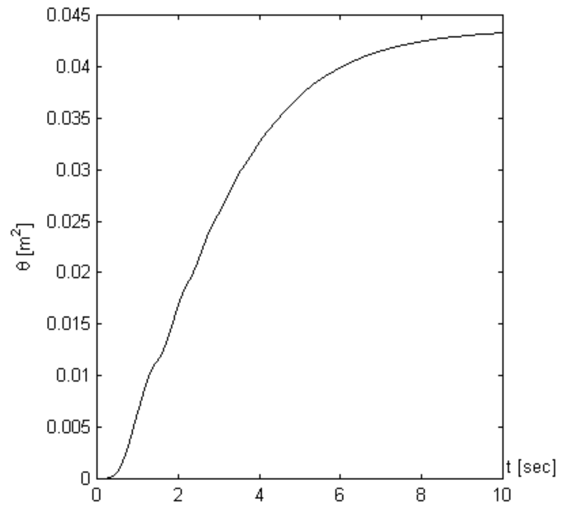

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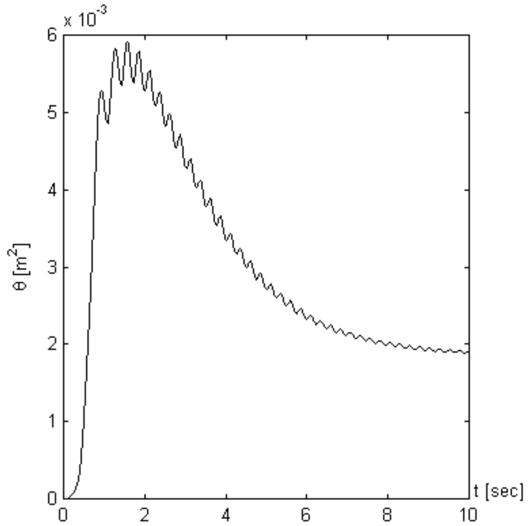


Fig. 3.2.

where $T_\theta > 0, T_\theta = \text{const}$ – algorithm parameter. The filtered variable $\theta(x, y, t)$ increases when the system is in the pre-resonance or resonance zone (see Fig. 3.1, M_0 = 0.6 [N·m]). In the post-resonance zone the value of $\theta(x, y, t)$ decreases significantly in comparison with the maximum value (see Fig. 3.2, M_0 = 0.7 [N·m]). Thus the variable $\theta(x, y, t)$ of the filter (2) allows to fix the moment of the passage through resonance. To synthesize the control algorithm we use the speed-gradient method [Fradkov *et. al.*, 1999]. At this stage we suppose that the control plant is conservative, i.e. the friction equals to zero. Then it is convenient to formalize the control goal as follows: To find controlling function $u(t)$ providing the goal equality $H(x, \dot{x}, y, \dot{y}, \varphi, \dot{\varphi}) = H^*$, where $H(t)$ is a current energy, H^* is the given energy level corresponding to the desired average rotation speed. Then it is possible to choose the goal functional as follows:

$$Q(z) = 1/2\left(H(z) - H^*\right)^2, \textit{where } z = [x, \dot{x}, y, \dot{y}, \varphi, \dot{\varphi}]^{\mathrm{T}}.$$

For the controller design purposes it is convenient to use Hamiltonian form

$$\dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q} + Bu,$$

where $p = p(t)$ is the vector of generalized momenta, $q = [\varphi, x, y]^T$ are generalized coordinates, $H = H(p, q)$ is the Hamiltonian function (total energy of the system), $B = [1, 0, 0]^T$. Then

$$\dot{Q}(z) = \left(H - H^*\right)\dot{\varphi}u.$$

and the speed-gradient method applies. One of the standard forms of speed-gradient algorithm is the "relay" one:

$$u = -M_0 \; sign\left[\left(H - H^*\right)\dot{\varphi}\right]. \qquad (3)$$

It is worth noticing that the algorithm (3) was designed neglecting the system dynamics. In case of 3-DOF oscillatory system such a design is not sufficient because of interaction between rotor and shaft, and because of the Sommerfeld phenomenon. It leads to appearance of fast oscillating motions that make difficult passing through resonance zone. In (Tomchina and Nechaev, 1999) new control algorithms were proposed facilitating passage through resonance by means of introducing additional low pass filter. Another pecularity of the algorithm (3) is large variability of the debalance angular velocity because of changes of potential energy due to gravity. Then the algorithm takes form:

$$\begin{cases} u = \begin{cases} M_0, & if \quad \left(H - H^*\right)(\dot\varphi - \psi) > 0, \\ 0, & else, \end{cases} \\ T_\psi \dot\psi = -\psi + \dot\varphi. \end{cases}$$

where $\psi(t)$ - filtered variable, $T_\psi > 0, T_\psi = \text{const}$.

However the efficiency of this algorithm is rather low because of high amplitude of rotor oscillations. So the value H^* may be achieved in the resonance zone. Also this algorithm requires choosing the value H^* for every set of plant parameters, and this task has no evident solution.

Thus we propose to exclude the factor $H - H^*$ having the negative sign in the post-resonance zone, from the algorithm. The modified algorithm will be again of speed-gradient type with respect to the goal function $Q = -H$. We also propose to switch off the control in the post-resonance zone and to leave only constant control torque. The algorithm is modified as follows.

The variable $\gamma_1(t)$:

$$\gamma_1(t) = \max_{[0,t]} \operatorname{sgn}\left[K \sup_{[0,t]} \theta(t) - \theta(t)\right],$$

is introduced, where $K > 0$ is the algorithm parameter. The properties of the variable $\theta(t)$ allows to say that $\gamma_1(t) = 0$ means that the system is in the pre-resonance or resonance zone (there was no significant decrease of $\theta(t)$). Also $\gamma_1(t) = 1$ means that the system is in the post-resonance zone. Thus, $\gamma_1(t)$ characterizes the current behavior of the system if K is properly chosen. The value of K should be sufficiently small to guarantee that the system is already in the post-resonance zone. At the same time the unjustified decrease of K may reduce the efficiency and transient time of the proposed algorithm.

Finally, the algorithm takes form:

$$\begin{cases} u(t) = \begin{cases} M_0, & if \quad \gamma_1(t) = 1, \\ M_0, & if \quad \gamma_1(t) = 0 \quad \& \quad (\dot\varphi - \psi) < 0, \\ 0, & else, \end{cases} \\ T_\psi \dot\psi = -\psi + \dot\varphi, \\ \gamma_1(t) = \max_{[0,t]} \operatorname{sgn}\left[K \sup_{[0,t]} \theta(t) - \theta(t)\right], \\ T_\theta \dot\theta(t) = -\theta(t) + x^2 + y^2, \quad \theta(0) = \dot\theta(0) = 0. \end{cases} \tag{4}$$

The value of T_ψ (time constant of the angular velocity filter) should be more then the period of the resonant oscillations. At the same time, if the value of T_ψ is too high, the algorithm works too slowly.

4. COMPUTER SIMULATION RESULTS

The designed control algorithm was numerically investigated to analyze the efficiency of the proposed algorithm for various values of plant and algorithm parameters. Numerical integration was made in MATLAB environment by means of Runge-Kutta method of second order. The value of the fixed step equal to 0.00025 [sec] was chosen so as the relative simulation error does not exceed 5%.

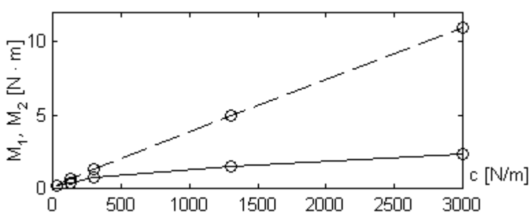
It was established by simulation that the worst values of simulation error are obtained when the damping factors are small. This is due to the increase of the oscillation amplitude when the damping factors decrease.

The nominal values of system parameters were chosen as follows: J = 0.014 [kg·m^2], m = 1.5 [kg], ε = 0.04 [m], k_φ = 0.02 [J·sec], $c = 1300$ [N/m], $k_x = k_y = 1$ [kg/sec].

The torques M_1 and M_2 were calculated for every series of experiments. M_1 is the value of the rotating torque of a motor, which allows system to pass the resonance zone for $u(t) \equiv M_1$, but not allows system to pass the resonance zone for any $M_0 < M_1$. M_2 is the value of the rotating torque of a motor, which allows the system to pass through the resonance zone for relay control algorithm (4), but not allows it for any $M_0 < M_2$.

Firstly, the influence of the shaft torsional stiffness c on system dynamics was investigated for nominal values of other plant parameters. The dependence of the minimal value of control action, allowing the passage through resonance, on the shaft torsional stiffness c is shown in Fig. 4.1. It is seen that the dependence of the constant control action M_1 on the stiffness c (dotted line) is linear. However some nonlinearity for the dependence of the torque M_2 on the stiffness c (solid line) is observed. The dependence of the efficiency of the proposed algorithm $\eta = M_1/M_2$ on the stiffness c is shown in Fig. 4.2. It is clear that the efficiency is small when c is small. However value of rotating torque M_0 can be reduced in 4-5 more times if the shaft torsional stiffness c increases. The time constant of the angular velocity filter T_ψ varies from 0.1 to 1.1 seconds.

Further the influence of the damping factor k_φ on system dynamics was investigated for nominal values of other plant parameters. The dependences



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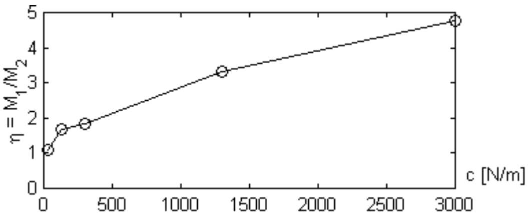

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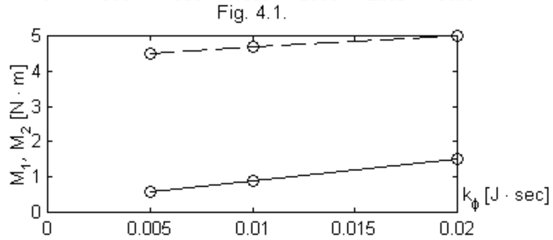
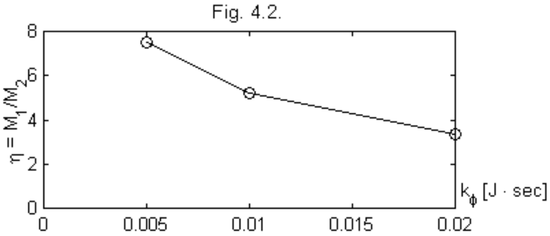


Fig. 4.3.



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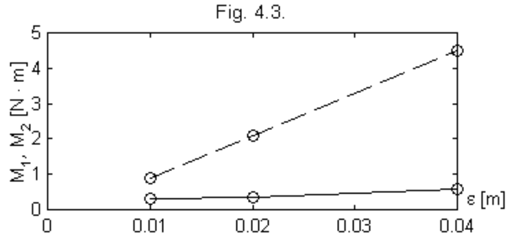

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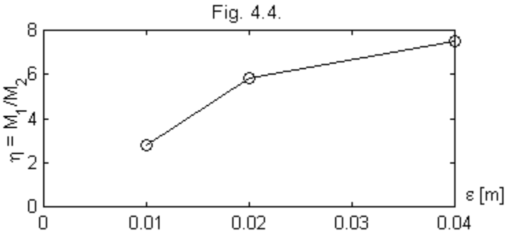

Fig. 4.6.

of the torques M_1 and M_2 on the damping factor k_φ are shown in Fig.4.3. It is seen that dependence on k_φ is almost linear both for the constant control action (dotted line) and relay control algorithm (solid line). The dependence of the efficiency of the proposed algorithm $\eta=M_1/M_2$ on the damping factor k_φ is shown in Fig. 4.4. It is clear that the efficiency is higher when k_φ is smaller. The value of rotating torque M_0 can be reduced in 7-8 times when k_φ decreases to 0.005 [J·sec].The time constant of the angular velocity filter T_ψ was equal to 0.1 seconds and was not varied.

Finally, the influence of the eccentricity of a rotor ε on system dynamics was investigated for nominal values of other plant parameters, k_φ= 0.005 [J·sec]. The dependence of the torques M_1 and M_2 on the eccentricity ε are shown in Fig. 4.5. It is seen that dependence on ε is almost linear both for the constant control action (dotted line) and relay control algorithm (solid line). The dependence of the efficiency of the proposed algorithm $\eta=M_1/M_2$ on the eccentricity ε is shown in Fig. 4.6. It is clear that the efficiency is higher when ε is higher and the value of rotating torque M_0 can be reduced in 7-8 times. The time constant of the angular velocity filter T_ψ was equal to 0.1 seconds and was not varied.

5. STUDY OF THE ALGORITHM ROBUSTNESS

The designed control algorithm was numerically investigated to analyze the robustness of the proposed algorithm for various values of plant parameters.

The algorithm has three design parameters: T_ψ - time constant of the angular velocity filter, T_θ - time constant of the additional variable θ filter, and K – the parameter allowing to fix the time of passing through resonance zone.

Simulation showed that the proper choice of the algorithms parameters $T_\theta = 1$ [sec], $K = 0.7$ allows to achieve satisfactory results described in Section 4. Further changing of these parameters does not increase the algorithm efficiency.

Further, the algorithm robustness for different values of the shaft torsional stiffness c was investigated for nominal values of other plant parameters J = 0.014 [kg·m^2], m = 1.5 [kg], ε = 0.04 [m], k_φ = 0.02 [J·sec], $k_x = k_y = 1$ [kg/sec]. The torques M_2 and M_3 were calculated for every value of the stiffness c. M_2 is the value of the rotating torque of a motor, which allows the system to pass through the resonance zone for relay control algorithm (4) and some value of T_ψ, but does not allow it for any $M_0 < M_2$ and any value of T_ψ (T_ψ varied in the range 0.1 ÷ 1.1 [sec] with the step 0.05 [sec]). M_3 is the value of the rotating torque of a motor, which allows the system to pass through the resonance zone for relay control

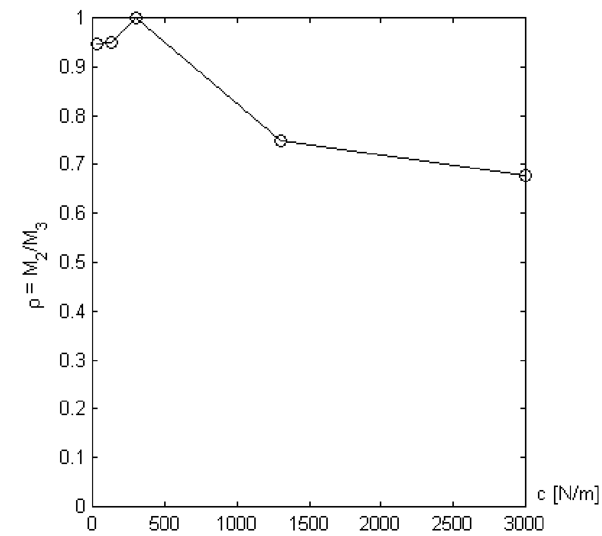

Fig. 5.1.

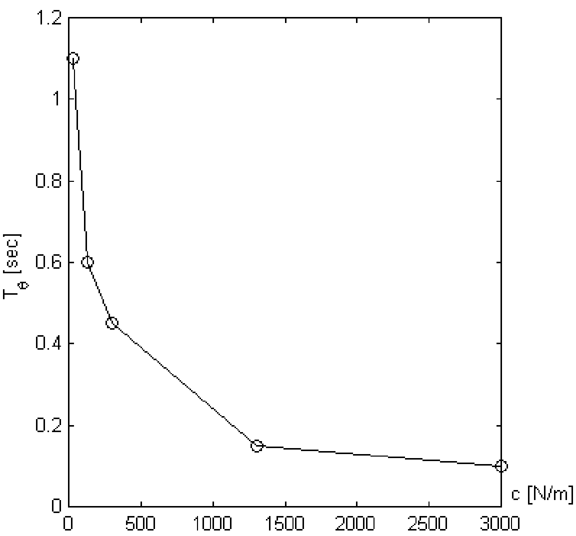

Fig. 5.2.

algorithm (4), but does not allow it for any $M_0 < M_2$; the value of T_ψ is fixed $T_\psi = 0.45$ [sec]. The dependence of $\rho = M_2/M_3$ on the stiffness c is shown in Fig. 5.1. It allows to evaluate the robustness of the proposed algorithm. The dependence of the best value of T_ψ on the stiffness c is shown on Fig. 5.2 (this dependence results from the fact that decrease of stiffness leads to decrease of period of oscillations, and it is desirable to increase the time constant).

CONCLUSION

Computer simulations show that the use of the proposed algorithm allows to significantly (sometimes by order) decrease the level of the controlling torque required to pass through the resonance zone.

To increase the algorithm efficiency it is sufficiently to change the only design parameter T_ψ. If the choice is proper, the algorithm efficiency is sufficiently high.

The robustness of the algorithm is significant. If the stiffness c changes from 30 to 3000 [N/m], then the constant value $T_\psi = 0.45$ [sec] provides the algorithm efficiency not less then 65% of the efficiency achieved for choice of T_ψ for every value of stiffness. Compared with the optimal control algorithm of (Malinin and Pervozvansky, 1993) the proposed algorithm is more simple for design and exhibits stronger robustness properties.

It is planned to test the proposed algorithm on the two-rotor vibrational set-up (Blekhman *et. al.*, 1999).

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