State estimation of passifiable Lurie systems via limited-capacity communication channel

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***Abstract*—State estimation problems for a class of nonlinear oscillatory systems under information constraints imposed by limited capacity of the communication channel are analyzed. A binary time-varying coder-decoder scheme is described and a theoretical analysis for state estimation for nonlinear plants represented in Lurie form (linear part plus nonlinearity depending only on measurable outputs) based on the Passification Theorem is provided. It is shown that the estimation error exponentially tends to zero for sufficiently high transmission rate (channel capacity). The results are illustrated by example of state estimation of chaotic Chua system via a communication channel with limited capacity.**

I. Introduction

In various emerging applications, such as automation industry or automotive systems, micro-electromechanical systems and mobile telephony, a growing number of complex remote operations should be performed. In large-scale control applications of the modern industry, the functional agents, such as sensors, actuators, and controllers are geographically distributed, therefore, observation and control signals are transmitted via communication channels [1]–[7]. For complex networked sensor systems containing a very large number of low-power sensors, the amount of data collected by the sensors is too large to be transmitted in full via the existing communication channel [5], [6], [8]. In these problems, only the sequence of finite-valued symbols, transmitted over a communication channel, are available to controller (observer). For instance, this issue may arise with the transmission of control signals when a large number of mobile units needs to be controlled remotely by a single decision maker. Since the radio spectrum is limited, communication constraints are a real concern.

Recently the limitations of control under constraints imposed by a finite capacity information channel have been investigated in detail in the control literature, see [4]–[6], [9]–[11], [11], [12], [12]–[15], [15]–[17] and the references therein. It has been shown that stabilization of linear systems under information constraints is possible if and only if the capacity of the information channel exceeds the entropy production of the system at the equilibrium (*Data Rate Theorem*) [11], [12], [16]. In [18], [19] a general statement was proposed, claiming that the difference between the entropies of the open loop and the closed loop systems cannot exceed the information introduced by the controller, including the transmission rate of the information channel.

For nonlinear systems only a few results are available in the literature [12], [20]–[24]. In the above papers only the problems of stabilization to a point are considered. The result of [12] is local, while the papers [20]–[24] deal only with equilibrium stabilization.

In the control literature there is a strong interest in control of oscillations, particularly in controlled synchronization problems [25]–[31]. However, most of the previous results on control systems analysis under information constraints do not apply to synchronization systems since in a synchronization problem trajectories in the phase space converge to a set (a manifold) rather than to a point, i.e. the problem cannot be reduced to simple stabilization. Moreover, the Data Rate Theorem is difficult to extend to nonlinear systems.

The first results on synchronization under information constraints were presented in [32]–[34], where the so called observer-based synchronization scheme was considered. It is shown that for first-order coder-decoder scheme the upper bound of limit synchronization error is proportional to the maximum rate of the coupling signal and inversely proportional to the information transmission rate (channel capacity) [32]. The analysis is extended to networks having a chain, star, or star-chain topology [33], [35] and to adaptive observer-based synchronization systems [34]. In [36] the controlled synchronization problem is analyzed by quadratic Lyapunov functions and Passification method [37]–[39]. It is shown, that for the case of an ideal channel and noncorrupted measurements, the proposed output feedback controlled synchronization strategy with full order coder/decoder pair ensures exponentially vanishing synchronization error if the channel capacity exceeds some threshold value.

Unlike [32]–[34], [36], in the present paper we consider an *output feedback controlled estimation* scheme. The control signal is computed based on a measurable innovation (error) signal transmitted over a communication channel. Our study is restricted to Lurie systems (linear part plus nonlinearity depending only on measurable outputs). It is shown that, in the spirit of [36], for the case of an ideal channel and non-corrupted measurements, the proposed state estimation strategy ensures exponentially vanishing estimation error if the data transmission rate (channel capacity) exceeds some threshold value. Similarly to [36] the proposed estimation algorithm is based on the Passification Theorem [37]–[39].

The paper is organized as follows. The state estimation strategy over the limited-band communication channel is described in Section II. Coding procedure is presented in Section III. Section IV is devoted to evaluation of the state estimation error. The example of state estimation of chaotic Chua system under communication constraints is given in Section V.

II. State estimation scheme

Let the plant model be presented in the following Lurie form (i.e. the right hand side is split into a linear part and a nonlinear part which depends only on the measurable outputs):

$$\dot{x}(t) = Ax(t) + B\varphi(y),\ y(t) = Cx(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the plant state variables vector; $y(t)$ is a scalar output variable; A is an $(n \times n)$-matrix; B is $n \times 1$ (column) matrix; C is an $1 \times n$ (row) matrix, $\varphi(y)$ is a continuous nonlinearity.

Let only the plant output $y(t)$ be measured. Matrices A, B and function $\varphi(\cdot)$ are assumed to be known. The problem is to produce an estimate of the unmeasured state vector $x(t)$ based on available sensor data, taking in a view limitations of communication channel capacity. Introduce the following *nonlinear state estimator of full order* (cf. [40]):

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\varphi(\hat{y}) + L\varepsilon(t),\ \hat{y}(t) = C\hat{x}(t), \tag{2}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is a vector of estimates; $\hat{y}(t)$ is a scalar output variables of the observer; $\varepsilon(t) = y(t) - \hat{y}(t) = Ce(t)$ is the error between outputs of the plant and observer; $n \times 1$ (column) matrix L is an observer gain (design parameter).

In our estimation scheme we assume that a smart sensor at the plant node, incorporating the nonlinear observer (2) may be used, i.e. that the state estimates $\hat{x}(t)$ may be generated by the sensor based on the full information about plant output (with the exception of sensor errors). In contrast to some other papers, see e.g. [41], where the full state vector estimate $\hat{x}(t)$ is transmitted over the channel, in the present work only the scalar variable is to be transmitted. Instead of transmission of the plant output signal $y(t)$ (this approach has been used in [32]–[34], [39], [42]), in the present work we employ transmission of the "innovation signal" $\varepsilon(t)$. It is shown below that this makes possible to ensure asymptotically vanishing on t state estimation error $e(t) = x(t) - \hat{x}(t)$ for finite, but sufficiently large, data transmission rate (we omit here the channel and measurement imperfections). Note, that this approach concurs with the results of [10], [43]–[47].

A key difficulty arises because the error signal between the master system and the slave systems is not available directly but only through a communication channel with a limited capacity. Despite no rate limitations in data transmission from the sensor to the coder is supposed, the observer (2) should not be directly employed due to the so-called "equi-memory condition" [44], [48]. This condition means that the encoder and decoder make decisions based on the same information. For the considered estimation problem this means that the coding-decoding scheme should be incorporated to the state estimation algorithm. Therefore, the estimation error $\varepsilon(t)$ should be coded with symbols from a finite alphabet at discrete sampling time instants $t_k = kT$, $k = 0, 1, 2, \ldots$, where T is the sampling time. The coded symbol $\bar{\varepsilon}[k] = \bar{\varepsilon}(t_k)$ is to be used instead of ε in the observer (2). Then (2) reads as

$$\begin{aligned} &\dot{\hat{x}}(t) = A\hat{x}(t) + B\varphi(\hat{y}) + L\bar{\varepsilon}(t),\ \hat{y}(t) = C\hat{x}(t), \\ &\bar{\varepsilon}(t) = \bar{\varepsilon}[k] \quad \text{as} \quad t \in [t_k, t_{k+1}), \quad t_k = kT, \quad k = 0, 1, 2, \ldots. \end{aligned} \tag{3}$$

The coded value of $\bar{\varepsilon}[k]$ is transmitted to the receiver side, where the state estimation process is replicated. Namely, the following sate estimation procedure, employing received coded signal $\bar{\varepsilon}[k]$ is realized by the decoder:

$$\begin{aligned} &\dot{\hat{x}}_d(t) = A\hat{x}_d(t) + B\varphi(\hat{y}_d) + L\bar{\varepsilon}(t),\ \hat{y}_d(t) = C\hat{x}_d(t), \\ &\bar{\varepsilon}(t) = \bar{\varepsilon}[k] \quad \text{as} \quad t \in [t_k, t_{k+1}), \quad t_k = kT, \quad k = 0, 1, 2, \ldots, \end{aligned} \tag{4}$$

where $\hat{x}_d \in \mathbb{R}^n$ is plant state estimation vector, generated by the decoder, $\hat{x}_d(0) = \hat{x}(0)$.

To simplify the analysis, we assume that the observations are not corrupted by observation noise; transmission delay and transmission channel distortions may be neglected. The coded symbols are assumed to be available at the receiver side at the same sampling instant $t_k = kT$, as they are generated by the coder. Assume that *zero-order extrapolation* is used to convert the digital sequence $\bar{\varepsilon}[k]$ to the continuous-time input of the observer $\bar{\varepsilon}(t)$, namely, that $\bar{\varepsilon}(t) = \bar{\varepsilon}[k]$ as $kT \le t < (k+1)T$. Then the *transmission error* is defined as follows:

$$\delta_\varepsilon(t) = \varepsilon(t) - \bar{\varepsilon}(t). \tag{5}$$

In what follows we restrict consideration the matrix L in the form

$$L = \kappa B, \tag{6}$$

where a scalar gain κ is a design parameter.

Our future consideration employs the constructive conditions for output feedback stabilization existing for the class of *passifiable* (or *feedback passive*) systems [36], [38], [39], [49], [50]. Since we are dealing with a nonlinear problem further complicated by information constraints, we restrict our attention to sufficient conditions for solvability of the problem and evaluate upper bounds for estimation error.

III. Coding procedures

In the paper [32] the properties of observer-based estimation for Lurie systems over a limited data rate communication channel with a one-step memory time-varying coder are studied. It is shown that an upper bound on the limit estimation error is proportional to a certain upper bound on the transmission error. Under the assumption that a sampling time may be properly chosen, optimality of binary coding in the sense of demanded transmission rate is established, and the relationship between estimation accuracy and an optimal sampling time is found. It is worth to mention that the tight data-rate bound for stabilizability of a scalar system was given in [51]. In this article is shown that even for the *binary control* the bound is achievable. The linear plant stabilization problem was further considered in [52], where it was shown that for the first-order linear plant, binary control is the most robust control strategy under varying data-rate constraint and asynchronism of sampling and control actuation. On the basis of these results, the present paper deals with a binary coding procedure.

Consider the memoryless (static) binary quantizer to be a discretized map $q : \mathbb{R} \to \mathbb{R}$ as

$$q(y,M) = M\,\mathrm{sign}(y), \tag{7}$$

where $\mathrm{sign}(\cdot)$ is the *signum* function: $\mathrm{sign}(y) = 1$, if $y \geq 0$, $\mathrm{sign}(y) = -1$, if $y < 0$. Parameter M may be referred to as the *quantizer range*. Notice that for a binary coder each codeword symbol contains one bit of information. Therefore the transmission rate is $R = 1/T$. The discretized output of the considered quantizer is given as $\bar{y} = q(y,M)$. We assume that the coder and decoder make decisions based on the same information. The output signal of the quantizer is represented as a one-bit information symbol from the coding alphabet $\mathscr{S}$ and transmitted over the communication channel to the decoder.

In *time-varying quantizers* [15], [20], [32], [46], [53] the range M is updated with time and different values of M are used at each step, $M = M[k]$. Using such a "zooming" strategy it is possible to increase coder accuracy in the steady-state mode and at the same time, to prevent coder saturation at the beginning of the process [46].

In the present paper we use the following time-based zooming strategy for a quantizer range

$$M[k] = M_0\rho^k, \quad k = 0,1,\ldots, \tag{8}$$

where $0 < \rho \leq 1$ is the decay parameter. The initial value M_0 should be large enough to capture the region of possible initial values of y_0. Equations (7), (8) describe the coder algorithm. A similar algorithm is realized by the decoder. Namely, the sequence $M[k]$ is reproduced at the receiver node utilizing (8) such that the values of $\bar{y}[k]$ are restored with the given $M[k]$ using the received codeword $s[k] \in \mathscr{S}$.

IV. Evaluation of estimation error

Let us evaluate the limit estimation error, taking into account transmission of the error signal over the communication channel and coding procedure. Since the estimation error signal is piecewise constant over sampling intervals $[t_k, t_{k+1}]$, the observer (3), (6) becomes

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\varphi(\hat{y}) + \kappa B\bar{\varepsilon}, \tag{9}$$

where $\bar{\varepsilon}(t) = \bar{\varepsilon}[k]$ as $t_k < t < t_{k+1}$, $\bar{\varepsilon}[k]$ is the result of transmission of the estimation error signal $\varepsilon(t) = y(t) - \hat{y}(t)$ over the channel, $t_k = kT$, $k = 0,1,\ldots$.

According to the quantization algorithm (7), the quantized error signal $\bar{\varepsilon}[k]$ becomes

$$\bar{\varepsilon}[k] = M[k]\,\mathrm{sign}(\varepsilon(t_k)), \tag{10}$$

where the range $M[k]$ is defined by (8).

The key point of the approach is application of the so-called *method of continuous models*: analysis of the hybrid nonlinear system via analysis of its continuous-time approximate model [54], [55], see also [56].

In order to analyze the estimation error we make two assumptions:

A1. Nonlinearity $\varphi(y)$ is Lipschitz continuous:

$$|\varphi(y) - \varphi(\hat{y})| \leq L_\varphi |y - \hat{y}| \tag{11}$$

for all y, $\hat{y}$ and some $L_\varphi > 0$.

A2. The linear part of (1) is strictly passifiable: according to the Passification Theorem [36], [38], [39], [49], [50], this means that the numerator $\beta(\lambda)$ of the transfer function $W(\lambda) = C(\lambda \mathrm{I} - A)^{-1}B = \beta(\lambda)/\alpha(\lambda)$ is a Hurwitz (stable) polynomial of degree $n-1$ with positive coefficients (the so-called hyper-minimum-phase (HMP) property).

It follows from condition A2 and the Passification Theorem, that the stability degree η_0 of the polynomial $\beta(\lambda)$ (a minimum distance from its roots to the imaginary axis) is positive and for any η: $0 < \eta < \eta_0$ there exist a positive definite matrix $P = P^{\mathrm{T}} > 0$ and a number κ such that the following matrix relations hold:

$$PA_\kappa + A_\kappa^{\mathrm{T}} P \leq -2\eta P, \quad PB = C^{\mathrm{T}}, \quad A_\kappa = A - \kappa BC. \tag{12}$$

Any sufficiently large real number can be chosen as the value of κ.

The main result of this Section is formulated as follows.

Theorem 1. Let A1, A2 hold, the controller gain K satisfies passivity relations (12) and the coder parameters ρ, T be chosen to meet the inequalities

$$\begin{aligned} &\exp(\eta T)\big(\exp(L_F T) - 1\big) \leq \frac{L_F}{\|C\|\big(K\|B\| + L_F\big)}, \\ &\exp(-\eta T) < \rho < 1, \end{aligned} \tag{13}$$

where $L_F = \|A\| + L_\varphi \|B\| \cdot \|C\|$, η is from (12). Let the coder range $M[k]$ be specified as

$$M[k] = M_0\rho^k. \tag{14}$$

Then for all initial conditions $e(0)$ such that $e(0)^{\mathrm{T}} Pe(0) \leq M_0^2$ the estimation error decays exponentially

$$|\varepsilon[k]| \leq \|e[k]\| \leq M_0\rho^k. \tag{15}$$

In addition, $|\varepsilon(t)| \leq |\varepsilon[k]|$ for $t_k \leq t \leq t_{k+1}$.

Proof of the Theorem is omitted due to the space limitation. It is fulfilled by the analogy with that of the paper [36].

Remark 1. In a stochastic framework the estimates of the mean square value of the estimation error can be obtained. There is a significant body of work in which the quantization error signal $\delta(t)$ is modeled as an extra additive white noise. This assumption, typical for digital filtering theory, is reasonable if the quantizer resolution is high [57], but it needs modification for the case of a low number of quantization levels [15].

Remark 2. For practice, it is reasonable to choose the coder range $M[k]$ separated from zero. The following zooming strategy for a quantizer range may be recommended instead of (8) [32]:

$$M[k] = (M_0 - M_\infty)\rho^k + M_\infty, \quad k = 0,1,\ldots, \tag{16}$$

where $0 < M_\infty < M_0$ stands for the limit value of $M[k]$.

Remark 3. Assumption A2 could limit the proposal when the channel adds delay. Also the limitations of the proposed estimation scheme for the case of the channel imperfections arise due to violation of the mentioned above equi-memory condition [44].

V. Example. State estimation of chaotic Chua system under communication constraints

Let us apply the above results to state estimation of chaotic Chua system over the limited-band communication channel.

Plant model. Let the plant (1) be represented by the following *Chua system*:

$$\begin{cases} \dot{x}_1 = p(-x_1 + \varphi(y) + x_2), & t \geq 0, \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -q x_2, \end{cases} \tag{17}$$
$$y(t) = x_1(t),$$

where $y(t)$ is the plant output, p, q are known parameters, $x = [x_1, x_2, x_3]^{\mathrm{T}} \in \mathbb{R}^3$ is the state vector; $\varphi(y)$ is a piecewise-linear function, having the form:

$$\varphi(y) = m_0 y + m_1(|y+1| - |y-1|), \tag{18}$$

where m_0, m_1 are given parameters.

Evidently, Chua system (17) may be represented in Lurie form (1) with the matrices:

$$A = \begin{bmatrix} -p & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix}, \quad B = \begin{bmatrix} p \\ 0 \\ 0 \end{bmatrix}, \quad C = [1, 0, 0]. \tag{19}$$

It is easy to check that the linear part of the Chua system satisfies the HMP condition. Indeed, for the triple (A, B, C) from (19) the transfer function $W(\lambda) = C(\lambda \mathrm{I} - A)^{-1} B = \beta(\lambda)/\alpha(\lambda)$ is as follows:

$$W(\lambda) = \frac{p(\lambda^2 + \lambda + q)}{\lambda^3 + (1+p)\lambda^2 + q\lambda + pq}.$$

The numerator $\beta(\lambda) = p(\lambda^2 + \lambda + q)$ is a Hurwitz polynomial of degree 2, i.e. the HMP condition holds for all $p > 0$, $q > 0$.

Observer. Correspondingly, the observer equations (3), (9) for the considered case becomes

$$\begin{cases} \dot{\hat{x}}_1 = p\big(-\hat{x}_1 + \varphi(\hat{x}_1(t)) + \hat{x}_2 + \kappa \bar{\varepsilon}(t)\big), & t \geq 0, \\ \dot{\hat{x}}_2 = \hat{x}_1 - \hat{x}_2 + \hat{x}_3 \\ \dot{\hat{x}}_3 = -q \hat{x}_2, \end{cases} \tag{20}$$

where $\varphi(\hat{y})$ is defined by (18).

Coding procedure has a form (8), (10). The input signal of the coder is $\varepsilon(t)$. The error signal $\bar{\varepsilon}(t)$ in (9) is found by holding the value of $\bar{\varepsilon}[k]$ over the sampling interval $[kT, (k+1)T)$, $k = 0, 1, \ldots$. The initial value M_0 of the coder range and the decay factor ρ in (8) are design parameters.

The following parameter values were used for the simulation:

- Chua system parameters: $p = 10$, $q = 15.6$, $m_0 = 0.33$, $m_1 = 0.945$. For chosen parameter values the system behavior is chaotic;
- the observer gain $\kappa = 10$. The gain κ is chosen according to relations (12) in Theorem 1. Feasibility of relations (12) for this value of κ and the given matrices A, B, C is checked by means of YALMIP package [58];
- the sampling time T was taken from the interval $T \in [0.02, 0.1]$ s for different simulation runs (a corresponding interval for the transmission rate R is $R \in [10, 50]$ bit/s);
- the coder parameter $M_0 = 5$ is chosen to cover the region of the initial values of y_0. This region is found based on the maximum value of $|y_0|$ over Chua system attractor;
- the coder parameter is taken for each sampling interval T as $\rho = \exp(-\eta T)$, where parameter $\eta = 0.3$. This value is chosen according to Theorem 1 to ensure inequalities (13);
- the initial conditions for the master and slave systems were: $x = [3, -1, 0.3]^{\mathrm{T}}$, $z = 0$;
- the simulation final time $t_{\text{fin}} = 1000$ s.

The normalized state estimation error

$$Q = \frac{\max\limits_{0.8 t_{\text{fin}} \leq t \leq t_{\text{fin}}} \|e(t)\|}{\max\limits_{0 \leq t \leq t_{\text{fin}}} \|x(t)\|}, \tag{21}$$

where $\delta_y(t) = y(t) - \bar{y}_1(t)$, $e(t) = x(t) - \hat{x}(t)$ was calculated.

Simulation results are plotted in Figs. 1–3.

As seen from the plots, the state estimation transient time is about 15 s, which agrees with the chosen value of the coder parameter η. The logarithmic graph of the normalized estimation error Q as a function of the transmission rate R is shown in Fig. 3. It is seen from this plot that if the transmission rate exceeds the *minimal bound* $R_{\min} \approx 23$ bit/s, the proposed state estimation strategy ensures asymptotic vanishing the estimation error. If the transmission rate is less that the bound $R_{\min}$, the estimation is not always possible.

Remark 3. An idealized problem has been considered in this paper to highlight the effect of the data-rate limitations in the state estimation of nonlinear systems. In real-world problems external disturbances, measuring errors and channel imperfections should be taken into account. Evidently in the presence of irregular non vanishing disturbances, asymptotic accurate estimation cannot be achieved.

VI. Conclusion

Limit possibilities of estimation over the limited rate communication channel are evaluated. It is shown that the framework proposed in [36], is suitable not only for controlled master-slave synchronization, but also for state estimation of nonlinear oscillatory (chaotic) system via a communication channel with limited information capacity.

We propose a simple coder-decoder scheme and provide theoretical analysis for multi-dimensional master-slave systems represented in Lurie form. An estimation algorithm is proposed based on the Passification Theorem [36], [38], [39], [49], [50]. It is shown that the estimation error exponentially tends to zero for sufficiently high transmission rate (channel capacity). The key point of the estimation analysis is comparison of the hybrid system in question with an auxiliary continuous-time system (*the continuous model*) possessing useful stability and passivity properties. Such an approach was systematically developed in the 1970s under the name of *the Method of Continuous Models* [54], [55].

The results are applied to state estimation of chaotic Chua system via a communication channel with limited capacity. Simulation results illustrate and confirm the theoretical analysis. Unlike many known papers on state estimation of nonlinear systems over a limited-band communication channel, we propose and justify a simple coder/decoder scheme, which does not require transmission of the full system state vector over the channel.

Future research is aimed at examination of more complex system configurations, where channel imperfections (drops, errors, delays) will be taken into account.

Fig. 1. Time histories of $x_2(t)$, $\hat{x}_{d,2}(t)$, $e_2(t) = x_2(t) - \hat{x}_{d,2}(t)$. $R = 25$ bit/s

Fig. 2. Time histories of $x_3(t)$, $\hat{x}_{d,3}(t)$, $e_3(t) = x_3(t) - \hat{x}_{d,3}(t)$. $R = 25$ bit/s

Fig. 3. Normalized synchronization error Q vs transmission rate R.

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