Proceedings of the 35th
Conference on Decision and Control
Kobe, Japan • December 1996

Trajectory Approximation Based Adaptive Control for Nonlinear Systems Under Matching Conditions.

E. Skafidas, R.J. Evans and I.M Mareels
Centre for Sensor,
Signal and Information Processing,
Department of Electrical Engineering,
University of Melbourne,
Parkville 3054 Australia.
{e.skafidas,r.evans,i.mareels@ee.mu.oz.au}

A. Fradkov
Institute for Problems of
Mechanical Engineering,
Academy of Sciences of Russia,
61 Bolshoy ave V.O., 199178,
St. Petersburg Russia.
alf@ccs.ipme.ru

Abstract

Uncertain nonlinear systems affine in the input are considered. It is shown that under some matching conditions these systems can be made practically stable by state feedback. We present an adaptive hybrid algorithm which renders the origin practically stable to any desired accuracy. The presented algorithms are simple and computationally cheap.

1 Introduction

State and output feedback controllers which guarantee both global and asymptotic tracking have been designed for nonlinear systems with uncertain parameters. Typically these results rely on the theory developed in [1, 2], and on extensions developed for adaptive systems, as in [3, 4, 5] for example. In these "adaptive" papers an assumption is made that the plant model depends on a finite number of unknown parameters. The control algorithms are based on tuning parameter estimates.

A different approach, based on approximating the trajectories of the non-linear systems was presented in [6] and extended to output feedback in [7]. Related results may be found in [8, 9]. In this approach the defining vector field is not estimated, but approximated locally in space and time.

The aim of this paper is to bring into focus the similarities (in the control design) and differences (in the identification and the hybrid nature of the closed loop) between the above mentioned approaches. It extends the work presented in [6] in that the nonlinear systems, presented in the more *traditional* affine in the input format are dealt with, rather that the input-output format used in [6, 7]. In this paper this extension is discussed under matching conditions, either an exact matching (hard constraint) or matching via a Lyapunov function (soft constraint).

The two approaches are equivalent and basically differ in the way that they use prior knowledge about the system. For "hard" matching the equation of an asymptotically stable reference model is assumed to be known and a control algorithm which is an extension of that in [6] is used. For the "soft" matching case two approaches are presented. In both cases only a goal function guaranteeing stability of the reference model is required. In this paper it is demonstrated that an arbitrarily high accuracy of control achievement, in the noiseless case, is possible if suitable parameters are chosen for the algorithms presented.

The paper is organised as follows. The problem is introduced in section 2. The identification algorithm is outlined in section 3 where analytical results depending on the design parameters bounding the estimation error are proved. The control algorithm is then presented together with results demonstrating control implementation accuracy and control input bounds. The main results of the paper are presented in section 4 where we establish the general applicability of the techniques presented for nonlinear twice differentiable plants affine in the control input. Lastly a number of simulations are described in section 6.

2 Problem Statement

Let the plant to be controlled be modeled as:

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})u;\ t \geq 0;\ \mathbf{x}(0) = x_0 \tag{1}$$

where $\mathbf{x}(t) \in \mathbf{R}^n$ is the plant state. The state is measurable and measurements are taken at the sample points $k\Delta$, where $k \in \{1, 2 \ldots\}$ and $\Delta > 0$ is the sampling interval. $u(t) \in \mathbf{R}$ is the control input and the vector fields $f, g : \mathbf{R}^n \to \mathbf{R}^n \in C^2$ are twice continuously differentiable functions of the state.

Let B_r^n denote a ball of radius r centered at the origin

0-7803-3590-2/96 $5.00 © 1996 IEEE

in $\mathbf{R}^n$:

$$B_r^n = \{\mathbf{x} \in \mathbf{R}^n \text{such that } \|\mathbf{x}\| \le r\}$$

Thus the vector fields f, g satisfy the following relationships $\forall \mathbf{x} \in B_r^n$ and $\{\mathbf{x}_1, \mathbf{x}_2\} \in B_r^n$

$$\begin{aligned} f(0) &= 0 \\ \|f(\mathbf{x})\| &< F_0(r), \\ \|g(\mathbf{x})\| &< G_0(r), \\ \|f(\mathbf{x}_1) - f(\mathbf{x}_2)\| &< F_1(r)\|\mathbf{x}_1 - \mathbf{x}_2\|, \\ \|g(\mathbf{x}_1) - g(\mathbf{x}_2)\| &< G_1(r)\|\mathbf{x}_1 - \mathbf{x}_2\|. \end{aligned} \tag{2}$$

Under the assumptions (2) standard results [10] guarantee the existence and uniqueness of solutions for the plant model (1) for all initial conditions in a compact set, all initial times and for all bounded control input u. Further requirements on the plant are that the function $g(\mathbf{x})$ does not vanish on $\mathbf{R}^n$

$$\|g(\mathbf{x})\| > 0; \ \ \forall \mathbf{x} \in B_r^n.$$

The control objective is to regulate the state to zero using a control input which is piecewise constant over the sampling intervals such that for any given $a > 0$

$$\exists T > 0 : \sup_{t>T} \|\mathbf{x}(t)\|_{2,[t,\infty]} \le a. \tag{3}$$

We consider the above control objective under two different matching conditions which are defined as follows.

Definition 2.1 *The plant (1) is said to satisfy a hard matching condition (HM) if there exists a function $f_m(\mathbf{x})$, i.e. $f_m(\mathbf{x}) \in C^2$, and a continuous at the origin smooth everywhere else control function $u_m(\mathbf{x})$ such that*

$$f(\mathbf{x}) + g(\mathbf{x})u_m(\mathbf{x}) = f_m(\mathbf{x}) \tag{4}$$

where the origin is a globally asymptotically stable equilibrium point of $\dot{\mathbf{x}} = f_m(\mathbf{x})$.

Definition 2.2 *A plant (1) is said to satisfy a soft matching condition (SM) if there exists a smooth proper positive definite scalar function, also referred to as a goal function, $Q(\mathbf{x})$ and a function $u_q(\mathbf{x})$ continuous at $x = 0$, with $u_q(0) = 0$, smooth everywhere else, such that:*

$$\nabla Q^T(\mathbf{x})(f(\mathbf{x}) + g(\mathbf{x})u_q(\mathbf{x})) < 0 \, \forall x \neq 0 \tag{5}$$

See also Theorem (1) in [11].

For notational convenience we define the quantities

$$\begin{aligned} a(\mathbf{x}) &= \nabla Q^T(\mathbf{x}) \cdot f(\mathbf{x}) && (6) \\ b(\mathbf{x}) &= \nabla Q^T(\mathbf{x}) \cdot g(\mathbf{x}). && (7) \end{aligned}$$

The two explicit forms for $u_q(\mathbf{x})$ we use in the sequel are either the control law introduced by Sontag [11]:

$$u_q(\mathbf{x}) = -\frac{a(\mathbf{x}) + \sqrt{a^2(\mathbf{x}) + b^4(\mathbf{x})}}{b(\mathbf{x})} \tag{8}$$

or alternatively the speed gradient control law introduced in [12, 13]

$$u_q(\mathbf{x}) = -\gamma b(\mathbf{x}). \tag{9}$$

The constant γ is calculated to ensure that condition(5) is satisfied.

Both "hard" and "soft" matching conditions are sufficient to guarantee that the control objective can be realised.

The adaptive control problem under the HM and SM condition is to achieve the control objective using full state measurements $\mathbf{x}(k\Delta)$ and full knowledge of $f_m(\mathbf{x})$ for the hard matching condition and $Q(\mathbf{x})$ in the soft matching condition case. As can be anticipated both matching conditions are closely linked via classical Lyapunov stability theorems. Equivalence between the two approaches is stated in the following proposition

Proposition 2.1 *For the plant given in (1) and stable reference model the "soft" and "hard" matching are equivalent.*

3 Adaptive Control Algorithms

The control algorithm generates an input, piecewise constant over the sampling intervals $[2(k-1)\Delta, 2k\Delta)$, $(k = 1, 2, \ldots)$ according to :

$$u(t) = \begin{cases} u_k - \kappa & 2(k-1)\Delta \le t < (2k-1)\Delta \\ u_k + \kappa & (2k-1)\Delta \le t < (2k)\Delta. \end{cases} \tag{10}$$

Here $\kappa > 0$ is a (small) test signal which is used in the identification step of the adaptation algorithm, while u_k should be chosen to perform the control task.

3.1 Identification

For notational convenience we introduce $f(\mathbf{x}_k) = f_k$, $g(\mathbf{x}_k) = g_k$ where $\mathbf{x}_k = \mathbf{x}(k\Delta)$. Estimates for f_{2k}, g_{2k}, namely $\hat{f}_{2k}$ and $\hat{g}_{2k}$ can be computed, [6], from:

$$\begin{aligned} \begin{bmatrix} \hat{f}_{2k} \\ \hat{g}_{2k} \end{bmatrix} &= -\frac{1}{2\Delta^2\kappa} \begin{bmatrix} \Delta(u_k - \kappa)\mathbf{I} & -\Delta(u_k + \kappa)\mathbf{I} \\ -\Delta\mathbf{I} & \Delta\mathbf{I} \end{bmatrix} \cdot \\ & \qquad \begin{bmatrix} \mathbf{x}_{2k} - \mathbf{x}_{2k-1} \\ \mathbf{x}_{2k-1} - \mathbf{x}_{2k-2} \end{bmatrix} \end{aligned} \tag{11}$$

We are now in a position to investigate the estimation accuracy of (11). However before doing so we prove the following lemmata and introduce the notation $C(\rho)$ and $C(\rho_x, \rho_u)$ to denote a positive valued function of its arguments. Furthermore it is assumed that $0 < \kappa, \Delta < 1$.

Let $x(t_0, t, x_0)$ denote the state trajectories of (1) for initial condition x_0 and for any $t \in U = [t_0, t_d)$ where t_d is the region of definition of this system.

Starting assumption: $\|\mathbf{x}(s)\| < \rho_x$ and $|u(s)| < \rho_u$, $\forall 0 \leq s \leq t_d$ for some $t_d > 0$. Note that $t_d > 0$ exists if we assume that, e.g. $\|\mathbf{x}(0)\| \leq \rho_x/2$. Since $f(\mathbf{x})$ and $g(\mathbf{x})$ are Lipschitz continuous in B_n^r, a conservative estimate for

$$t_d \geq T_1 = \min\left(\frac{\rho_x}{2\max(\rho_x, F_0(\rho_x) + G_0(\rho_x)u_m}, \frac{\rho_u}{\max(\rho_u, u_m)}\right). \tag{12}$$

The sampling period Δ is hence chosen

$$\Delta \leq \frac{t_d}{6} \tag{13}$$

Lemma 3.1 *For all $|t - s| \leq \Delta$, $0 \leq t, s \leq 2l\Delta < t_d$ we have $\|\mathbf{x}(t) - \mathbf{x}(s)\| \leq C(\rho_x, \rho_u)\Delta$*

Now we are in a position to bound the estimation error for the estimates of $f(\mathbf{x}(t))$ and $g(\mathbf{x}(t))$ as obtained from (11).

Lemma 3.2 *On any interval $2(k-1)\Delta \leq t \leq 2(k+1)\Delta$, $t \in [0, t_d)$ the estimate error is bounded by*

$$\begin{aligned} \|\hat{f}_{2k} - f(\mathbf{x}(t))\| &\leq C(\rho_x, \rho_u, \kappa)\Delta \\ \|\hat{g}_{2k} - g(\mathbf{x}(t))\| &\leq C(\rho_x, \rho_u, \kappa)\Delta. \end{aligned} \tag{14}$$

3.2 Control Step

In this section we introduce the two variants of the control law which will be implemented to control the plant and investigate the control implementation for each case. For the hard matching condition the control law is chosen to be

$$u_{k+1} = \begin{cases} \hat{g}_{2k}^T(f_m(\mathbf{x}_{2k}) - \hat{f}_{2k})/\hat{g}_{2k}^T\hat{g}_{2k}, & \|\hat{g}_{2k}\| > \beta \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

$0 < \beta < 1$ is a design parameter. $u_1 = 0$ i.e. control action starts at sample time 2Δ. (15) amounts to choosing u_{k+1} as the least squares solution of the equation $\hat{f}_{2k} + \hat{g}_{2k}u = f_m(\mathbf{x}_{2k})$

For the soft matching condition the law is chosen either as

$$u_{k+1} = \begin{cases} -\frac{a_k + \sqrt{a_k^2 + b_k^4}}{b_k} & \text{if } |b_k| > \beta \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

or speed gradient

$$u_{k+1} = -\gamma b_k \tag{17}$$

where

$$a_k = \hat{f}_{2k}^T \nabla Q(\mathbf{x}_{2k}) \tag{18}$$

$$b_k = \hat{g}_{2k}^T \nabla Q(\mathbf{x}_{2k}) \tag{19}$$

and $0 < \beta < 1$, $\gamma > 0$ are design parameters. We take $u_1 = 0$. In the next sections the control implementation accuracy is investigated for the above control laws and it is proven that the control input for the proposed laws is bounded. The existence of γ may be seen from

$$\gamma \geq \max_{\mathbf{x} \in B_r} \frac{a(\mathbf{x}) + \sqrt{a(\mathbf{x}) + b^4(\mathbf{x})}}{b^2(\mathbf{x})}. \tag{20}$$

3.3 Control Implementation Accuracy

Lemma 3.3 *HM control case: If $\|\mathbf{x}(t)\| \leq \rho_x$ and $|u(t)| \leq \rho_u$ on $0 \leq t \leq 2l\Delta$, $t \in [0, t_d)$ then for all $t \in [2\Delta, 2l\Delta)$ we have*

$$\|f_m(\mathbf{x}(t)) - f(\mathbf{x}(t)) - g(\mathbf{x}(t))u(t)\| \leq C(\rho_x, \rho_u, \kappa)(\beta + \kappa + \Delta) \tag{21}$$

Lemma 3.4 *SM control case: If $\|\mathbf{x}(t)\| \leq \rho_x$ and $|u(t)| \leq \rho_u$ on $0 \leq t \leq 2l\Delta$, $t_d \in [0, t_d)$ then for all $t \in [2\Delta, 2l\Delta)$ we have*

$$\begin{aligned} &\nabla Q^T(\mathbf{x}(t))(f(\mathbf{x}(t)) + g(\mathbf{x}(t))u(t)) \leq \\ &\qquad C(\rho_x, \rho_u, \kappa)(\beta + \Delta + \kappa) \\ &\quad - \sqrt{(\nabla Q^T(\mathbf{x}(t))f(\mathbf{x}(t)))^2 + (\nabla Q^T(\mathbf{x}(t))g(\mathbf{x}(t)))^2} \end{aligned} \tag{22}$$

3.4 Control Input bounds

We now investigate the input bounds for the two proposed control laws. For the hard matching case we have

Lemma 3.5 *Assume $\|\mathbf{x}(t)\| \leq \rho$ on $0 \leq t \leq \Delta$. Let $\kappa < 1$ and $\rho^* = \sup_{\|x\| < \rho} |u_m(\mathbf{x})|$, then there exists a $\Delta_1(\rho, \kappa, \beta) > 0$ such that for all $0 < \Delta < \Delta_1(\rho, \kappa, \beta)$, we have $|u(t)| \leq \rho_* + 1$ on $0 \leq t < 2l\Delta$.*

Similarly for the soft matching case we have

Lemma 3.6 *Assume $\|\mathbf{x}(t)\| \leq \rho$ on $0 \leq t \leq \Delta$. Let $\kappa < 1$ and $\rho^* = \sup_{\|\mathbf{x}\| < \rho} |u_q(x)|$, then there exists a $\Delta_1(\rho, \kappa, \beta) > 0$ such that for all $0 < \Delta < \Delta_1(\rho, \kappa, \beta)$, we have $|u(t)| \leq \rho_* + 1$ on $0 \leq t < 2l\Delta$.*

4 Main Result

The main result of this paper, a proof showing the general applicability of the techniques outlined in previous sections for all twice continuously differentiable nonlinear plants affine in the control input, is encompassed in the following theorem:

Theorem 4.1 *Consider the system defined by (1) under HM (alternatively SM). For any $a > 0$ and any $R > 0$ there exist a positive (small) constant $\Delta_o(a, R) > 0$ such that all trajectories of the closed loop system described by equations (1), (10), (11) and (15) (alternatively equations (1), (10), (11), (16) (18) and (19)) with initial condition $\|\mathbf{x}_o\| \leq R$ are bounded and achieve the control objective for any sample period $\Delta \in (0, \Delta_o \kappa)$ when the design parameters are chosen as $\beta \in (0, \Delta_o)$ and $\kappa \in (0, \Delta_o)$.*

Theorem 4.2 *Consider the system defined by (1) under SM. For any $a > 0$ and any $R > 0$ there exist a positive (small) constant $\Delta_o(a, R) > 0$ and $\gamma >> 0$ such that all trajectories of the closed loop system described by equations (1), (10), (11) and (17) (18) with initial condition $\|\mathbf{x}_o\| \leq R$ are bounded and achieve the control objective for any sample period $\Delta \in (0, \Delta_o \kappa)$ when the design parameters are chosen as $\beta \in (0, \Delta_o)$ and $\kappa \in (0, \Delta_o)$.*

Proof: All that is required to be proven is that there exists a positive definite function $W(x)$ such that $V(x_{2k}) - V(x_{2k-1}) \leq -W_0(x_{2k-2})$. The rest follows directly from Theorem 1. ∎

5 Simulation Example

In this section we utilise the techniques described in this paper and illustrate their general applicability by attempting to control the trajectories of a continuous time nonlinear system to converge to a reference model. The particular example we consider

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \alpha(\mathbf{y}) & +\beta(\mathbf{y}) \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad (23)$$

where $\alpha(\mathbf{y})$ and $\beta(\mathbf{y})$ are unknown locally Lipschitz continuous functions of the state. In this particular simulation example they are set to $\alpha(\mathbf{y}) = \frac{1}{2}\cos^2(y_1)$ and $\beta(\mathbf{y}) = 3+\sin(y_2)$. It is emphasised that the structure of α and β are unknown.

In order to ensure synchronisation we introduce the control aim

$$\lim_{t\to\infty} \sup(|y_1(t) - y_{1r}(t)| + |y_2(t) - y_{2r}(t)|) < \epsilon \quad (24)$$

where $y_{1r}(t)$ and $y_{2r}(t)$ are a solution of the reference model

$$\begin{bmatrix} \dot{y}_{1r} \\ \dot{y}_{2r} \end{bmatrix} = \begin{bmatrix} -2y_{1r} + y_{2r} \\ y_{1r} - y_{2r} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_r \quad (25)$$

and the initial conditions in (25) are arbitrary but fixed. The aim (24) can be reformulated by introducing the state vector

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 - y_{1r} \\ y_2 - y_{2r} \end{bmatrix} \quad (26)$$

Then the synchronisation error can be expressed as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u - v_r) + \begin{bmatrix} 0 & 0 \\ (\frac{1}{2}\cos^2(x_1 + y_{1r}) - 1) & (4 + \sin(x_2 + y_{2r})) \end{bmatrix} \cdot \begin{bmatrix} x_1 + y_{1r} \\ x_2 + y_{2r} \end{bmatrix} \quad (27)$$

where the components $1/2\cos^2(x_1 + y_{1r})$ and $3 + \sin(x_2 + y_{2r})$ are assumed to be unknown. The obtained equation is in a form for which the trajectory approximation algorithms presented in this paper are directly applicable. The identification step can be performed exactly as (11). We are now in a position to formulate the appropriate control laws for (27).

5.1 HM control law synchronisation

A candidate for $f_m(\mathbf{x})$ is

$$f_m(\mathbf{x}) = \begin{bmatrix} -2 & 1 \\ a_1^* & a_2^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \quad (28)$$

In this example a_1^* and a_2^* are set to $1, -1$ respectively. It can be seen that $\dot{\mathbf{x}} = f_m(\mathbf{x})$ is asymptotically stable and the "ideal" control law which satisfies (4) can be shown to be equal to

$$\begin{aligned} u_m(\mathbf{x}) &= -v_r + (1 - (1/2)\cos^2(x_1 + y_{1r}))(x_1 + y_{1r}) \\ &\quad -(4 + \sin(x_2 + y_{2r}))(x_2 + y_{2r}) + x_1 - x_2 \quad (29) \end{aligned}$$

This control law cannot be used because it depends on unknown plant parameters. Hence the hard matching control law was chosen according to (15).

The simulation was performed for the following sets of parameters. The update interval was $\Delta = 0.01s$, control excitation $\kappa = 0.01$ and $\beta = 0.01$. The plant initial conditions were set to $y_1 = -0.1$ and $y_2 = 0.1$. Our aim is to synchronise this plant with the reference model, initial conditions $y_{1r} = 0$ and $y_{2r} = 0.1$ and $v_r = 0.1$. Figure(2) displays the phase diagram of synchronisation error where x_2, the velocity synchronisation error is plotted against x_1.

5.2 SM control law synchronisation

The soft matching control law can also be used to control the system (27). As outlined in the previous sections we utilised the following two variants: Sontag's form (16) or speed gradient (17). More references can be found in [11],[12]. For both of these variants an appropriate goal function needs to be determined. In this simulation example the goal function investigated was

$$Q(\mathbf{x}) = x_1^2 + x_2^2 \tag{30}$$

It can be shown that $Q(\mathbf{x})$ is a control Lyapunov functions. Our aim as with the hard matching condition is to synchronise the plant with the reference model.

Figure [3]displays results obtained using control law (16) with goal function Q for plant and reference models with parameters similar to the hard matching condition. As is demonstrated in figure [3] the system exhibits order epsilon chattering behaviour. This chattering behaviour can be attributed to the following three factors. Firstly errors in the estimates of the dynamics of the plant which are specified in our algorithm by $\hat{f}$ and $\hat{g}$. Secondly when $b(\mathbf{x}_{2k}) < \beta$ the control input is prematurely switched off, since $b(\mathbf{x}) \neq 0$ there is no requirement that $a(\mathbf{x}) < 0$, the plant begins to lose synchronisation until the next control input update. The chattering effect can be controlled via the selection of Δ or the control excitation level κ.

Figure [4] displays the results obtained using the speed gradient approach, with goal function Q, for plant and reference models with parameters similar to those in the hard matching section with the exception that $\beta = 0$. For this simulation the value of γ was set to 100. Increasing γ increases the region of stability. In our simulations it was found that if γ is increased, the update interval must be decreased. Through our simulations we found that the update interval should be set in the order of $\Delta\gamma \leq 1$ otherwise the system may exhibit large undesirable oscillations.

6 Conclusions

In this paper we consider uncertain twice continuously differentiable nonlinear systems affine in the input. It is shown that under certain matching conditions these systems can be made globally asymptotically stable by state feedback. We presented adaptive algorithms which render the origin practically stable to any desired degree under both "hard" and "soft" matching conditions. The algorithms presented were demonstrated on a simulation example. It was shown that if an asymptotically stable reference is known then the hard matching control law could be used yielding very satisfactory results. If no such reference model exists then a goal function is required. It was shown that this goal function must be a control Lyapunov function.

7 Acknowledgements

This work was supported by the Centre for Sensor Signal and Information Processing and the Australian Research Council.

References

[1] H. Nijmeijer and Van der Schaft. *Nonlinear dynamical control systems.* Springer-Verlag, New York, 1990.

[2] A. Isidori. *Nonlinear Control Systems.* Springer-Verlag, London, third edition, 1994.

[3] I. Kanellakopoulos, P. Kokotovic, and A.S. Morse. Systematic design of adaptive controllers for feedback linearizable systems. *IEEE Trans. on Aut. Contr.*, 36:1241–1253, 1991.

[4] R. Marino and P. Tomei. Global adaptive output-feedback control of nonlinear systems, part i: linear parameterization. *IEEE Trans. on Aut. Contr.*, 38:17–32, 1992.

[5] M. Kristic, P. Kokotovic, and I. Kanellakopoulos. Adaptive nonlinear output-feedback control with an observer based identifier. In *Proceedings of the American Control Conference*, pages 2821–2835, San Francisco, 1993.

[6] I.M.Y. Mareels, H.B. Penfold, and R.J. Evans. Controlling nonlinear time varying systems via euler approximations. *Automatica*, 28:681–696, 1992.

[7] H.B. Penfold, I.M.Y. Mareels, and R.J. Evans. Adaptively controlling nonlinear systems using trajectory approximations. *Int. Journal of Adapt. Contr. Signals*, 6:394–411, 1993.

[8] A.S. Kulnich and G.D. Penev. Parametric optimization of multi-link system equations of motion and adaptive control algorithm. *Automation and Remote Control*, pages 1793–1803, 1979.

[9] V.Y. Tertychnyi. Synthesis of adaptive systems for stabilizing nonlinear dynamic plants using integral transform. *Automation and Remote Control*, pages 420–429, 1992.

[10] J. Dieudonee. *Foundations of Mathematical Analysis.* Academic Press, New York, 1978.

[11] E.D. Sontag. A 'universal' construction of artstein's theorem on nonlinear stabilization. *Systems and Control Letters*, 13:117–123, 1989.

[12] A.L. Fradkov. Speed gradient scheme and its application in adaptive control. *Automation and Remote Control*, 40:1333–1342, 1979.

[13] A.L. Fradkov, P.Yu. Guzenko, D.J. Hill, and A.Yu. Pogromsky. Speed-gradient control and passivity of nonlinear oscillators. In *IFAC Symposium on Nonlinear Control Systems.*, pages 660–665, Tahoe City, CA, 1995.

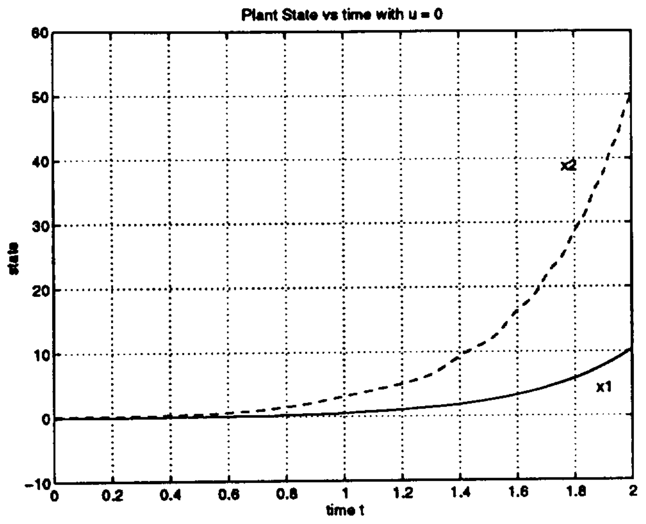

Figure 1: Evolution of the states for uncontrolled plant with initial conditions $x_1 = -0.1$ and $x_2 = 0.1$

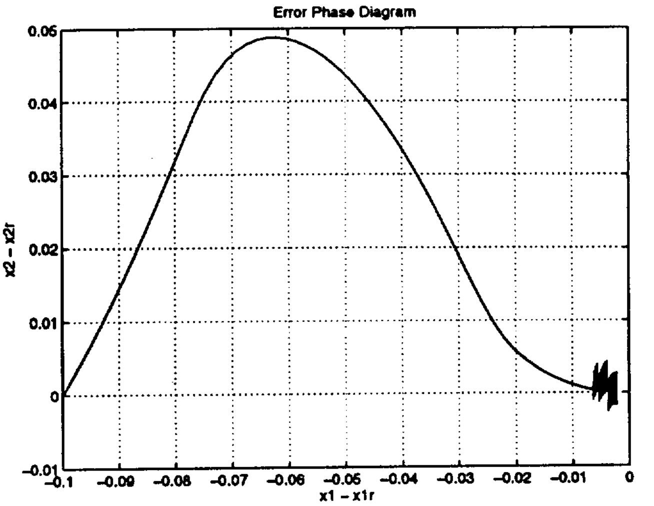

Figure 3: Phase diagram of synchronisation error under SM control with goal function Q.

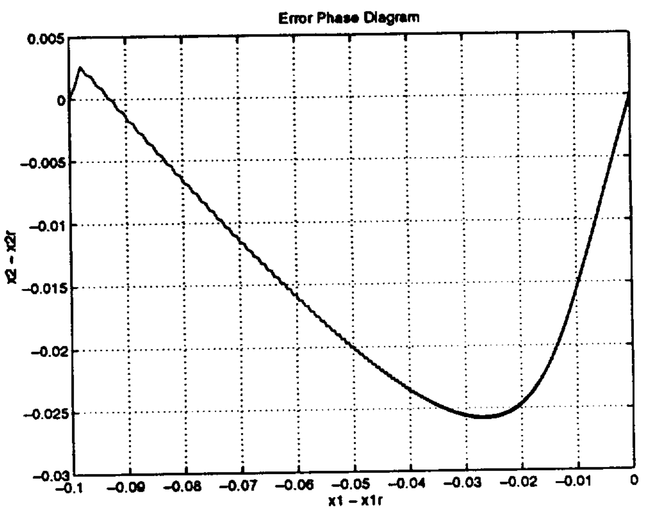

Figure 2: Phase diagram of synchronisation error under HM control.

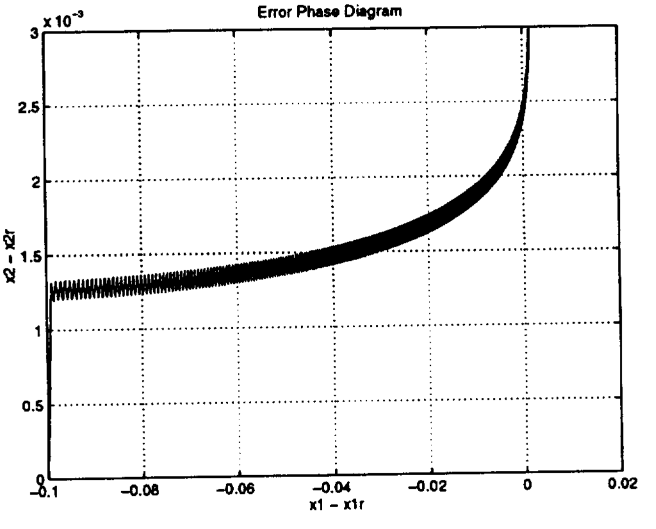

Figure 4: Phase diagram of synchronisation error under SG control with goal function Q.