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Adaptive Passification of Interconnected Nonlinear Systems

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Abstract

In this paper, we provide some passivity-based tools for adaptive control design of interconnected nonlinear systems. Sufficient conditions are given under which an interconnected nonlinear system with parametric uncertainty is rendered adaptively feedback passive with adaptive regulation.

1 Introduction

Recently adaptive stabilization of nonlinear systems has received considerable attention (see [4] and references cited therein). Different adaptive control algorithms have been available for a reasonably large class of nonlinear systems. The feedback linearizability condition is a common feature in most previous work. One of our motivations is to remove this condition and prove that previous adaptive stabilization results on the basis of Lyapunov approach may also be obtained using passivity arguments.

While the Lyapunov method has been much employed to design adaptive controllers, the alternative passivity approach has been less explored ([2], [4], [5], [6]). Motivated by recent work on global feedback stabilization of nonlinear systems using passivity techniques (see [1] and references therein), the purpose of this paper is to present some key design ingredients towards the development of an adaptive controller for a complex interconnected system.

It is worth noting that Rodriguez and Ortega sought in their previous work [5] to relax the condition of feedback linearizability by means of passivity analysis tools. However, a restrictive matching condition is required in [5]. As in our previous work [6, 2], the key concept used in this paper is the *feedback passivity* introduced in Byrnes *et al.* [1]. Actually we will prove that the feedback passivity property can be propagated through adding any relative-degree-one nonlinear system with linearly appearing parametric uncertainty. It is important to note that an iterative use of these design ingredients gives a simpler reinterpretation of the backstepping design method along with tuning functions proposed by Krstić *et al.* [4]. The interested reader is referred to [3] for more details.

2 Definition and main results

Consider a nonlinear control system with parametric uncertainty:

$$\dot{x} \;=\; f(x,\theta) \;+\; G(x,\theta)u \tag{1}$$

where $x \in \mathbb{R}^n$, $u\,, y \in \mathbb{R}^m$ and $\theta \in \mathbb{R}^p$ is a vector of unknown constant parameters. Assume that the functions f and G are C^∞ with $f(0,\theta) = 0$ for all θ.

Definition 1 A system (1) is said to be *adaptively feedback passive (AFP)* with a quintuple $(V, \vartheta, \tau, h, \eta)$, if there exist a positive semidefinite function V, functions ϑ and τ, all of class C^∞, and a C^0 function h such that the resulting system composed of (1) and $\dot{\widehat{\theta}} = \tau(x,\widehat{\theta}) + \overline{\tau}$ and $u = \vartheta(x,\widehat{\theta}) + v$ is passive with respect to input $(v,\overline{\tau})$ and output h and the storage function $\overline{V} = V(x,\widehat{\theta}) + \frac{1}{2}(\widehat{\theta}-\theta)^T\Gamma^{-1}(\widehat{\theta}-\theta)$. That is,

$$\dot{\overline{V}} \;\le\; -\eta(x,\widehat{\theta}) \;+\; h^T(x,\widehat{\theta},\theta)\begin{pmatrix} v \\ \overline{\tau} \end{pmatrix} \tag{2}$$

where η is a nonnegative function.

Note that (2) is a *differential dissipation inequality* characterizing the passivity for the closed-loop system. If all unforced solutions $(x(t),\widehat{\theta}(t))$ (i.e. $v = 0$, $\overline{\tau} = 0$) are bounded and satisfy $x(t) \to 0$ as $t \to \infty$, then system (1) is said to have *AFP property with adaptive regulation.*

In the following, we show that the AFP property can be propagated through any minimum-phase nonlinear system with relative degree one.

For this, consider an interconnected nonlinear system with linearly appearing parametric uncertainty:

$$\begin{aligned}\dot{\xi} &= f_{10}(\xi) + f_1(\xi)\theta + (G_{10}(\xi) + \Delta G_1(\xi)\theta)y \\ \dot{z} &= q(\xi, z, y) \\ \dot{y} &= f_{20}(x) + f_2(x)\theta + (G_{20}(x) + \Delta G_2(x)\theta)u\end{aligned} \tag{3}$$

with $x = (\xi^T, z^T, y^T)^T$, $\xi \in \mathbb{R}^{n_o}$, $z \in \mathbb{R}^{n-m}$ and y, $u \in \mathbb{R}^m$. Denote $G_2(x,\theta) = G_{20}(x) + \Delta G_2(x)\theta$.

Proposition 1 *If the ξ-subsystem of (3) with y considered as input possesses AFP property with a quintuple $(V_1, \vartheta_1, \tau_1, h_1, \eta_1)$, if G_2 is globally invertible for each θ, then the interconnected system (3) possesses AFP property with $(V_2, \vartheta_2, \tau_2, h_2, \eta_2)$.*

Proof: see [3] for an idea of proof. □

Sufficient conditions under which system (3) possesses AFP property with adaptive regulation are given in the following.

Proposition 2 *Under the conditions of Proposition 1, assume that '$V_1(t)$ bounded' implies '$\xi(t)$ bounded' and $\eta_1 \equiv 0$ implies '$\xi(t) \to 0$'. If z-subsystem in (3) is BIBS stable and is GAS at $\xi = 0$ when $(z, y) = (0, 0)$, then the interconnected system (3) has AFP property with adaptive regulation.*

Proof: It suffices to prove that the unforced solutions $(x(t), \widehat{\theta}(t))$ are bounded and $x(t)$ converges to 0. The boundedness property follows from the assumptions and Prop. 1. The convergence part follows from LaSalle's invariance principle and [7, Proposition]. □

Remark 1 The BIBS (bounded-input bounded-state) stability condition is necessary for the global stabilization of interconnected systems. A lack of this condition may lead to the impossibility of globally stabilizing the system in question (cf. [3] for a counterexample).

An iterative application of Props. 1 and 2 gives rise to the following result via integrator backstepping.

Corollary 1 *[4] Any system in strict-feedback form*

$$\begin{aligned}\dot{x}_i &= x_{i+1} + \phi_i(x_1, \ldots, x_i)\theta\ , \quad 1 \le i \le n-1 \\ \dot{x}_n &= u + \phi_n(x_1, \ldots, x_n)\theta\end{aligned}$$

is globally adaptively stabilizable.

Example: Consider the interconnected system

$$\begin{aligned}\dot{z}_1 &= -\frac{1}{3}z_1^3 + y_1 \\ \dot{y}_1 &= -\theta_1\psi_1(z_1) + y_2 \\ &\cdots\cdots\cdots\cdots \\ \dot{z}_2 &= -\frac{1}{3}z_2^3 + y_2 \\ \dot{y}_2 &= -\theta_2\psi_2(z_2) + u\end{aligned} \tag{4}$$

It is easily checked that the conditions of Proposition 2 hold for each (z_i, y_i)-subsystem.

A direct application of the above control design procedure yields a global adaptive stabilizer for (4):

$$\begin{aligned}\dot{\widehat{\theta}}_1 &= -\gamma_1\left(2y_1 + y_2 - \widehat{\theta}_1\psi_1(z_1)\right)\psi_1(z_1) \\ \dot{\widehat{\theta}}_2 &= -\gamma_2(y_1 + y_2 - \widehat{\theta}_1\psi_1(z_1))\psi_2(z_2) \\ u &= -2y_1 - 2y_2 + \widehat{\theta}_1\frac{\partial\psi_1}{\partial z_1}(z_1)y_1 + \widehat{\theta}_2\psi_2(z_2) \\ &\quad + (2\widehat{\theta}_1 + \dot{\widehat{\theta}}_1)\psi_1(z_1) - \frac{1}{3}\widehat{\theta}_1\frac{\partial\psi_1}{\partial z_1}(z_1)z_1^2\end{aligned}$$

where γ_1 , $\gamma_2 > 0$.

In conclusion, a passive systems framework was presented in this paper for adaptive stabilization of a class of interconnected multiinput nonlinear systems. By employing the feedback passivity concept, we derive a simple reformulation of the important backstepping design with tuning functions in [4] for multiinput nonfeedback linearizable systems.

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