Passification-based robust flight control system design ⋆

Alexander L. Fradkov and Boris Andrievsky *

* *Institute for Problems of Mechanical Engineering of RAS and St. Petersburg State University of IT, Mechanics and Optics, Saint Petersburg, Russia*
e-mail: {alexander.fradkov, boris.andrievsky}@gmail.com

Abstract: A robust autopilot for attitude control of the flexible aircraft under parametric uncertainty is designed. The high gain controller with forced sliding motions is used to secure adaptability in the wide region of the aircraft model parameters. The shunting method is applied to ensure the closed-loop system stability in the face of the lack of the aircraft state information. The sequential reference model is used to assign the desired closed-loop system performance. An example illustrating a typical design procedure for aircraft attitude control in the horizontal plane for different flight conditions is given. The system robustness with respect to uncertain plant parameters is studied. The simulation results demonstrate efficiency and high robustness of the suggested control method.

Keywords: flight control, uncertain dynamic systems, shunt compensation, robustness

1. INTRODUCTION

Modern high-maneuverable aircrafts, such as fighters, operate over a wide range of flight conditions, which vary with altitude, Mach number, angle-of-attack, and engine thrust. The mechanical characteristics of the airframe, such as the center of gravity, change as well. The aircraft autopilot has to be able to produce a response that is accurate and fast despite severe variations in speed and altitude of the airframe or, in the other words, in the face of large parametric uncertainty (Tsourdos and White, 2001; Singh et al., 2003; Belkharraz and Sobel, 2007). The promising way to fulfill these requirements is application of the adaptive control technique. The adaptation method has to meet the conflicting requirements on the tuning rate and performance quality under the conditions of lack of the aircraft state measurements (Andrievsky et al., 1996b; Schumacher and Kumar, 2000; Fradkov and Andrievsky, 2005; Wise et al., 2006; Singh et al., 2003; Yaesh et al., 2004).

Implicit Reference Model Adaptive Control (IMRAC) (or Passification-based Adaptive Control) was proposed in the 1970s (Fradkov, 1974, 1976; Fomin et al., 1981). Theoretical background of passification-based design of adaptive control systems may be found in (Fradkov, 1974; Fradkov et al., 1999; Fradkov and Andrievsky, 2005; Fradkov, 2003; Andrievskii and Fradkov, 2006). Some essentials of the method are outlined in Sec. 2. Later related structures were used in the so-called *Simple Adaptive Control* (SAC) systems (Kaufman et al., 1994; Iwai and Mizumoto, 1994; Yaesh et al., 2004; Barkana, 2007; Belkharraz and Sobel, 2007; Mizumoto et al., 2010). Connection between two approaches was studied in (Andrievsky et al., 1994).

⋆ The work was supported by the Russian Foundation for Basic Research (Proj. Nos 08-01-00775, 09-08-00803), by the Russian Federal Program “Research and Teaching Cadres”, contract No 02.740.11.5056, and the Program of basic research of OEMPPU RAS No 2 “Control and safety in energy and technical systems”.

2. PASSIFICATION METHOD AND ADAPTIVE CONTROLLERS WITH IMPLICIT REFERENCE MODEL

2.1 Passification Theorem

Consider a linear time invariant (LTI) single-input multiple-output (SIMO) system

$$\dot{x} = Ax + Bu, \quad z = Cx, \tag{1}$$

where $x = x(t) \in \mathbb{R}^n$ is a state vector, $u = u(t) \in \mathbb{R}^1$ is a scalar control variable, $z = z(t) \in \mathbb{R}^l$ is a measured output vector, A, B, C are constant real matrices of sizes $n \times n$, $n \times 1$, $l \times n$ respectively.

Passification problem for the system (1) is understood as finding an $(l \times 1)$-matrix K such that the closed loop system with feedback $u = -K^{\mathrm{T}}z + v$ is strictly passive with respect to an auxiliary output $\sigma = Gz$ (G is $(1 \times l)$-matrix): inequality $\int_0^T (\sigma v - \rho|x|^2)\,\mathrm{d}\,t \geq 0$ for some $\rho > 0$ and all $T > 0$ holds for all trajectories of (1) starting from $x(0)=0$. This is equivalent (as follows from Kalman–Yakubovich–Popov Lemma) to finding a matrix K satysfying the *strict positive realness* (SPR) condition: transfer function $W(\lambda)=GC(\lambda I_n - A + BK^{\mathrm{T}}C)^{-1}B$ of the closed-loop system [1] from input v to the output $\sigma = Gz$ satisfies the relations

$$\operatorname{Re} W(i\omega) > 0 \ \text{ for all } \omega \in \mathbb{R}^1,\ i^2 = -1$$
$$\text{and } \lim_{\omega \to +\infty} \omega^2 \operatorname{Re} W(i\omega) > 0. \tag{2}$$

[1] I_n denotes $n \times n$ identity matrix.

Definition 1. System (1) is called *minimum phase* with respect to the output $\sigma = Gz$, if the polynomial

$$\varphi_0(s) = \det \begin{bmatrix} sI_n - A & -B \\ GC & 0 \end{bmatrix} \qquad (3)$$

is Hurwitz; *hyper minimum phase* (HMP), if it is minimum phase and $GCB > 0$.

Theorem 2. (*Passification Theorem*, or *Feedback Kalman–Yakubovich–Popov Lemma*), Fradkov et al. (1999); Fradkov (2003).

The following statements are equivalent:

(A1) There exist a positive definite $(n \times n)$-matrix H and an $(l \times 1)$-matrix K such that the relations

$$H(A + BK^{\mathrm{T}}C) + (A + BK^{\mathrm{T}}C)^{\mathrm{T}}H < 0, \quad HB = C^{\mathrm{T}}G^{\mathrm{T}} \qquad (4)$$

hold.

(B1) The system (1) is hyper minimum phase with respect to the output $\sigma = Gz$.

(C1) There exists a feedback

$$u = K^{\mathrm{T}}z + v \qquad (5)$$

rendering the closed-loop system (1), (5) strictly passive with respect to the output $\sigma = Gz$.

Note, that if the condition (B1) is satisfied then the matrix K in (4) can be found in the form $K = -\kappa G^{\mathrm{T}}$ where κ is a sufficiently large positive real number. Extension of Theorem 2 to MIMO case can be found in Fradkov (2003).

Remark 3. For MIMO case an additional requirement of symmetry $(GCB)^{\mathrm{T}} = GCB$ is included in the HMP definition. Recently, Barkana et al. (2006) have shown that if the non-symmetric positive definite matrix $HGCB$ is diagonalizable, an unknown positive definite symmetric matrix R exists that makes the product $RGCB$ positive definite symmetric. Then the original adaptive controllers can be used without additional G (or R).

Passification Theorem (Theorem 2) gives conditions of solvability of matrix inequalities related to feedback version of classical Kalman–Yakubovich–Popov (KYP) Lemma (Fradkov, 2003). Passification Theorem provides also solvability of the system passification problem by means of static output feedback. It has various applications in control design since the 1970s, e.g. design of adaptive controllers with Implicit Reference Model, see below.

2.2 Passification-based Design of Adaptive Controllers with Implicit Reference Model

Based on the papers (Fradkov, 1974; Andrievskii and Fradkov, 2006), let us present application of the Passification Theorem to design the adaptive control systems with the Implicit Reference Model. Let the LTI SISO system (1) be described in the input-output form as follows:

$$A(p)y(t) = B(p)u(t), \; t \geq 0, \qquad (6)$$

where u, y are the scalar variables, $A(p) = p^n + a_{n-1}p^{n-1} + \cdots + a_1 p + a_0$, $B(p) = b_m p^m + b_{m-1}p^{m-1} + \cdots + b_1 p + b_0$ are polynomials with *a priori* unknown plant model parameters a_i, b_j $(i = 0, 1, \ldots, n-1, \; j = 1, 2, \ldots, m)$, $p \equiv d/dt$ denotes the time derivative.

Consider the control problem of tracking the reference signal $r(t)$. For the disturbance free case the control goal is $\lim_{t\to\infty} \big(r(t) - y(t)\big) = 0$. To solve the posed problem introduce the secondary goal (the *adaptation goal*) prescribing the desired tracking error behavior:

$$|\sigma_t| \leq \Delta \quad \text{if} \;\; t \geq t_*, \qquad (7)$$

where $\sigma_t = G(p)y(t) - D(p)r(t)$ is the *auxiliary error* signal, $G(p) = p^{l-1} + \cdots + g_1 p + g_0$, $D(p) = d_s p^s + d_{s-1}p^{s-1} + \cdots + d_1 p + d_0$ are given polynomials specifying the desired properties of the closed-loop system. $G(\lambda)$ is assumed to be stable (Hurwitz) polynomial. Note, that the signal σ_t may be treated as equation error for the equation

$$G(p)y_*(t) = D(p)r(t), \qquad (8)$$

because $\sigma_t = G(p)\varepsilon(t)$, where $\varepsilon(t) = y(t) - y_*(t)$. Supposing that $\sigma \equiv 0$ one sees that the controlled variable $y(t)$ satisfies equation (8). Hence, (8) may be interpreted as the *reference equation*, representing reference model implicitly. In other words, (8) may be called the *Implicit Reference Model* (IRM). Let us take the *control law of the main loop* in the form

$$u(t) = k_r(t)\big(D(p)r(t)\big) - \sum_{i=0}^{l-1} k_i(t)\big(p^i y(t)\big), \qquad (9)$$

where $k_r(t)$, $k_i(t)$ $(i = 0, \ldots, l-1)$ are tunable parameters. Take the *adaptation algorithm* as follows:

$$\dot{k}_r(t) = -\gamma_r \sigma_t D(p)r(t) - \alpha_r\big(k_r(t) - k_r^0\big), \quad k_r(0) = k_r^0,$$
$$\dot{k}_i(t) = \gamma_i \sigma_t p^i y(t) - \alpha_i\big(k_i(t) - k_i^0\big), \; k_i(0) = k_i^0, \qquad (10)$$

where $\gamma_r > 0$, $\gamma_i > 0$ are the *adaptation gains*; $\alpha_r \geq 0$, $\alpha_i \geq 0$ are the *parametric feedback gains*; k_r^0, k_i^0 are prior estimates of the appropriate values of the tunable parameters. Passification Theorem and results of (Fradkov, 1974; Andrievskii and Fradkov, 2006) give the following applicability conditions for the adaptive controller (9), (10):

(C1) $B(p)$ is Hurwitz polynomial,

(C2) $l = n - m$, where $n - m$ is a relative degree of the plant model (6).

These conditions imply that the plant must be minimum phase and a sufficient number l of the derivatives of its output must be used in the control law. The value of l is defined by the relative degree of the plant transfer function. Therefore, the order of the reference model may be small even if the control plant obeys a high-order equation. Note, that neither degree s of the polynomial $D(p)$, nor its coefficients appear in the above conditions. The degree of $D(p)$ is bounded by the amount of available derivatives of $r(t)$ and is subjected to designer's decision. Additionally, the order of the plant model may be unknown when designing the control algorithm, which is a peculiarity of the systems with IRM as compared with the traditional *model reference adaptive control* (MRAC) (Landau, 1979; Ioannou and Fidan, 2006; Barkana, 2007). Another feature of these systems lies in the possibility of using the IRM not only for tracking, but also for stabilization. The model output, that is its response to the reference signal, is used in the systems with explicit reference model (RM). Finally, on choosing the main loop of the traditional MRAS, an important part is usually played by the *matching condition* (Landau, 1979; Ioannou and Fidan, 2006), which means existence of the controller coefficients providing coincidence of the closed-loop system equations with the explicit RM equations. In many cases, such a condition is rather restrictive. Summarizing, the described IRM adaptive control design procedure is simpler than conventional MRAC (Feuer and Morse, 1978; Landau, 1979; Ioannou and Fidan, 2006) and robust LTI design (e.g., H_∞ design) (Zhou and Doyle, 1998) under condition of strong uncertainty.

2.3 Passification-based design of VSS and signal-parametric adaptive controllers

Describe application of Passification Theorem to design variable-structure systems (VSS) (Utkin, 1992) and signal-parametric adaptive controllers (Andrievsky et al., 1989; Andrievskii and Fradkov, 2006).

Consider the LTI plant (1) for $m = 1$ and the control objective $\lim_{t\to\infty} x(t) = 0$. Let the auxiliary objective be chosen as maintaining the *sliding mode* on the plane $\sigma = 0$, where $\sigma = Gz$ is the auxiliary variable, G is the $(1 \times m)$-matrix. Let us define the control law as follows:

$$u(t) = -\gamma \operatorname{sign} \sigma_t, \quad \sigma_t = Gz(t) \qquad (11)$$

where $\gamma > 0$ is the gain parameter. As it is shown in (Fradkov et al., 1999; Fradkov, 1979), the goal $x(t) \to 0$ may be achieved in the system (1), (11) if there exist matrix $P = P^{\mathrm{T}} > 0$ and vector K_* such that $PA_* + A_*{}^{\mathrm{T}}P < 0$, $PB = C^{\mathrm{T}}G^{\mathrm{T}}$, $A_* = A + BK_*{}^{\mathrm{T}}C$. As it is clear from the Passification Theorem, the mentioned condition is fulfilled if and only if the function $W(s)$ is HMP, where $W(s) = GC\big(sI_n - A\big)^{-1}B$, and the sign of the high frequency gain GCB is known. In that case for the sufficiently large γ the relation $\lim_{t\to\infty} x(t) = 0$ holds. To eliminate dependence of system stability from initial conditions and plant parameters, the following "signal-parametric", or "combined", adaptive control law may be used instead of (11) (Andrievsky et al., 1989; Andrievskii and Fradkov, 2006):

$$u(t) = -K(t)^{\mathrm{T}}z(t) - \gamma \operatorname{sign} \sigma_t, \quad \sigma_t = Gz(t) \qquad (12)$$

$$\dot{K}(t) = \sigma_t \Gamma z(t), \qquad (13)$$

where $\Gamma = \Gamma^{\mathrm{T}} > 0$, $\gamma > 0$ are design parameters.

It should be noticed that convergence of σ_t to zero at the finite time t_* is essential for the VSS systems. It can be shown (see, e.g. (Fradkov et al., 1999; Fradkov, 1990)) that this property is valid for any bounded region of initial conditions for the system (1), (12), (13). To impart boundness to the gain $K(t)$ on practice, the parametric feedback may be added to the algorithm. This robustification of the adaptation algorithm (13) leads to the following adaptation law:

$$\dot{K}(t) = \sigma_t \Gamma z(t) - \alpha\big(K(t) - K_0\big), \quad K(0) = K_0, \qquad (14)$$

where $\alpha > 0$ is chosen parametric feedback gain, K_0 is some "guessed" value of the gain matrix K.

2.4 Parallel feedforward compensator

Note that the HMP condition is valid only for the case of plant relative degree $k = n - m = 1$. However, the design and analysis for general case $k > 1$ involve well known difficulties. Standard solutions based on explicit reference models (Feuer and Morse, 1978; Landau, 1979; Fomin et al., 1981) provide adaptive controllers of high order which are both difficult to implement and sensitive to noise. It was shown in (Andrievsky et al., 1996a; Andrievsky and Fradkov, 1994; Fradkov, 1994) that FKYL allows to design the simplified adaptive controller based on the so called "*shunt*" – the parallel feedforward compensator (see also (Iwai and Mizumoto, 1992; Kaufman et al., 1994; Bar-Kana, 1994; Mareels, 1984)). Below the algorithm of (Andrievsky et al., 1996a; Fradkov, 1994) is described containing few design parameters even for MIMO case. The solution is based on the following statement (see (Fradkov, 1994)).

Theorem 4. Assume the plant with transfer function $G^{\mathrm{T}}W(s)$ is minimum phase with scalar relative degree $k > 1$ for some $l \times m$ matrix G, the matrix $-G^{\mathrm{T}}CA^{r-1}B$ being Hurwitz. Let $P(s)$, $Q(s)$ be Hurwitz polynomials of degrees $k-2$, $k-1$, correspondingly and all three polynomials $P(s)$, $Q(s)$, $\varphi(s) = \delta(s) \det G^{\mathrm{T}}W(s)$ have the same signs of coefficients. Denote

$$W(s) = G^{\mathrm{T}}W(s) + \kappa\varepsilon(\varepsilon s)/Q(s)I_m \qquad (15)$$

Then there exist scalar $\kappa_0 > 0$ and function $\varepsilon(\kappa) > 0$ such, that matrix $W(s)$ is HMP for $\kappa > \kappa_0$, $0 < \varepsilon < \varepsilon_0(\kappa)$.

For SISO plants, the shunt transfer function has the following form (Andrievsky et al., 1996b; Fradkov and Andrievsky, 2005; Fradkov, 1994):

$$W_c(s) = \frac{\kappa\varepsilon\,(\varepsilon s + 1)^{k-2}}{(s + \lambda)^{k-1}} = \frac{B_c(s)}{A_c(s)}, \qquad (16)$$

where k is the relative degree of the plant transfer function ($k = n - m$).

Consider the LTI SISO plant (A_p, B_p, C_p) with the state vector $x_p(t) \in \mathbb{R}^n$, the scalar control and output signals $u(t)$, $y_p(t)$. The plant transfer function is

$$W_p(s) = C_p\big(sI_n - A_p\big)^{-1}B_p = \frac{B_p(s)}{A_p(s)} \qquad (17)$$

where $s \in \mathbb{C}$ denotes Laplace transform variable, $A_p(s) = s^n + a_1 s^{n-1} + \ldots + a_n$, $B_p(s) = b_0 s^m + b_1 s^{n-1} + \ldots + b_m$; $k = n - m$ is the plant relative degree.

The following Theorems give the desired property of *augmented plant* (AP) (15) transfer function $W(s)$ (Andrievsky et al., 1996b; Fradkov and Andrievsky, 2005):

Theorem 5. Let $W_p(s)$ (17) be minimum-phase with the relative degree $r > 1$ and $b_0 > 0$. Then there exist $\kappa_0 > 0$ and function $\varepsilon_0(\kappa) > 0$ such that the AP transfer function $W(s) = W_p(s) + W_c(s)$ is HMP for all $\kappa > \kappa_0$ and $0 < \varepsilon < \varepsilon_0(\kappa_0)$.

Theorem 6. (Andrievsky et al., 1996b). Let $W_p(s)$ be stable ($A_p(s)$ be a Hurwitz polynomial) with the relative degree $k > 1$ and $b_0 > 0$. Then for every $\varepsilon > 0$ there exists sufficiently large κ_0, such that $W(s) = W_p(s) + W_c(s)$ is HMP for all $\kappa \geq \kappa_0$.

Theorem 5 shows that one can introduce the shunt (16) with order $\deg(A_c(s)) = k - 1 = n - m - 1$ providing for sufficiently large κ and small ε the augmented plant (15) satisfying the HMP condition for arbitrary given minimum-phase plant parameters domain. As it is follows from the Theorem 6, another way of shunt (16) parameters choosing provides the HMP condition for stable (and, possible, nonminimum-phase) plants. For this case, the shunt equation can be simplified; namely $W_c(s) = \dfrac{\kappa}{s+\lambda}$ may be taken instead of (16).

3. ROBUST AUTOPILOT DESIGN

In this Section, the yaw controller for flexible aircraft is designed based on the approach presented in Sec. 2.

3.1 Airframe and onboard equipment modeling

In the sequel the following model of the lateral motion of the aircraft as a rigid body is used:

$$\begin{cases} \dot{\beta}(t) = \omega_y(t) + a_z^{\beta}\beta(t) + a_z^{\delta}\delta_r(t), \\ \dot{\omega}_y(t) = a_m^{\beta}\beta(t) + a_m^{\omega}\omega_y(t) + a_m^{\delta}\delta_r(t), \\ \dot{\psi}(t) = \omega_y(t), \end{cases} \quad (18)$$

where $\psi(t)$, $\omega(t)$ are the yaw angle and the yaw rate respectively, $\beta(t)$ denotes the sideslip angle; $\delta_r(t)$ is the rudder angle; a_i^j denote the aircraft model parameters. Values of a_i^j depend on the flight conditions (such as flight altitude, Much number, etc.) and are changing in the wide range during the flight.

The rudder actuators control loop modeled as the following second order LTI system

$$W_{\text{servo}}(s) = \frac{\delta_r(s)}{\sigma_{\psi}(s)} = \frac{k_{\text{servo}}}{T_{\text{servo}}^2 s^2 + 2\xi_{\text{servo}} T_{\text{servo}} s + 1}, \quad (19)$$

where k_{servo} is an actuator steering servosystem gain (further on $k_{\text{servo}} = 1$ is taken), T_{servo} stands for the servosystem response time factor, ξ_{servo} is the damping ratio, σ_{ψ} denotes the commanded rudder deflection angle, generated by the autopilot, $s \in \mathbb{C}$ stands for the Laplace transform variable.

The first mode of the airframe bending is taken into account and modeled as

$$W_{\text{bend}}(s) = \frac{\Delta\psi(s)}{\delta_r(s)} = \frac{k_{\text{bend}}}{T_{\text{bend}}^2 s^2 + 2\xi_{\text{bend}} T_{\text{bend}} s + 1}, \quad (20)$$

where k_{bend} is the bending mode transition factor; T_{bend} is the response time factor, $T_{\text{bend}} = \omega_{\text{bend}}^{-1}$, where ω_{bend} is the 1st bending mode natural frequency; ξ_{bend} is the damping ratio ($\xi_{\text{bend}} \approx 0$).

The signal ψ_{g}, measured by the gyros, is the sum of the yaw and bending angles:

$$\psi_{\text{g}}(t) = \psi(t) + \Delta\psi(t). \quad (21)$$

Equations (18)–(21) describe the seven-order plant model with the uncertain parameters a_i^j. Note that the aircraft is the weathercock unstable if $a_m^{\beta} < 0$.

3.2 Shunt transfer function

The plant (18)–(21) relative degree $k = nm = 4$, therefore the HMP condition does not valid for the considered system. Let us apply the shunting method of Sec. 2.4 and pick up the shunt transfer function (16) in the form

$$W_{\text{s}}(s) = \frac{y_{\text{s}}(s)}{\sigma_{\psi}(s)} = \frac{\kappa}{s+\lambda}, \quad (22)$$

where κ, λ are the shunt parameters, $\lambda > 0$, $\operatorname{sign}\kappa = \operatorname{sign} b_0$. The *extended plant* output $y(t)$ is the sum $y(t) = \psi_{\text{g}}(t) + y_{\text{s}}(t)$, where $y_{\text{s}}(t)$ is the shunt (22) output. The extended plant (18)–(22) transfer function $W(s)$ is of order $n = 8$, the numerator $B(s)$ has degree $m = 7$, hence the relative degree k is equal to one, $k = n - m = 1$. Therefore, to secure the HMP requirement for the extended plant model (18)–(22) only the Hurwitz condition for the numerator $B(s)$ must be fulfilled.

3.3 HMP analysis

Let us find the HMP domain numerically for a typical area of the light jet aircraft model (18)–(21) parameters. The rudder servosystem and 1st bending mode parameters are taken as follows: $T_{\text{servo}} = 0.05$ s, $\xi_{\text{servo}} = 0.7$, $k_{\text{bend}} = 1.5 \cdot 10^3$, $\omega_{\text{bend}} = 65$ s^1, $\xi_{\text{bend}} = 0.01$. The following shunt (22) parameters are taken: $\lambda = 14$ s^{-1}, $\kappa = -2$. The "tightened" HMP domain is found, which corresponds to the strengthened Hurwitz condition of polynomial $B(s)$ with the stability margin $\eta = 0.5$. This domain in the space of the principal aerodynamic parameters of the aircraft model (18) is shown in Fig. 1 (an interior of the domain bounded by the surfaces). It is seen that the HMP domain is wide and covers the cases of weathercock stable and weathercock unstable aircrafts as well.

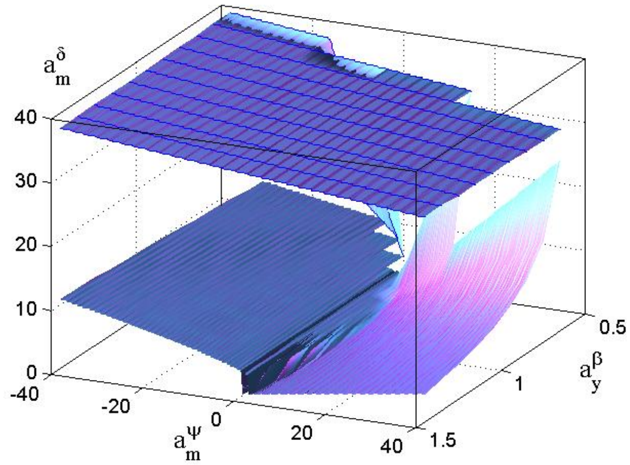

Fig. 1. The HMP domain for system (18)–(22).

3.4 Variable-structure robust autopilot design

The fulfillment of the HMP condition makes possible to use the VSS or the high-gain robust autopilot for the aircraft attitude control (see above Sec. 2.3). Let us use the following VSS control law:

$$\sigma_{\psi} = \bar{\sigma}\,\text{sat}\big(Ke(t)\big), \quad e(t) = y(t) - r(t), \quad (23)$$

where $\bar{\sigma}$ denotes the maximal rudder deflection angle, sat(·) stands for the *saturation function*, K is the high gain coefficient, $r(t)$ is the reference signal, $y(t)$ is the extended plant output. The control law (23) ensures the stable sliding motion with an accurate tracking the reference input $r(t)$ by the output $y(t)$.

For ensuring the desired closed-loop system performance with respect to the command yaw angle $\psi^*(t)$, the following second-order *sequential reference model* is employed:

$$W_{\text{M}}(s) = \frac{r(s)}{\psi^*(s)} = \frac{\Omega_{\text{M}}^2}{s^2 + 1.4\Omega_{\text{M}} s + \Omega_{\text{M}}^2}. \quad (24)$$

The natural frequency Ω_M gives the desired transient time of the closed-loop attitude control system.

3.5 Simulation results

Results of the yaw control system (18)–(24) simulation for various aircraft model parameters are pictured in Fig. 4. Parameter values are given in Table 1.

Table 1. Aircraft model parameters

No	a_m^ω, s^{-1}	a_z^β, s^{-1}	a_m^β, s^{-2}	a_m^δ, s^{-2}
1	1.3	0.75	33	19
2	0.9	1.0	22	6.5
3	0.13	0.6	3.2	3.6
4	0.45	1.2	−11	8.3
5	0.6	0.8	−15	8.0

Note that the sets of parameters Nos. 1–3 correspond the weathercock stable aircraft, whereas the sets Nos. 4, 5 correspond the unstable aircraft. The plant (18)–(21) properties in the frequency domain for different flight conditions are demonstrated by Figs. 2, 3, where the corresponding Bode diagrams and Nichols charts are depicted.

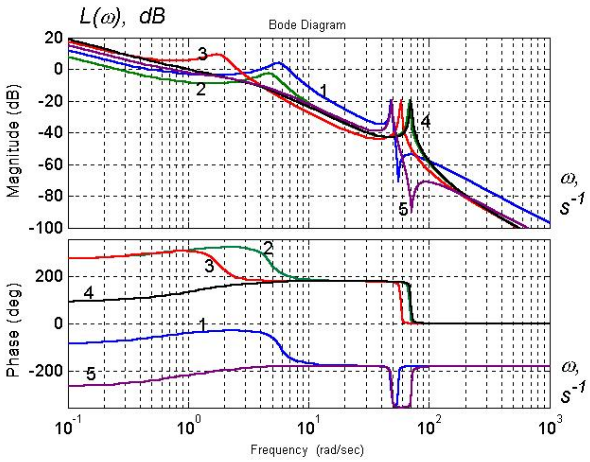

Fig. 2. Bode diagrams for various aircraft model (18)–(21) parameters.

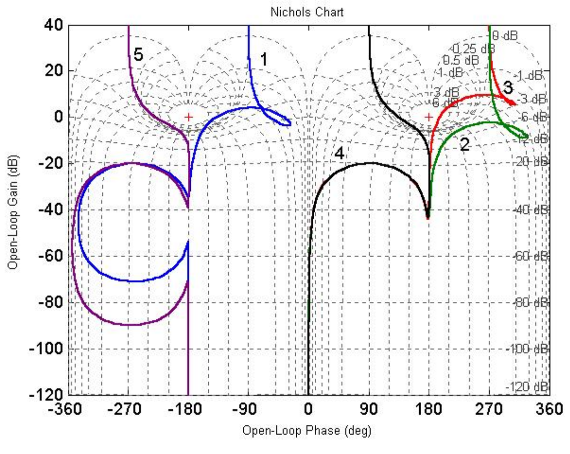

Fig. 3. Nichols charts for various aircraft model (18)–(21) parameters.

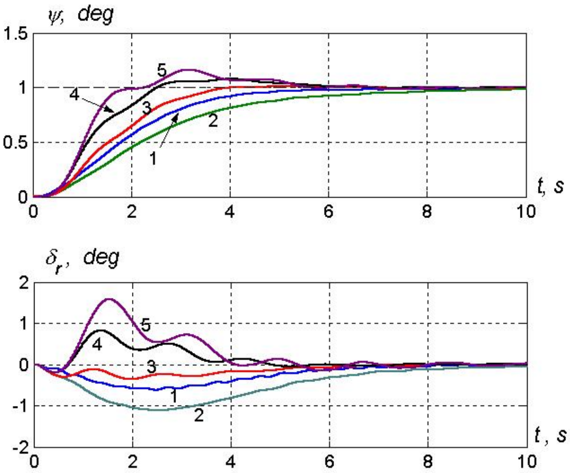

Fig. 4. Yaw angle and rudder deflection angle step responses for various aircraft model parameters.

The following autopilot (23), (24) parameters are taken: $\bar{\sigma} = 20$ deg, $K = 100$, $\Omega_M = 1\ s^{-1}$. The step signal of one degree in a magnitude is taken as a command yaw angle ψ^*. It should be noticed that since the shunt parameter κ is small, the steady-state error is close to zero despite the fact that the extended plant output $y(t)$ does not equal to the yaw angle $\psi(t)$) due to adding the shunt output $y_s(t)$.

One may compare the results obtained with those for employing the standard PD-control law with unchangeable parameters. The step responses for the same parameter values and a typical PD-controller

$$\sigma_\psi(t) = k_\psi\big(\psi^*(t) - \psi_g(t)\big) - k_{\dot\psi}\dot\psi_g(t) \qquad (25)$$

are depicted in Fig. 5. The autopilot gains $k_\psi = 2$, $k_{\dot\psi} = 1.5$ s has been found satisfying closed-loop system performance requirements for the regime No 4, but these parameters give unsatisfactory results for other regimes.

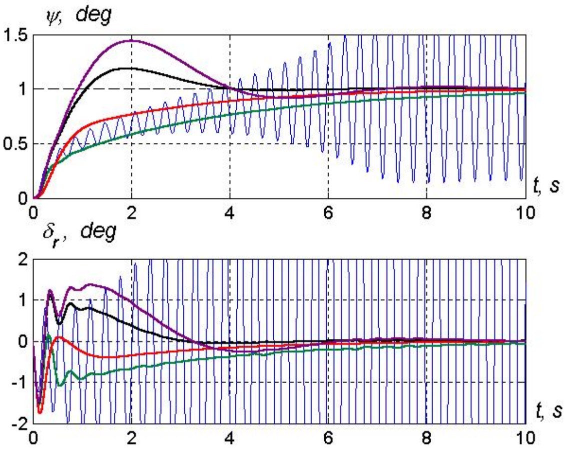

Fig. 5. Standard PD-controller (25). Yaw angle and rudder deflection angle step responses for various aircraft model parameters.

4. CONCLUSIONS

The Passification-based method applied for robust flight control system design. The shunting method is used ensuring the closed-loop system stability in the face of the lack of the aircraft state information. An example illustrating a typical design procedure for aircraft attitude control in the horizontal plane for wide range of the aircraft parameters is given, demonstrating efficiency and high robustness of the suggested control method.

REFERENCES

Andrievskii, B.R. and Fradkov, A. (2006). Method of passification in adaptive control, estimation, and synchronization. *Autom. Remote Control*, 67(11), 1699–1731.

Andrievsky, B., Fradkov, A.L., and Kaufman, H. (1994). Necessary and sufficient conditions for almost strict positive realness and their application to direct implicit adaptive control systems. In *Proc. American Control Conference*, 1265–1266. Baltimore, USA.

Andrievsky, B.R., Churilov, A., and Fradkov, A. (1996a). Feedback Kalman–Yakubovich Lemma and its applications to adaptive control. In *Proc. 35th IEEE Conf. Dec. Contr.*, 4537–4542. Kobe, Japan.

Andrievsky, B.R. and Fradkov, A.L. (1994). Adaptive controllers with Implicit Reference Models based on Feedback Kalman–Yakubovich Lemma. In *Proc. 3rd IEEE Conf. on Control Applications*, 1171–1174. Glasgow, UK.

Andrievsky, B., Fradkov, A., and Stotsky, A. (1996b). Shunt compensation for indirect sliding-mode adaptive control. In *Proc. 13th Triennial IFAC World Congress*, volume K, 193–198. San Francisco, US.

Andrievsky, B.R., Stotsky, A.A., and Fradkov, A.L. (1989). Velocity gradient algorithms in control and adaptation problems. *Autom. Remote Control*, 1533–1564.

Bar-Kana, I. (1994). Adaptive control can robustify uncertain control systems. In *Proc. American Control Conference (ACC 1994)*, 63–67. Baltimore, Maryland.

Barkana, I. (2007). Simple adaptive control: A stable direct model reference adaptive control methodology – brief survey. In *Proc. 9th IFAC Workshop on Adaptation and Learning in Control and Signal Processing*. Saint Petersburg, Russia. URL `http://www.ifac-papersonline.net/Detailed/30246.html`.

Barkana, I., Teixeira, M., and Hsu, L. (2006). Mitigation of symmetry in positive realness for adaptive control. *Automatica*, 42(9), 1611–1616.

Belkharraz, A.I. and Sobel, K. (2007). Simple adaptive control for aircraft control surface failures. *IEEE Trans. Aerosp. Electron. Syst.*, 43(2), 600–611.

Feuer, A. and Morse, A.S. (1978). Adaptive control of single-input, single-output linear systems. *IEEE Trans. Automat. Contr.*, 23(3), 557–570.

Fomin, V.N., Fradkov, A.L., and Yakubovich, V. (1981). *Adaptive control of dynamical plants*. Nauka, Moscow. (in Russian).

Fradkov, A.L. (1974). Synthesis of an adaptive system for linear plant stabilization. *Autom. Remote Control*, 35(12), 1960–1966.

Fradkov, A.L. (1976). Quadratic Lyapunov functions in the adaptive stability problem of a linear dynamic target. *Siberian Mathematical Journal*, 17(2), 436–445.

Fradkov, A.L. (1990). *Adaptive control in large-scale systems*. Nauka, Moscow. (in Russian).

Fradkov, A.L. (1994). Adaptive stabilization for minimum-phase multi-input plants without output derivatives measurement. *Physics-Doklady*, 39(8), 550–552.

Fradkov, A.L. (2003). Passification of nonsquare linear systems and Feedback Kalman–Yakubovich–Popov Lemma. *Europ. J. Contr.*, 9(11).

Fradkov, A.L. and Andrievsky, B. (2005). Combined adaptive controller for UAV guidance. *Europ. J. Contr.*, 11(1), 71–79.

Fradkov, A.L., Miroshnik, I.V., and Nikiforov, V.O. (1999). *Nonlinear and Adaptive Control of Complex Systems*. Kluwer, Dordrecht.

Fradkov, A. (1979). Speed-gradient scheme and its applications in adaptive control. *Autom. Remote Control*, 40(9), 1333–1342.

Ioannou, P. and Fidan, B. (2006). *Adaptive Control Tutorial*. SIAM, Philadelphia, USA.

Iwai, Z. and Mizumoto, I. (1992). Robust and simple adaptive control systems. *International Journal of Control*, 55, 1453–1470.

Iwai, Z. and Mizumoto, I. (1994). Realization of simple adaptive control by using parallel feedforward compensator. *Int. J. Control*, 59(6), 1543–1565.

Kaufman, H., Bar-Kana, I., and Sobel, K. (1994). *Direct Adaptive Control Algorithms*. Springer Verlag, New York.

Landau, J.D. (1979). *Adaptive control systems. The model reference approach*. Dekker, New York.

Mareels, I. (1984). A simple self-tuning controller for stable invertible system. *Syst. Contr. Letters*, 4, 5–16.

Mizumoto, I., Ikeda, D., Hirahata, T., and Iwai, Z. (2010). Design of discrete time adaptive PID control systems with parallel feedforward compensator. *Control Engineering Practice*, 18(2), 168–176.

Schumacher, C.J. and Kumar, R. (2000). Adaptive control of UAVs in close-coupled formation flight. In *Proc. American Control Conference (ACC 2000)*, volume 2, 849–853. DOI:10.1109/ACC.2000.876619.

Singh, S.N., Steinberg, M.L., and Page, A.B. (2003). Nonlinear adaptive and sliding mode flight path control of F/A-18 model. *IEEE Trans. Aerosp. Electron. Syst.*, 39(4), 1250–1262.

Tsourdos, A. and White, B.A. (2001). Modern missile flight control design: an overview. In *Proc. 15th IFAC Symposium on Automatic Control in Aerospace*, 425–430. Bologna/Forli, Italy.

Utkin, V.I. (1992). *Optimization and Control using Sliding Modes*. Springer-Verlag, London.

Wise, K.A., Lavretsky, E., and Hovakimyan, N. (2006). Adaptive control of flight: theory, applications, and open problems. In *Proc. American Control Conference (ACC'06)*, 6 p. DOI: 10.1109/ACC.2006.165767.

Yaesh, I., Shaked, U., and Yossef, T. (2004). Simplified adaptive control of F16 aircraft pitch and angle-of-attack loops. In *Proc. 44st Israel annual conference on Aerospace sciences*. Haifa, Israel.

Zhou, K. and Doyle, J. (1998). *Essentials of Robust Control*. Prentice-Hall, New Jersey.