Proceedings of the 35th
Conference on Decision and Control
Kobe, Japan • December 1996

ADAPTIVE SYNCHRONIZATION OF CHAOTIC GENERATORS BASED ON TUNNEL DIODES

A. Yu. Markov, A. L. Fradkov, G. S. Simin
Institute for Problems of Mechanical Engineering
Academy of Sciences of Russia
61 Bolshoy ave V.O., St.Petersburg, 199178, RUSSIA
E-mail: alf@ccs.ipme.ru

Abstract

Two synchronization algorithms for a pair of semiconductor chaotic generators (transmitter and receiver) are proposed. It is shown, that both of them are efficient under uncertainty of parameters of the transmitter. Computer simulation results are presented confirming the theoretical analisis.

1 Introduction

Recently, much attention has been attracted to control of chaotic motions of electrical circuits, particulary to synchronization of chaotic generators [1, 2].

It was shown in [7], that autonomous broad-band chaotic generators could be realized on the basis of semiconductor devices with negative differential conductivity, in particular based on a "classical" semiconductor devices - tunnel diodes [3].

The paper presents a solution to the synchronization problem for two chaotic generators based on tunnel diodes, under assumption, that some parameters of the transmitter are unknown (or known inexactly). This condition is very important, because semiconductor devices may have a significant scattering of parameters. Synchronization algorithms are obtained using one of approaches of adaptive control theory - so-called speed gradient method [8]. It has been applied successfully to adaptive control of such chaotic systems as Duffing equation [4] and Chua circuit [5, 6].

Section 2 contains the mathematical model of the chaotic generators (transmitter and receiver) and the procedure of synchronization law design is shown in section 3. Section 4 presents the results of computer simulation of the designed system.

2 Problem statement

Electrical scheme of the generator based on tunnel diode is shown in Fig.1. As it was shown in [7], the scheme demonstrates chaotic behavior for some values of parameters. An important factor which determines the possibility of chaotic behavior of the circuit is the nonlinear voltage versus current characteristics of the tunnel diode. In this paper we use a simple cubic approximation of the NDC (negative differential conductivity) region in the form:

$$i = i(u_1) = a_1(u_1 - b)^3 - a_2(u_1 - b) + a_3 \qquad (1)$$

where b characterizes the initial shift on tunnel diode. The circuit is described by [7]:

$$\begin{cases} \dot{u}_1 = C_{21}(u_2 - u_1 - i(u_1)R)/q, \\ \dot{u}_2 = (u_1 - u_2 + i_3 R)q, \\ \dot{i}_3 = q(E - u_2)/R, \end{cases} \qquad (2)$$

where $C_{21} = C_2/C_1$, $q = R/\omega_2 L$, $R > 0$ are system parameters, u_1, u_2, i_3 are state parameters.

For typical GaAs tunnel diode [7] approximation characteristic (1) gives the following values of parameters: $a_1 = 192.3$, $a_2 = 43.6$, $a_3 = 12$. If the initial bias b is $0.37V$., system behavior becomes chaotic for the following set of parameters: $C_{21} = 8.25$, $q = 3.75$, $R = 0.03$.

In order to achieve synchronization of the two generators, introduce a control element into the receiver circuit: replace linear resistor by a field-effect transistor, operating in the linear regime. Then resistance of the transistor channel is described by:

$$R = \frac{R_0}{1 - \sqrt{\frac{u - u_b}{u_0}}} \qquad (3)$$

where R_0 - is an undepleted channel resistance, u_b - is a built-in voltage, u_0 - is a pinch off voltage, u - is the control voltage. One can see that control voltage is bounded:

$$u_b \leq u \leq u_0. \qquad (4)$$

Suppose that control voltage satisfies the inequality (4) (we can achieve it by choosing the parameters of the control algorithms, which would be obtained later). Extracting controlled parameter $R(u)$ in the system

0-7803-3590-2/96 $5.00 © 1996 IEEE

(2), and denoting

$$G = \frac{1}{R} = \frac{1 - \sqrt{\frac{u - u_b}{u_0}}}{R_0} \quad (5)$$

we obtain controlled system model in the form:

$$\begin{cases} \dot{u}_1 = p((u_2 - u_1)G - i(u_1)), \\ \dot{u}_2 = ((u_1 - u_2)G + i_3)/d, \\ \dot{i}_3 = d(E - u_2), \end{cases} \quad (6)$$

where $p = C_{21}R/q = C_2\omega_2 L/C_1,\ d = q/R_1 = 1/\omega_2 L$ do not depend on R.

The second system (the reference model) differs from the controlled system (6) by parameters values and by linear resistor R_m (denote $G_m = 1/R_m$), introduced instead of controlled field-effected transistor:

$$\begin{cases} \dot{u}_{1m} = p_m((u_{2m} - u_{1m})G_m - i_m(u_{1m})), \\ \dot{u}_{2m} = ((u_{1m} - u_{2m})G_m + i_{3m})/d_m, \\ \dot{i}_{3m} = d_m(E_m - u_{2m}), \end{cases} \quad (7)$$

where $i_m(u_{1m}) = a_{1m}(u_{1m}-b_m)^3 - a_{2m}(u_{1m}-b_m)+a_{3m}$ is the voltage versus current characteristic of the tunnel diode.

Suppose that p_m is unknown and choose the control aim as follows:

$$Q(u_1 - u_{1m}) = 0.5(u_1 - u_{1m})^2 \to 0, \text{ when } t \to \infty. \quad (8)$$

3 Design of synchronization algorithms

To design control algorithm use the speed-gradient method [8, 9]. Calculate the derivative of $Q(\cdot)$ along the trajectory of the system (6), (7):

$$\begin{aligned} \omega &= \dot{Q}(\cdot) = (u_1 - u_{1m})\Big(p((u_2 - u_1)G - i(u_1)) \\ &- p_m((u_{2m} - u_{1m})G_m - i_m(u_{1m}))\Big). \end{aligned} \quad (9)$$

Synchronization goal (8) could be achieved if the value of the control parameter $G = G_*$ satisfy inequality:

$$\omega(\cdot, G_*) < 0. \quad (10)$$

This inequality will be fulfilled if $u_1 \neq u_2$ and

$$\begin{aligned} G_* &= \Big(p_m((u_{2m} - u_{1m})G_m - i_m(u_{1m})) \\ &- \beta(u_1 - u_{1m}) + pi(u_1)\Big)/\Big(p(u_2 - u_1)\Big) \end{aligned} \quad (11)$$

where $\beta > 0$.

To simplify the control algorithm design, linearize the nonlinear characteristics of the transistor (3) near the operation point $u_0/2$:

$$G = \frac{1}{R} = \frac{1 - \alpha(u - u_b)/u_0}{R_0}, \quad (12)$$

where α could be found from the condition: derivative $\partial G/\partial u$ should be equal before and after linearization.

Using (12) and (11) we obtain the ideal control law in the form:

$$\begin{aligned} u_* &= \Big(p_m((u_{1m} - u_{2m})G_m + i_m(u_{1m})) + \beta(u_1 \\ &- u_{1m}) - pi(u_1)\Big)\frac{u_0 R_0}{p(u_2 - u_1)\alpha} + \frac{u_0}{\alpha} + u_b. \end{aligned} \quad (13)$$

It means that there exists such $u = u_*$, that inequality (10) is fulfilled[1]. However this algorithm is inapplicable because of its dependence on unknown parameter p_m.

According to speed gradient method the real control, indipendent of p_m and ensuring the synchronization aim (8), could be obtained in the two forms.

1. Nonadaptive synchronization algorithm. According to speed-gradient method [8] we obtain the following control algorithm:

$$u = -\gamma \nabla_u \omega(\cdot) = \gamma \frac{(u_1 - u_{1m})(u_2 - u_1)p\alpha}{u_0 R_0} \quad (14)$$

where $\gamma > 0$ is a gain coefficient.

2. Synchronization algorithm with parametric adaptation. According to adaptive control theory [8] the ideal control (13) also can be applied after some transformation. We replace the unknown parameter p_m in (13) by its estimate k:

$$\begin{aligned} u &= k\frac{i_m - (u_{2m} - u_{1m})G_m}{u_2 - u_1} + \frac{u_0}{\alpha} + u_b \\ &+ \frac{(\beta(u_1 - u_{1m}) - pi)u_0 R_0}{p\alpha(u_2 - u_1)}. \end{aligned} \quad (15)$$

and obtain the adaptation algorithm for the tunable parameter k useing the differential form of speed-gradient algorithm [8] (we can use differential form because the ideal value of k is constant). According to [8] calculate the derivative of $\omega(\cdot)$ along k taking into account (15) and (12):

$$\dot{k} = \gamma\alpha p(u_1 - u_{1m})(i_m - (u_{2m} - u_{1m})G_m)/u_0 R_0. \quad (16)$$

where $\gamma > 0$.

Note that for implementation of the algorithm (15), (16) u_{2m} mast be measurable together with u_{1m}, which is undesirable.

In order to simplify the algorithm (15), (16) replace the expression $(u_{2m} - u_{1m})$ by its mean value $\delta = const$ (it

[1] We assume that $u_2 = u_1$. It is possible because the trajectory of the reference model strongly influences the trajectory of the controlled system and can be be chosen in such way that this situation would not appear at all

is supposed that the error would be cancelled by tuning the parameter k):

$$\begin{aligned} u &= k\frac{i_m - \delta G_m}{u_2 - u_1} + \frac{(\beta(u_1 - u_{1m}) - pi)u_0R_0}{p\alpha(u_2 - u_1)} \\ &+ \frac{u_0}{\alpha} + u_b, \end{aligned} \quad (17)$$

$$\dot{k} = \gamma\alpha p(u_1 - u_{1m})(i_m - \delta G_m)/u_0R_0. \quad (18)$$

The achievement of the syncronization goal by the algorithms (14) and (15), (16) follows from stability theorems for speed-gradient algorithms. However stability and performance of simplified algorithm (17), (18) needes further analysis by means of computer simulation.

4 Computer simulation results

To exemine the efficiency of the proposed algorithms computer simulations were carried out. The overall system consisted of:

1. Reference model (7) with parameters $G_m = 100/3$, $p_m = 0.066$, $d_m = 125$, $E_m = 0.73$, $u_{1m}(0) = 0.2$, $u_{2m}(0) = 0.75$, $i_{3m}(0) = 20$, $a_{1m} = 192.3$, $a_{2m} = -43.6$, $a_{3m} = 12$, $b_m = -0.37$;
2. Controlled system (6) with parameters $p = 0.08$, $d = 125$, $E = 0.73$, $G(0) = 0.03$, $u_1(0) = 0.5$, $u_2(0) = 0.67$, $i_3(0) = 7$, $a_1 = 192.3$, $a_2 = -43.6$, $a_3 = 12$, $b = -0.37$, and R_0 is given in table 1;
3. Characteristic of the transistor channel resistance: nonlinearized (5) or linearized (12), parameters see table 1;
4. Control algorithms: nonadaptive (14) or adaptive (17),(18), parameters see table 1 (all initial values, except $G(0)$ are chosen zero) .

Some simulation results for the cases mentioned in table 1 are shown on Fig.2-5. Here: a) - trajectory of outputs u_1 and u_{1m}; b) - trajectory of control signal $u(t)$; c) - trajectory of the norm state vector $y(t) = \sqrt{u_1^2(t) + u_2^2(t) + i_3^2(t)}$; d) - trajectory plot of $k(t)$.

The results may be summarized as follows:

1. The controll signal u may be put into the region $u_b \le u(t) \le u_0$ by proper choice of parameters γ, β, R_0.
2. Algorithm (14) provides a good synchronization of the controlled generator with the drive signal $u_{1m}(t)$, which can be generated by the same generator with other values of parameters or by some other system (both in case of the linearized (12) and nonlinearized (5) characteristic of the transistor).

Control algorithm	(14)		(17),(18)	
N° of simulation	1	2	3	4
Transistor characteristic	(12)	(5)	(12)	(5)
γ	10	10	5	2
β	-	-	5	30
α	0.7	0.7	0.7	0.7
R_0	0.015	0.0166	0.02	0.01
u_0	1	1	1	1
u_b	0.001	0.001	0.001	0.001
Figure N°	2	3	4	5

Table 1: Computer simulation results

3. Algorithm (17), (18) provides an excellent synchronization when the characteristics of the transistor resistance is linearized (12), and satisfactory synchronization when it is not linearized (5). The synchronization is better if the structure of the transmitter and receiver coincide.
4. The regulation error does not vanish because of the boundedness of the control signal $u(t)$.

5 Conclusions

The paper presents a solution to the synchronization problem for two semiconductor chaotic generators (transmitter and receiver), which differs from the existing ones, by uncertainty of systems parameters. This advantage allows to synchronize circuits with semiconductor devices having significant scattering of parameters. The proposed adaptive and nonadaptive synchronization algorithms provide a small value of the error Δ between the outputs of the transmitter and receiver after a few cycles of oscillations.

References

[1] Ogorzalek M. J. Taming chaos - part I: synchronization; pat II: control. IEEE Trans. circ. & syst. I. 40. 639-706. 1993.

[2] Carroll T. L., Pecora L. M. Synchronizing chaotic syrcuits. IEEE Trans. circ. & syst. 38. 453-456. 1991.

[3] Sze S. M. Phtysics of semiconducttor devices. John Wiley and Sons, New York - Chichester - Brishbane - Toronto - Singapore, 1981.

[4] Fradkov A. L. Nonlinear adaptive control: regulation, tracking, oscillations. Preprint of the 1st IFAC Workshop "New trends in the design of control systems". Smolenice, Sept. 1994. 392-397.

[5] A.L. Fradkov, A.Yu. Pogromsky, A.Yu. Markov. "Adaptive control of chaotic continuous-time systems", 3rd European Contr. Conf. Rome, 1995, pp.3062-3067.

[6] A.L.Fradkov. Adaptive synchronization of hyper-minimum-phase systems with nonlinearities", Proc.

3rd IEEE Mediterranean Symposium on New Directions in Control and Automation, Limassol, July 1995, v.1, pp.272-277.

[7] G. S. Simin, A. L. Fradkov, S. A. Shadchin. Chaotic oscillations emission by NDC semiconductor devices. Proc. of ISDRS-95. Sharlotteswille, Wa, USA. 211-224.

[8] Fradkov A. L. "Adaptive control in complex systems", Moscow, Nauka, 1990 (in Russian).

[9] Fradkov A.L., Guzenko P.Yu., Hill D., Pogromsky A.Yu. Speed gradient control and passivity of nonlinear oscillators, IFAC Symposium on Nonlinear Control Systems, NOLCOS'95, Tahoe City, USA, pp.555-559.

[10] Fradkov A.L., Pogromsky D.Yu. "Speed gradient adaptive control of chaotic continuous-time systems". Academy of Science of Russia. Institute of Problems of Mechanical Engineering, Preprint 107, Design of nonlinear oscillating systems, 1994.

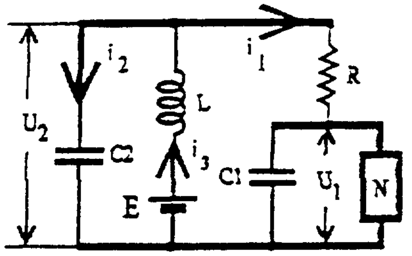

Fig. 1a.Electrical circuit for modelling chaotic oscillations with N-type devices

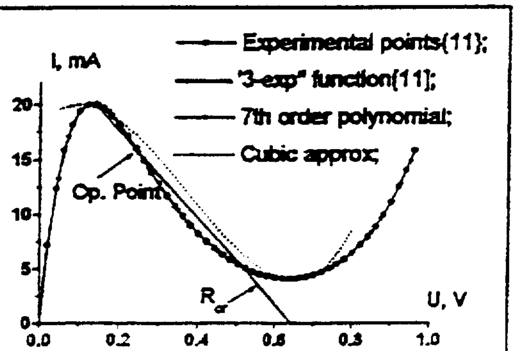

Fig. 1b.I-V Curves of the GaAs Tunnel Diode. R_{cr} shows approx. maximal load for chaos .

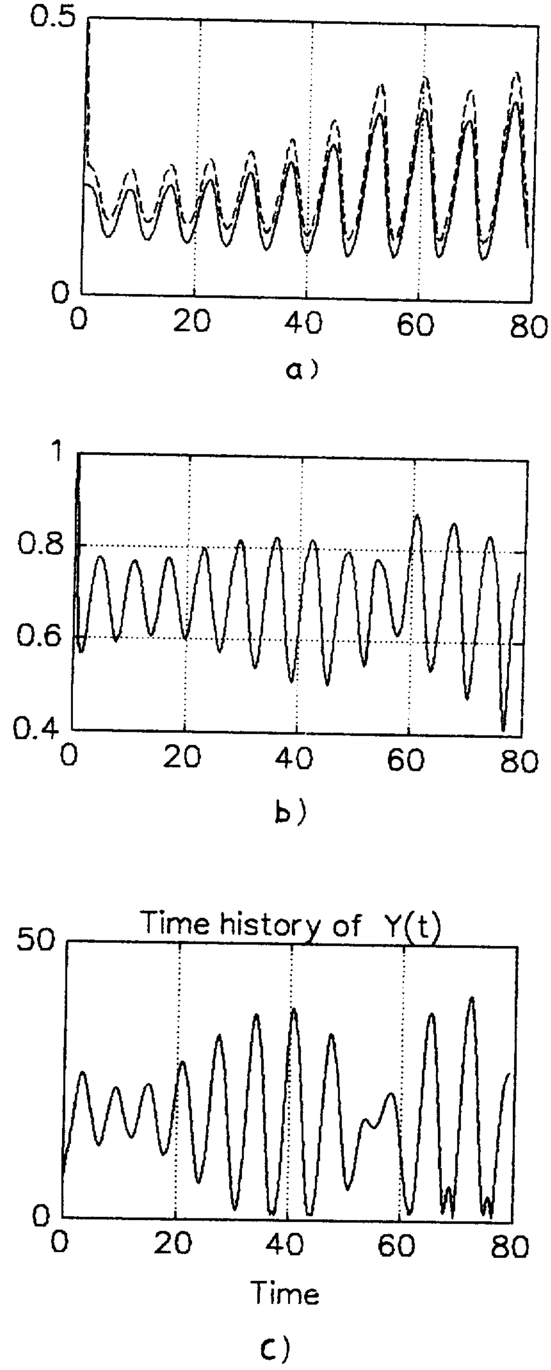

Fig. 2

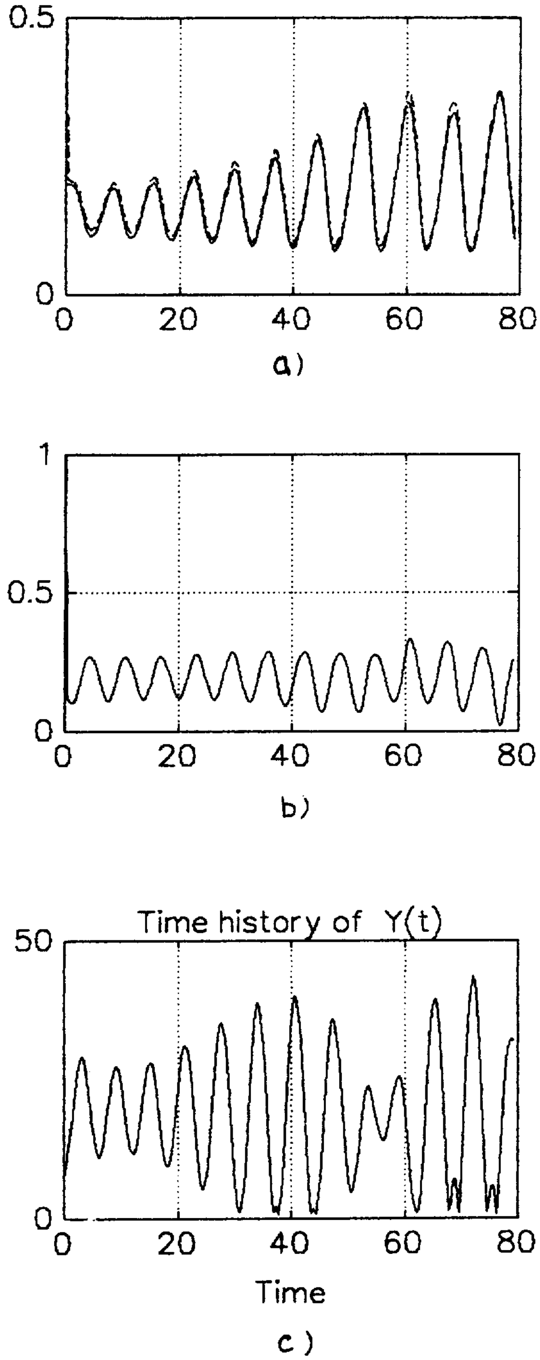
0.5
0
0
20
40
60
80
a)
1
0.5
0
0
20
40
60
80
b)
Time history of Y(t)
50
0
0
20
40
60
80
Time
c)

Fig. 3

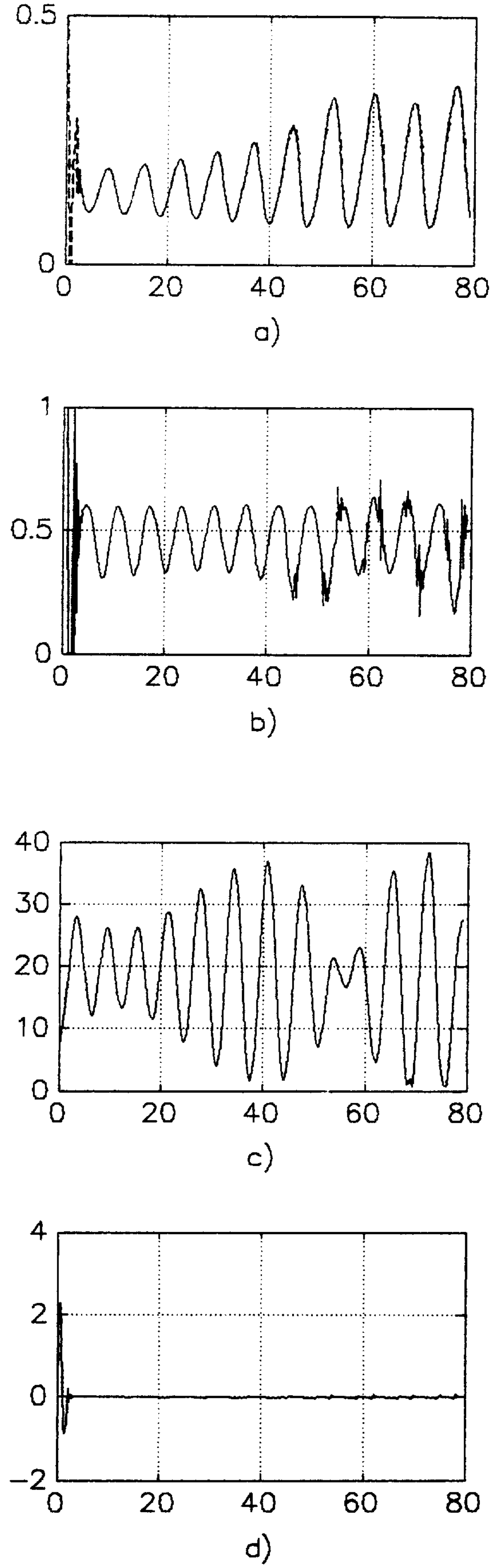
0.5
0
0
20
40
60
80
a)
1
0.5
0
0
20
40
60
80
b)
40
30
20
10
0
0
20
40
60
80
c)
4
2
0
−2
0
20
40
60
80
d)

Fig. 4

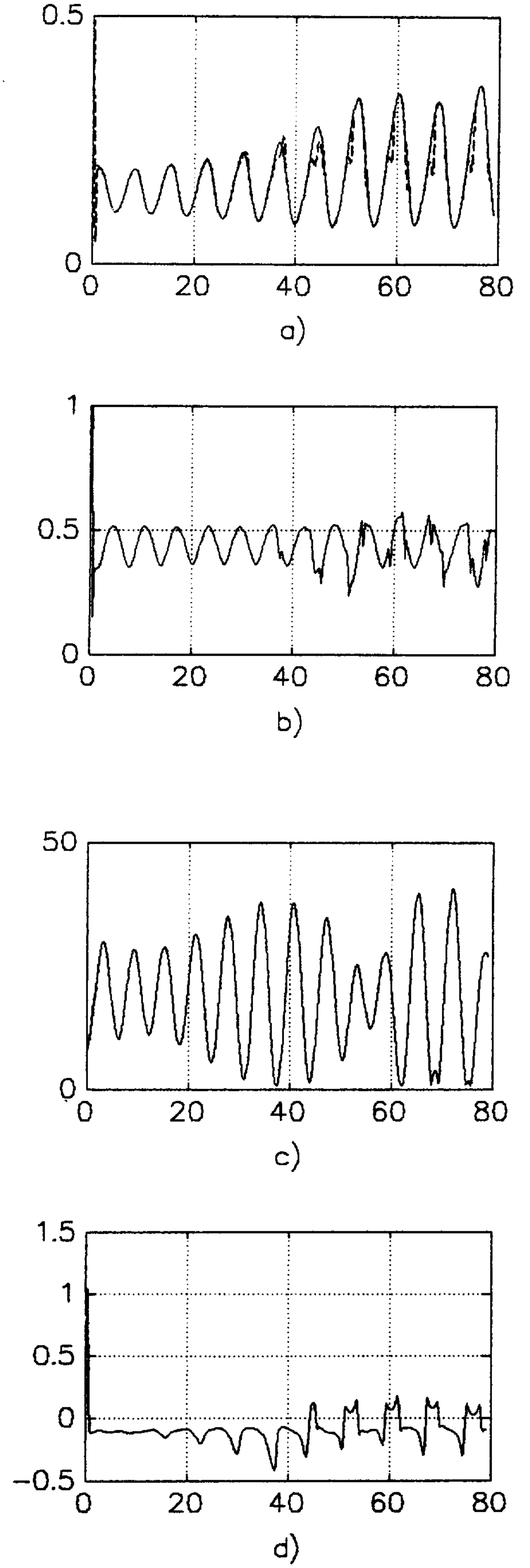
0.5
0
0 20 40 60 80
a)
1
0.5
0
0 20 40 60 80
b)
50
0
0 20 40 60 80
c)
1.5
1
0.5
0
−0.5
0 20 40 60 80
d)

Fig. 5