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Stabilization of Invariant Sets of Cascaded Nonlinear Systems[1]

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Abstract. In this paper the problem of invariant set stabilization for cascaded nonlinear system is considered. New stabilizability conditions and new feedback control laws are suggested. The energy stabilization problem for controlled pendulum with actuator is examined in detail.

1 Introduction

An interest in nonlinear control problems has been continuously growing during last decade. It was motivated both by growing maturity of nonlinear control theory and by applications (control of mechanical systems, biochemical processes, *etc*). More attention has been recently attracted by control of oscillations (see, e. g. [4]).

The control of oscillations problems has some specific features compared with conventional problems of stabilization and tracking. Namely the control goal is formulated as approaching some manifold rather than a prescribed point (position) or a curve (trajectory). It corresponds to the transition from a conventional stability to partial stability. Another feature is a "small control" requirement. The existing methods of the partial stabilization (e. g. [15]) use mainly the feedback linearization technique that often leads to the local results and makes it difficult to meet the "small control" requirement. Another approach based on passivity was proposed in [3] for the problem of stabilization of a number of first integrals of the unforced Hamiltonian system. It was extended to more general problems in [4, 12].

The main difficulty of the partial stabilization algorithm design is to find constructive conditions localizing the limit set of the closed loop system. Usually the desired limit set is contained in $\mathcal{D}_0 = \{x : V(x) = 0\}$, where $V \geq 0$ is a given goal function. Obviously, $\mathcal{D}_0$ does not necessarily coincide with $\mathcal{D}_1 = \{x : \dot{V}(x) = 0\}$. In this case, standard arguments based on La Salle principle require the evaluation of the largest invariant set containing in $\mathcal{D}_1$. Therefore they do not provide explicit conditions ensuring that the trajectories of the closed loop system approach the set $\mathcal{D}_0$. On the other hand the existing conditions of stability of the sets (e. g. [9]) assume that the limit set is known which is not case in our problem.

The above obstacle was overcome in [3, 13] based on the concept of V-detectability [12], which extends the zero-state detectability, see [2], to the case of the partial stabilization.

In the present paper the results of [3, 13, 12] are extended to the class of cascaded systems. Important results concerning with the stabilization of the cascaded systems were established previously in [1, 14, 7, 11, 10, 2, 6, 8]. In [6, 8] a systematic design of stabilizing controllers for nonlinear cascades, the so called "integrator backstepping", was proposed. However the results of [6, 8] related to the set stabilization problems (e. g. Lemma 2.8, p. 34, and Corollary 2.22, p. 51, of [8]) state convergence to the largest invariant set $\mathcal{M}$ contained in the set $\{(x, v) : \dot{V}(x) = 0, v = 0\}$. The evaluation of the set $\mathcal{M}$ is a separate problem. Even the establishing the relation $\mathcal{M} \subset \{(x, v) : V(x) = 0\}$, meaning the achievement of the control goal, needs special consideration.

The main results of the present paper (Theorems 3.1, 3.4) together with the sufficient conditions of V-detectability (Propositions 2.3–2.5) give constructive

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conditions ensuring that the limit set $\mathcal{M}$ is inside the set $\{(x,v) : V(x) = 0\}$, i. e. the goal is achieved.

Theorems 3.1, 3.4 hold for fully nonlinear cascades. Their application, in view of Propositions 2.3–2.5, requires the evaluation of rank of some distribution depending on the system to be controlled. Application of Theorems 3.1, 3.4 is demonstrated by example of swinging the pendulum with actuator.

2 Problem Statement and Preliminaries

We will consider a cascaded nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)v, && (2.1)\\ \dot{v} &= a(x,v) + b(x,v)u, && (2.2)\\ y &= h(x), && (2.3) \end{aligned}$$

where $x \in \mathbf{R}^n$, $v \in \mathbf{R}^m$ are state vectors of subsystems, $u \in \mathbf{R}^m$ is control input vector, $y \in \mathbf{R}^m$ is output vector; $f(x)$, $g(x)$, $a(x,v)$, $b(x,v)$ are smooth vector fields of appropriate dimensions. It is assumed that in the state space of the system (2.1) a goal set $\mathcal{S}$ is defined and that this set $\mathcal{S}$ is contained in the set of zeros of known smooth scalar function V, i. e. $\mathcal{S} \subset \{x : V(x) = 0\}$. The problem is to find output feedback regulator and sufficient conditions which guarantee the control goal

$$\lim_{t\to+\infty} V(x(t)) = 0. \quad (2.4)$$

In addition, it will be assumed that the subsystem (2.1), (2.3) is passive (or passifiable by some smooth feedback) with the storage function V. For the completeness recall the notions.

Definition 2.1 [1]Let $V(x)$ be a C^r, $r \geq 1$, smooth nonnegative function. The system (2.1), (2.3) is said to be *passive* with storage function V if

$$\begin{aligned} L_f V(x) &\leq 0, \quad \forall x \in \mathbf{R}^n && (2.5)\\ L_g V(x) &= h(x)^T, \quad \forall x \in \mathbf{R}^n. && (2.6) \end{aligned}$$

Definition 2.2 ([12]) System (2.1), (2.3) is said to be *locally V-detectable* if there exists positive number $c > 0$ such that for all $x_0 \in \mathcal{U} = \{x \in \mathbf{R}^n : V(x) < c\}$

$$h(\Phi(t,x_0,0)) = 0\ \forall t \geq 0 \Rightarrow \lim_{t\to+\infty} V(\Phi(t,x_0,0)) = 0,$$

where $\Phi(t,x_0,0)$ is a solution of subsystem (2.1) with $v(t) \equiv 0$. If $\mathcal{U} = \mathbf{R}^n$, the system (2.1), (2.3) is *V-detectable.*

[1]This definition is just the differential version of the standard one [2]

As it will be shown the V-detectability condition of the system (2.1), (2.3) plays important role in the solution of stabilization problem (2.4), but this notion is nonconstructive and requires an additional checkable criterion. In the following statements some conditions, which imply V-detectability, are presented. Introduce the set

$$\begin{aligned} S = \{x \in R^n : L_f^m L_\tau V(x) = 0,\\ \text{for all} \quad \tau \in D,\ 0 \leq m \leq r-1\}, \end{aligned} \quad (2.7)$$

where D is a distribution defined by the relation

$$D = \text{span}\{ad_f^k g_i, 0 \leq k \leq n-1, 1 \leq i \leq m\}$$

where g_i, $1 \leq i \leq m$ are the vector components of the smooth vector field $g(x)$. Recall that a nonnegative function V is said to be *proper* if $\forall c \geq 0$ the set $V_c = \{x \in \mathbf{R}^n : V(x) \leq c\}$ is compact.

Proposition 2.3 ([12]) *Suppose that the system (2.1), (2.3) is passive with C^r, $r \geq 1$, smooth proper nonnegative function V. If the set*

$$S \setminus \{x \in \mathbf{R}^n : V(x) = 0\} \quad (2.8)$$

does not contain any whole trajectory of the unforced system $\dot{x} = f(x)$ then the system (2.1), (2.3) is V-detectable.

To formulate another criterion of V-detectability, assume that the C^r-smooth scalar function $V(x)$, $r \geq 1$, admits the factorization

$$V(x) = \frac{1}{2}|w(x)|^2, \quad (2.9)$$

where $w : R^n \to R^l$, $l \leq n$, is a C^r-smooth vector function. Introduce the distribution $\hat{S}(x)$ as follows

$$\hat{S}(x) = \text{span}\{L_f^k L_g w(x), \quad k = 0,1,\ldots,r-1\}. \quad (2.10)$$

It is easy to see that the dimension of distribution $\hat{S}(x)$ cannot be greater than l for any point $x \in R^n$. Denote the set

$$\begin{aligned} \Lambda_1 = \{x \in \mathbf{R}^n :\ (L_f w(x))^T L_f^k L_g w(x) = 0,\\ k = 0,1,\ldots,r-1\}, \end{aligned}$$

and consider its subset P_1, where the distribution $\hat{S}(x)$ has the maximal dimension, i. e.

$$P_1 = \{x \in \Lambda_1 : \dim \hat{S}(x) = l\}.$$

Let a set Ω_1 be defined as follows

$$\begin{aligned} \Omega_1 = \{x_0 \in X = \mathbf{R}^n \setminus (P_1 \cup \{x \in \mathbf{R}^n : V(x) = 0\}) :\\ \text{the whole trajectory } x = x(t,x_0) \text{ of (2.1)}\\ \text{with } v = 0 \text{ lies in } X\}. \end{aligned} \quad (2.11)$$

Proposition 2.4 ([12]) *Suppose that the system (2.1), (2.3) is passive with a proper C^r, $r \geq 1$, smooth function V which can be represented in the form (2.9). If the set Ω_1 is empty then the system (2.1), (2.3) is V-detectable.*

Denote the sets

$$\Lambda_2 = \{x \in R^n : w(x)^T L_f^k L_g w(x) = 0, k = 0, 1, \ldots, r-1\},$$

$$P_2 = \{x \in \Lambda_2 : \dim \hat{S}(x) = l\},$$

$$\Omega_2 = \{x_0 \in X = \mathbf{R}^n \setminus (P_2 \cup \{x \in \mathbf{R}^n : V(x) = 0\}) : \text{the whole trajectory } x = x(t, x_0) \text{ of (2.1) with } v = 0 \text{ lies in } X\}. \quad (2.12)$$

Proposition 2.5 ([12]) *Suppose that the system (2.1), (2.3) is passive with a proper C^r, $r \geq 1$, smooth function V which can be represented in the form (2.9). If the set Ω_2 is empty then the system (2.1), (2.3) is a V-detectable.*

3 Main results

3.1 Case $a(v, x) \equiv 0$, $b(v, x) \equiv I_m$

First consider a special case of problem, namely, suppose that the system of interest has the form

$$\dot{x} = f(x) + g(x)v, \quad (3.1)$$
$$\dot{v} = u, \quad (3.2)$$
$$y = h(x), \quad (3.3)$$

that is the first system in cascade is the block of integrators.

Theorem 3.1 *Suppose that the system (3.1), (3.3) is passive with C^r, $r \geq 1$, smooth proper storage function V. If the system (3.1), (3.3) is V-detectable then for any solution of the closed loop system (3.1), (3.2), (3.3) with the regulator*

$$u = -Pv - y, \quad (3.4)$$

where P is an arbitrary positive definite $m \times m$ matrix, $P = P^* > 0$, the goal relation (2.4) is valid. Moreover,

$$\lim_{t \to +\infty} |v(t)|^2 = 0. \quad (3.5)$$

Proof. Fix a regulator of the form (3.4) and consider the solution $[x(t), v(t)] = [x(t, x_0, v_0), v(t, x_0, v_0)]$ of the closed loop system (3.1)–(3.4). Take the Lyapunov function candidate as

$$W(x, v) = \frac{1}{2}|v|^2 + V(x). \quad (3.6)$$

Its derivative along the trajectory $[x(t), v(t)]$ has the form

$$\frac{d}{dt}W(x, v) = \frac{\partial V}{\partial x} f(x) - v^T P v \leq 0. \quad (3.7)$$

By assumption the function W is nonnegative and proper. Therefore the trajectory $[x(t), v(t)]$ possesses the non-empty ω-limit set Ω, which is closed and consists of whole trajectories of the closed loop system (3.1)–(3.4), see [5].

Take an arbitrary point $[\hat{x}_0, \hat{v}_0] \in \Omega$ and consider the trajectory $\hat{x}(t) = \hat{x}(t, \hat{x}_0)$, $\hat{v}(t) = \hat{v}(t, \hat{v}_0)$ of (3.1), (3.2), (3.3). Along this trajectory

$$\frac{d}{dt}W(\hat{x}(t), \hat{v}(t)) = \frac{\partial V(\hat{x}(t))}{\partial x} f(\hat{x}(t)) - \hat{v}(t)^T P \hat{v}(t) = 0.$$

Due to the passivity of (3.1), (3.3) and the positive definiteness of P, one has that $\forall t \geq 0$

$$\frac{\partial V(\hat{x}(t))}{\partial x} f(\hat{x}(t)) = 0 \quad \text{and} \quad \hat{v}(t) = 0.$$

Differentiating the last equality, one concludes that for all $t \geq 0$

$$0 = \dot{\hat{v}}(t) = -P\hat{v}(t) - y(t) = -y(t).$$

Then due to V-detectability assumptions along the trajectory $[\hat{x}_0, \hat{v}_0]$ the relation

$$\lim_{t \to +\infty} V(\hat{x}(t)) = 0$$

holds. Therefore the inclusion $(\hat{x}_0, \hat{v}_0) \in \Omega$ implies $\hat{v}_0 = 0$ and $V(\hat{x}_0) = 0$. This means that for trajectory $x(t) = x(t, x_0, v_0)$ the goal limit equality

$$\lim_{t \to +\infty} V(x(t)) = 0$$

is fulfilled. Theorem 3.1 is proved. ■

The combination of Theorem 3.1 and one of Propositions 2.3–2.5 leads to the following results.

Theorem 3.2 *Suppose that the system (3.1), (3.3) is passive with C^r, $r \geq 1$, smooth proper storage function V. If the set*

$$S \setminus \{x \in \mathbf{R}^n : V(x) = 0\}$$

does not contain any whole trajectory of the unforced system $\dot{x} = f(x)$. Then for any solution $[x(t), v(t)]$ of the closed loop system (3.1)–(3.3) with the regulator (3.4) the goal relation (2.4) is valid. Moreover, $\lim_{t \to +\infty} |v(t)|^2 = 0$.

Theorem 3.3 *Suppose that the system (3.1), (3.3) is passive with C^r, $r \geq 1$, smooth proper storage function V. If the set Ω_2 (or Ω_1) is empty, then for any solution $[x(t), v(t)]$ of the closed loop system (3.1)–(3.3) with the regulator (3.4) the goal relation (2.4) is valid. Moreover, $\lim_{t \to +\infty} |v(t)|^2 = 0$.*

3.2 General case

Theorem 3.4 *Consider the system (2.1), (2.2), (2.3) under the following assumptions:*
1). the 2nd subsystem is passifiable, i. e. there exists smooth locally bounded function $\alpha(x)$ *such that*

$$\frac{\partial V}{\partial x}[f(x)+g(x)\alpha(x)] \leq 0, \quad L_g V(x) = h(x)^T, \quad \forall x \in \mathbf{R}^n, \tag{3.8}$$

where $V(x)$ *is some nonnegative proper function;*
2). $\det b(v,x) \neq 0 \ \forall v, x$*;*
3). the system

$$\dot{x} = (f(x) + g(x)\alpha(x)) + g(x)v, \quad y = h(x) \tag{3.9}$$

is V-detectable.
Then for any solution $[x(t), v(t)]$ *of the closed loop system (2.1), (2.2), (2.3) with the regulator*

$$u = b(v,x)^{-1} \Big[-P(v - \alpha(x)) + \frac{\partial \alpha}{\partial x}(f(x) + g(x)v) - y - a(v,x)\Big] \tag{3.10}$$

where P is an arbitrary positive definite $m \times m$ *matrix, the goal relation (2.4) is valid. Moreover,* $\lim_{t\to+\infty} |v(t) - \alpha(x(t))|^2 = 0$.

Proof. Make the coordinate change $(x, v) \to (x, \hat{v})$, where $\hat{v} = v - \alpha(x)$. Then the system (2.1), (2.2), (2.3) takes the form

$$\begin{aligned} \dot{x} &= (f(x) + g(x)\alpha(x)) + g(x)\hat{v} = \\ &\quad \hat{f}(x) + g(x)\hat{v}, & (3.11)\\ \dot{\hat{v}} &= \hat{a}(\hat{v}, x) + \hat{b}(\hat{v}, x)u, & (3.12)\\ y &= h(x), & (3.13)\end{aligned}$$

where

$$\hat{a}(\hat{v}, x) = \left[a(v,x) + \frac{\partial \alpha}{\partial x}(f(x) + g(x)v)\right],$$

$$\hat{b}(\hat{v}, x) = b(v, x).$$

Put

$$u = \hat{b}(\hat{v}, x)^{-1}[\hat{u} - \hat{a}(\hat{v}, x)].$$

Then the system (3.11), (3.12), (3.13) is transformed to the following one:

$$\begin{aligned} \dot{x} &= \hat{f}(x) + g(x)\hat{v}, & (3.14)\\ \dot{\hat{v}} &= \hat{u}, & (3.15)\\ y &= h(x). & (3.16)\end{aligned}$$

The analysis of this system was made in Theorem 3.1, so to complete the proof it is sufficient to point out that by assumption all conditions of theorem 3.1 for the system (3.14), (3.15), (3.16) are obviously valid. ∎

Remark 3.5 As it was mentioned above the condition of V-detectability of the system (3.9) can be verified by Propositions 2.3–2.5. It is easy to formulate the analogs of Theorem 3.4 replacing the assumption of V-detectability by one of the sufficient conditions stated in Propositions 2.3–2.5.

Remark 3.6 All statements of this paper can be extended in natural way to the case when the state space of the system is n-dimensional smooth manifold and the function $V(x)$ is proper and nonnegative on this manifold. For example such a situation takes place for the problem of local (global) energy-level stabilization of Hamiltonian control systems with some periodic coordinates.

4 Example: stabilization of energy level of pendulum with actuator

In this section the stabilization problem of a given energy level for a pendulum with actuator is considered. The controlled plant model is

$$\begin{aligned} \dot{p}(t) &= -mgl \cdot \sin(q(t)) + ml \cdot v(t), & (4.1)\\ \dot{q}(t) &= \frac{1}{ml^2}p(t), & (4.2)\\ T \cdot \dot{v}(t) &= u(t) - v(t), & (4.3)\end{aligned}$$

where (q,p) are generalized coordinate and momenta of the pendulum, u is a control function, m, l are mass and length of the pendulum, g is the gravity acceleration, T is the time constant of actuator. The problem is to define the regulator and to describe the set of initial conditions such that along any solution of the closed loop system the goal relation

$$\lim_{t\to+\infty} V(q(t), p(t)) = \lim_{t\to+\infty} \frac{1}{2}[H_0(q,p) - H_*]^2 = 0, \tag{4.4}$$

is valid. Here H_* is an arbitrary nonnegative number, $H_* \geq 0$, and $H_0(q,p)$ is Hamiltonian function of the unforced (i. e. with $v = 0$) pendulum

$$H_0(q,p) = \frac{1}{2ml^2}p^2 + mgl \cdot (1 - \cos q).$$

It is convenient to choose the cylindrical phase space (q,p) with the circle of the radius l in a base, $\pi < q \leq \pi$, as a state space of the pendulum motions. On this state space the function V is proper and nonnegative. Since $H_0(q,p)$ is the first integral of the unforced pendulum, then the system (4.1), (4.2) is passive with the storage function V and the auxiliary output

$$y = \frac{1}{l} \cdot p \cdot [H_0(q,p) - H_*]. \tag{4.5}$$

However the theorem 3.1 cannot be applied directly. Indeed, the unforced subsystem (4.1), (4.2), (4.5) has two points of equilibrium $(q,p)=(0,0)$, $(q,p)=(\pi,0)$ and, therefore, for any energy level $H_* \geq 0$ the system (4.1), (4.2), (4.5) is not V-detectable. So the presented results do not globally solve the problem of any energy level stabilization of pendulum. One can hope that its local version takes place. The following statement shows that this is the case.

Proposition 4.1 *Consider the system (4.1), (4.2), (4.3), (4.5). Let $H_* \geq 0$ be given desired energy level of the unforced pendulum, and let the positive number C be defined as follows*

$$C = \frac{1}{2}\overline{\min}\{[H_* - 2mgl]^2, H_*^2\}, \qquad (4.6)$$

where

$$\overline{\min}\{a_1,a_2\} = \begin{cases} \min\{a_1,a_2\}, \text{ if } a_1>0,\ a_2>0 \\ a_1, \text{ if } a_2 = 0 \\ a_2, \text{ if } a_1 = 0 \end{cases},$$

and δ be any positive number, $\delta < C$. Then for an arbitrary initial point $(q_0,p_0) \in \Omega_{(C-\delta)} = \{(q,p) : V(q,p) < (C-\delta)\}$ with $|v_0| < \sqrt{2\delta}$ along the solution $(q(t),p(t),v(t)) = (q(t,q_0),p(t,p_0),v(t,v_0))$ of closed loop system (4.1), (4.2), (4.3), (4.5) with the regulator

$$u = -\gamma \cdot v - T\cdot y, \qquad (4.7)$$

where $\gamma > -1$, the goal relation (4.4) is valid, i. e. the pendulum motion approaches the desired energy level H_*.

Proof. Choose any point of the initial condition (q_0,p_0,v_0) such that $(q_0,p_0) \in \Omega_C$, $|v_0| < \delta$, and consider the solution $(q(t),p(t),v(t)) = (q(t,q_0),p(t,p_0),v(t,v_0))$ of the closed loop system (4.1)–(4.3), (4.5), (4.7). Along this trajectory the derivative of the function

$$W(q,p,v) = V(q,p) + \frac{1}{2}v^2$$

has the form

$$\frac{dW}{dt} = \frac{\partial V}{\partial q}\dot q + \frac{\partial V}{\partial p}\dot p + v\dot v = -\frac{\gamma+1}{T}\cdot v^2. \qquad (4.8)$$

In particular, the function $W(q,p,v)$ does not increase along the considered trajectory and $\forall t \geq 0$

$$\begin{aligned} W(q(t),p(t),v(t)) &= V(q(t),p(t)) + \frac{1}{2}v(t)^2 \\ &\leq V(q_0,p_0) + \frac{1}{2}v_0^2. \qquad (4.9)\end{aligned}$$

The function $V(q,p)$ is proper on the cylindrical state space. Hence $W(q,p,v)$ is also proper on the product of cylindrical state space and R^1. Therefore the trajectory $(q(t),p(t),v(t))$ has a non-empty ω-limit set Ω.

Take an arbitrary point $(\hat q_0,\hat p_0,\hat v_0) \in \Omega$ and consider the solution $(\hat q(t),\hat p(t),\hat v(t))$ of the closed loop system with an initial conditions in this point. The relation (4.8) implies $\hat v(t) \equiv 0$. Then

$$0 = \frac{d\hat v}{dt} = -\frac{\gamma+1}{T}\cdot \hat v - y = -y.$$

Due to the relations (4.9) and (4.6)

$$\begin{aligned} V(\hat q(t),\hat p(t)) &\leq V(q(t),p(t)) \leq V(q_0,p_0) + \frac{1}{2}v_0^2 \\ &< (C-\delta)+\delta = C.\end{aligned}$$

To complete the proof let us show that for the trajectory $(\hat q(t),\hat p(t))$ of the unforced system (4.1), (4.2), (4.5) with the initial condition $(\hat q_0,\hat p_0) \in \{(q,p) : V(q,p) < C\}$ for which $y(t)=0$, $\forall t \geq 0$, the goal relation (2.4) is valid.

Indeed, along the trajectory $(\hat q(t),\hat p(t))$ the output function $y(t)$ is equal identically to zero, $y(t)\equiv 0$. Suppose that $\lim_{t\to+\infty} V(\hat q(t),\hat p(t)) = a \neq 0$. From (4.5) one concludes that along this trajectory $\hat p(t) = 0$ for all $t \geq 0$. The unforced system (4.1), (4.2) has only two trajectories with this property, namely: $(q(t),p(t)) = (0,0)$, $(q(t),p(t)) = (\pi,0)$. By choice of constant C these equilibrium points do not belong to the set $\{(q,p) : Q(q,p) < C\}$, where the trajectory $(\hat q(t),\hat p(t))$ evolves. Thus $a=0$ and Proposition 4.1 is proved. ∎

Remark 4.2 It follows from the proof of Proposition 4.1 that for the closed loop system (4.1)–(4.3), (4.5), (4.7) any solution is bounded. Moreover, it has only three attractors: the equilibriums

$$(q,p,v) = (0,0,0), \quad (q,p,v) = (\pi,0,0)$$

and the compact invariant set

$$V_0 = \{(q,p,v) : H_0(q,p) = H_*,\ v=0\}.$$

Essentially Proposition 4.1 states, that V_0 is asymptotically stable, and describes the area of attraction of the set V_0. But it is easy to show that the regulator (4.7) will never stabilize the set V_0 in any global or semi-global sense. Indeed, one can prove by taking the linear approximation that the point $(q,p,v)=(0,0,0)$ is asymptotically stable for any regulator (4.7), while $(q,p,v) = (\pi,0,0)$ is always hyperbolic.

5 Conclusions

In this paper the problem of global asymptotic stabilization of invariant compact set for passive nonlinear cascaded system was considered. This problem is widely investigated for the special case of equilibrium point stabilization [2, 10] and others. In this paper the stabilization of invariant set, which may differ from the equilibrium, is suggested. To emphasize the novelty of our solution let us recall that following the idea of the control-Lyapunov function to stabilize the given invariant set Γ it is assumed that this desired attractor Γ is known and there exists nonnegative function V such that for all vectors x, being away from Γ, there exists control vector v with $(\partial V(x)/\partial x)f(x,v) < 0$.

In contrast to these standard assumptions we only assume that there exists the nonnegative smooth proper function V such that $\Gamma \subset \{x : V(x) = 0\}$; there exists feedback regulator which provides the nonstrict inequality $(\partial V(x)/\partial x)f(x,p(x)) \leq 0 \; \forall x$.

The contribution of this paper are the new sufficient conditions and the new stabilizing state feedback regulator which guarantees that along the closed loop system solution the value of the function V tends to zero. If the set $\{x : V(x) = 0\}$ does not contain any whole motion of the system except lying in Γ then this new regulator establishes stability of the set Γ and in this case, obviously, the function V may not serve the strict control-Lyapunov function.

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