Adaptive coding for position estimation in formation flight control ⋆

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Abstract: We propose the data processing algorithm based on adaptive coding procedure for sample data, measured by binary sensors of individual UAVs in the formation. The numerical example of following in the wake UAVs is presented, demonstrating applicability of the proposed scheme.

Keywords: Formation flight control, Position estimation, Communication network, Information capacity

1. INTRODUCTION

With the advent of modern wireless and Internet technologies, complex systems are becoming more networked, and access to information is more critical than ever. Serial communication networks are used to exchange system information and control signals between various components of the systems that may be physically distributed. *Networked control systems* (NCS) are real-time systems where sensor and actuator data are transmitted through shared or switched communication networks, see e.g. (Ishii and Francis, 2002; Goodwin et al., 2004; Abdallah and Tanner, 2007; Matveev and Savkin, 2008; Andrievsky et al., 2010).

Cooperative control of a collection of vehicles performing a shared task using intervehicle coordinating communication is considered in (Fax and Murray, 2004), where a method for decentralized information exchange between vehicles is proposed. The approach of (Fax and Murray, 2004) realizes a dynamical system that supplies each vehicle with a common reference to be used for cooperative motion. Adaptive formation control algorithms, compensating uncertain leader commands for two-dimensional point-mass models of aircraft dynamics are proposed in (Bošković et al., 2002).

The role of the tactical network operations communication coordinator in mobile UAV networks is studied in (Jeoun, 2004). An adaptive, mobile network with UAV relays is well-suited to support the ad hoc nature of special operations. The increasing utilization of special operations forces in ad hoc, dynamic operations poses a need for adaptable communications to support the unit. Effective communication within the unit and critical information exchange with the command center affect the overall outcome of the mission.

Due to the digital nature of the communication channel, every transmitted signal is quantized to a finite set (Ishii and Francis, 2002). Hence, we argue that the finite set nature of the data should be explicitly taken into account in the design of cooperative flight control NCS. Recently the limitations of estimation and control under constraints imposed by a finite capacity information channel have been investigated in detail in the control theoretic literature, see (Wong and Brockett, 1997; Nair and Evans, 2003; Nair et al., 2004; Bazzi and Mitter, 2005; Nair et al., 2007; Andrievsky et al., 2010; Andrievsky and Fradkov, 2010) and references therein. It has been shown that stabilization of linear systems under information constraints is possible if and only if the capacity of the information channel exceeds the entropy production of the system at the equilibrium (*Data Rate Theorem*) (Nair and Evans, 2003, 2004; Nair et al., 2004). In (Touchette and Lloyd, 2004) a general statement was proposed, claiming that the difference between the entropies of the open loop and the closed loop systems cannot exceed the information introduced by the controller, including the transmission rate of the information channel. For nonlinear systems only a few results are available in the literature (Liberzon, 2003; Nair et al., 2004; De Persis, 2005, 2006; De Persis and Nešić, 2005; Savkin and Cheng, 2007). In the above papers only the problems of stabilization to the point are considered. In a number of engineering applications (e.g. in distributed sensor networks, or remote surveillance systems), there is no possibility to mount advanced measurement/estimation devices on the transmitter (target) side (Liberzon, 2003; Nair and Evans, 2003; Matveev and Savkin, 2004; La Scala and Evans, 2005; Evans et al., 2005; Malyavej et al., 2006). In these cases only measurements of some scalar output variable of the transmitter system are available. Such a problem was studied in (Fradkov et al., 2006), where results on observer-based synchronization of chaotic systems, represented in the Lurie form are given, and optimality of the binary coding for coders with one-step memory is established. In the present paper the results of (Fradkov et al., 2006) are applied to the problem of state

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estimation via the limited-band communication channel for transmission of position information. The system considered consists of sensor node, which contains a tracking filter and encoder, bandwidth limited communication channel, and receiver node with a decoder. Some simplifying assumptions are made in the paper. It is assumed that the sensor uses linear Kalman filter for tracking, and that the receiver knows all the sensor parameters. Further simplifications are achieved by assuming that there are no clutter measurements, that probability of detection equals one, and thus there are no data association issues. The source (sensor) node performs a simple Kalman filter based tracking. The channel is assumed bandwidth limited, but otherwise noiseless. In the present paper the results of (Mušicki et al., 2006) are expanded to *adaptive* coding.

A communication scheme minimizing transmission data rate over bandwidth limited communication channel by sending encrypted innovations is proposed in (Andrievsky, 2007), where the observer-based full-order adaptive coding procedure of (Andrievsky et al., 2007; Fradkov et al., 2010) is applied for target position transmission with data-rate limitations. The adaptive tuning procedure for the coder range parameter is proposed and its efficiency for target tracking in the case of channel data rate limitations is demonstrated. In the present paper this scheme is used for designing a local navigation system of individual vehicles in the formation. Reducing the data rates in formation may be helpful for the sake of security, noise immunity and energy saving.

The paper is organized as follows. The data transmission scheme is described in Section 2. Its application to UAV formation flight control is given in Section 3. Concluding remarks and the future research work intentions are given in Section 4. Theoretical background of the applied state estimation scheme is presented in more details in (Fradkov and Andrievsky, 2009).

2. DATA TRANSMISSION SCHEME

2.1 Process model

Consider the following LTI discrete-time process model

$$x_{k+1}=Ax_k+E\varphi_k, \quad y_k=Cx_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the process state vector, $y_k \in \mathbb{R}$ is the scalar signal, measured by the sensor, $k = 0, 1, \ldots$. The matrices A, C, E are assumed to be known both at the transmitter and the receiver nodes, the sensor output y_k is subjected to coding/decoding procedure to be transmitted over the communication channel with limited capacity. The *external disturbance* vector $\varphi_k \in \mathbb{R}^s$ is an irregular process, unmeasured by the sensor. The pair (A, C) is assumed to be observable.

In the paper (Fradkov et al., 2006) the properties of observer-based synchronization for Lurie systems over a limited data rate communication channel with a one-step memory time-varying coder are studied. It is shown that an upper bound on the limit synchronization error is proportional to a certain upper bound on the transmission error. Under the assumption that a sampling time may be properly chosen, optimality of binary coding in the sense of demanded transmission rate is established, and the relationship between synchronization accuracy and an optimal sampling time is found. It is worth to mention that the tight data-rate bound for stabilizability of a scalar system was given in (Baillieul, 1999). In this article is shown that even for the *binary control* the bound is achievable. The linear plant stabilization problem was further considered in (Li and Baillieu, 2004), where it was shown that for the first-order linear plant, binary control is the most robust control strategy under varying data-rate constraint and asynchronism of sampling and control actuation. The paper (Fradkov et al., 2006) deals with synchronization problem of nonlinear n-th order systems. On the basis of the mentioned results, the present paper deals with a *binary* coding procedure.

2.2 Coding procedure

Consider the memoryless (static) binary quantizer to be a discretized map $q : \mathbb{R} \to \mathbb{R}$ as

$$q(y, M) = M \operatorname{sign}(y), \tag{2}$$

where $\operatorname{sign}(\cdot)$ is the *signum* function: $\operatorname{sign}(y) = 1$, if $y \geq 0$, $\operatorname{sign}(y) = -1$, if $y < 0$. Parameter M may be referred to as the *quantizer range*. Notice that for a binary coder each codeword symbol contains one bit of information. Therefore the transmission rate is $R = 1/T_s$, where T_s is a *sampling time*. The discretized output of the considered quantizer is given as $\bar{y} = M \operatorname{sign}(y)$. We assume that the coder and decoder make decisions based on the same information. The output signal of the quantizer is represented as a one-bit information symbol from the coding alphabet $\mathcal{S}$ and transmitted over the communication channel to the decoder.

In *time-varying quantizers* (Brockett and Liberzon, 2000; Liberzon, 2003; Tatikonda and Mitter, 2004; Fradkov et al., 2006; Nair et al., 2007) the range M is updated with time and different values of M are used at each step, $M = M_k$. Using such a “zooming” strategy it is possible to increase coder accuracy in the steady-state mode and at the same time, to prevent coder saturation at the beginning of the process (Brockett and Liberzon, 2000). Zooming is effective for coders with *memory* (Liberzon, 2003). To describe this kind of coders, introduce the sequence of *central numbers* c_k, $k = 0, 1, 2, \ldots$ with initial condition $c_0 = 0$. At step k the coder compares the current measured output y_k with the number c_k, forming the deviation signal $\partial y_k = y_k - c_k$. Then this signal is discretized with a current $M = M_k$ according to (2). The output signal

$$\bar{\partial} y_k = M_k \operatorname{sign}(\partial y_k) \tag{3}$$

is transmitted over the communication channel to the receiver. Then the central number c_{k+1} and the range parameter M_k are renewed based on the available information about the driving system dynamics. The following update algorithm is used in one-step memory coders:

$$c_{k+1} = c_k + \bar{\partial} y_k, \quad c_0 = 0, \quad k = 0, 1, \ldots. \tag{4}$$

The values of M_k may be precomputed (the *time-based* zooming), or, alternatively, current quantized measurements may be used at each step to update M_k (the *event-based* zooming).

2.3 Adaptive coding

In the present work the *adaptive tuning* of a coder, proposed in (Andrievsky et al., 2007; Andrievsky, 2007; Fradkov et al., 2010), is used: if the coder is not saturated, the quantizer range M is exponentially decreased; in the case of saturation the quantizer range M is increased. Note that the binary coder saturation leads to a chain of identical bits at the coder output. Therefore the moving average of the output signal may be used as an indicator of saturation. Such an adaptive tuning makes it possible to maintain the minimal range M and, at the same time, to prevent fail in tracking the signal y_k due to saturation. Formerly the adaptive quantizer with a memoryless (static) coder was proposed in (Goodman and Gersho, 1974). The adaptive coding for a special case of first-order system is analyzed in (Gomez-Estern et al., 2007).

The proposed method for adaptive tuning the quantizer range M_k realizes the following recurrent algorithm:

$$\lambda_k = (\bar{\partial} y_k + \bar{\partial} y_{k-1})/2,$$
$$M_{k+1} = m + \begin{cases} \rho M_k, & \text{if } |\lambda_k| \le 0.5 \\ M_k/\varrho, & \text{otherwise,} \end{cases} \qquad (5)$$

where $0 < \varrho \le 1$ is a decay parameter; $m = (1-\rho)M_{\min}$, $M_{\min}$ assigns the possible minimal value for M_k, $k = 1, 2, \ldots$. Unlike the case of time-based zooming (Liberzon, 2003), the initial value $M_0 > 0$ can be chosen arbitrarily, since it is adjusted during the zoom-out stage. In practice the value M_0 should correspond to the uncertainty region for y_0. The procedure (5) leads to time-based decreasing of M_k while signs of the successive values of ∂y_k alternate. The sort of discrete-time sliding-mode tracking the plant output appears in that case, and M_k recursively tends to the limiting value $M_{\min}$. When the moving average of the transmission error exceeds the threshold, the second alternative of algorithm (5) is realized, and the quantizer range M_k increases.

Equations (3), (5) describe the coder algorithm. The same algorithm is realized by the decoder. Namely, the decoder calculates the variables $\tilde{c}_k$, $\tilde{M}_k$ based on received codeword flow similarly to c_k, M_k.

The described observation procedure may be effectively used for transmission of the observations over the limited-band communication channel. Firstly, observer generates estimates of the unmeasured state variables. Secondly, implementation of the state observation algorithm in the data transmission procedure, leads to the full-order memory coder/decoder pair, which makes possible, in the ideal case, to achieve vanishing the transmission error. It should be noticed that because the coder and decoder use the same information, the observer should be realized both at the coder and decoder nodes. Despite the measurements are assumed to be accurate at the transmitter end, the observer, implemented at this end should use the quantized signal in the feedback, making possible to follow the state estimation procedure at the decoder node.

Let us apply the following full-order (Kalman) observer to the process model (1)

$$\hat{x}_{k+1} = A\hat{x}_k + L_k \bar{\partial} y_k, \quad \hat{y}_k = C\hat{x}_k, \qquad (6)$$

where the error signal $\bar{\partial} y_k$ is defined by (3), where $\partial y_k = y_k - \hat{y}_k$, $L_k \in \mathbb{R}^{n\times 1}$ is the observer matrix gain (column vector), $k = 0, 1, \ldots$.

3. UAV FORMATION FLIGHT CONTROL IN THE COMMUNICATION CONSTRAINTS CASE

We propose to use the data transmission algorithm of Sec. 2 for navigation of the individual vehicles in the formation. It is assumed that the relative position measurement between UAVs in the formation is made at the sample instants kT_s $(k = 0, 1, \ldots)$ by a coarse (binary) sensor. The measured data are processed by the algorithm (2), (3), (5) for obtaining information about UAV relative position with the desirable accuracy to be used at the individual UAV autopilots for ensuring the desired formation.

Consider the following numerical example. Three UAVs should follow in the wake with a prescribed distance 25 m. The UAV # 1 is a *leader*, its altitude $h_1(t)$ and the side position $z_1(t)$ should be reproduced by the *followers* (UAVs # 2, 3). The cruise ground-speed of UAVs is 250 m/s, cruise altitude h is 6 km. Initial coordinates of the UAVs are following: $x_1(0) = 70$ m, $x_2(0) = 0$ m, $x_3(0) = -70$ m, $h_1(0) = 6000$ m, $h_2(0) = h_3(0) = 6100$ m. Starting at $t_s = 100$ s, UAV #1 ascends to the altitude 6500 m, following the exponential reference signal. The relative positions $\Delta x_i(t) = x_i(t) - x_{i-1}(t)$, $\Delta h_i(t) = h_i(t) - h_{i-1}(t)$ $(i = 2, 3)$ are measured by the binary sensors of UAVs # 2,3 with the sample time $T_s = 0.02$ s (note that the data rate is 50 bit/s for each channel). The quantized errors and used in the algorithm (2), (3), (5) to restore the true relative positions of the UAVs. The simulation results are presented in Fig. 1, showing the desired behavior of UAVs in the formation.

4. CONCLUSIONS

The data processing algorithm based on adaptive coding procedure for sample data, measured by binary sensors to be employed by individual UAVs in the formation is proposed. The numerical example of following in the wake UAVs is presented, demonstrating applicability of the proposed scheme. Future research work is intended for consideration other kinds of flight formation and taking into account measuring errors, channel noise, delays and dropout.

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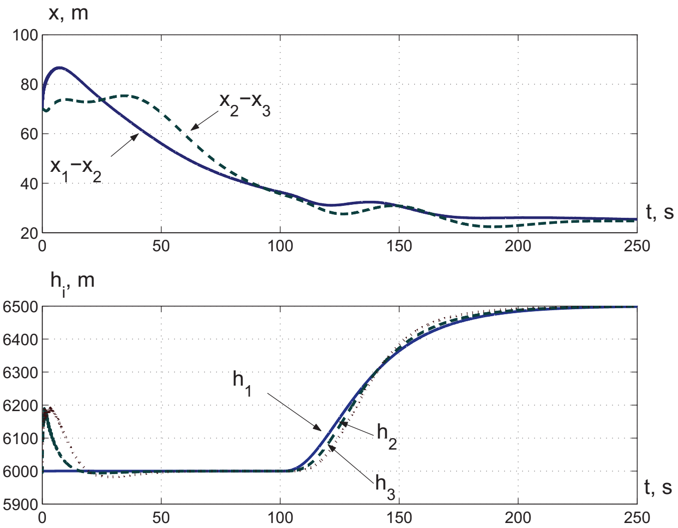

Fig. 1. UAVs trajectories in the formation.

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