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Nonlinear problems in control of manufacturing systems ⋆

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Abstract:
Nowadays, production control problems has been widely studied and a lot of valuable approaches have been implemented. This paper addresses the problem of tracking the uncertain demand in case of uncertain production speeds. The uncertainties are described by deterministic inequalities and the performance is analyzed in from of the worst-case scenario. First, simple mathematical models are introduced and the control problem is formulated. In continuous-time, the cumulative output of a manufacturing machine is the integral of the production speed over time. At the same time, the production speed is bounded from below and above, and hence the manufacturing process can be modeled as an integrator with saturated input. Since the cumulative demand (which is the reference signal to track) is a growing function of time, it is natural to consider control policies that involve integration of the mismatch between the current output and current demand. In the simplest consideration it results in models similar to a double integrator closed by saturated linear feedback with an extra input that models disturbances of a different nature. This model is analyzed and particular attention is devoted to the integrator windup phenomenon: lack of global stability of the system solutions that correspond to the same input signal. As a systematic design procedure to prevent the windup phenomenon we present observer based techniques with full and reduced-order observers. The next part of the paper deals with a similar control problem in discrete-time under the surplus-based policy: each machine in the production network tracks the demand trying to keep the downstream buffer at some specified safe level. The performance of manufacturing networks with different topologies is analyzed via the second Lyapunov method, while the disturbances are modeled by deterministic inequalities. The nature of the approach leads to performance analysis in the form of a worst case scenario and allows to find a trade-off between the inventory for each machine in the system and the demand tracking accuracy. The final part of the paper illustrates how to make the theoretical findings operational with the experimental setup called Liquitrol. The experimental setup consists of a number of water tanks and pumps that can be interconnected via flexible piping. Due to its flexibility and mobility, the setup allows not only to verify theoretical results via experiments, but it also can be used in an educational process to illustrate different phenomena in tandem and re-entrant manufacturing networks.

Keywords: Manufacturing systems, performance analysis, Lyapunov function, nonlinear systems

1. INTRODUCTION

The production control of manufacturing systems, i.e. how to control the production rates of machines such that the system tracks a certain customer demand while keeping a low inventory level, has been a field of interest for several decades. Early control strategies based on simple push and pull concepts, such as material requirements planning (MRP), enterprize resources planning (ERP), and just-in-time (JIT), see e.g. (Hopp and Spearman, 2000), can provide an adequate solution if the system requirements are not very strict and a fast reaction to possible disturbances/failures is not required (e.g. since such disturbances/failures hardly occur). However, as manufacturing systems become more complex and the system's performance

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must constantly improve in order to stay competitive in today's global economy, these control strategies become less effective.

A possible way to tackle the problem is to describe manufacturing systems using so-called flow models, see e.g. (Alvarez-Vargas et al., 1994). These models, which are based on ordinary differential/difference equations (ODEs), or sometimes partial differential equations (PDEs), see e.g. (van den Berg et al., 2008; Lefeber et al., 2005), form a continuous approximation of the discrete-event manufacturing systems and therefore result in a simpler control problem. Moreover, various (advanced) control theories are already available for ODEs, which makes these models attractive to work with.

In this paper we study nonlinear problems that arise in control of manufacturing systems. First under quite general assumptions we derive a very simple model for a manufacturing machine and present some nonlinear phenomena caused by saturation in the control loop. In particular we will show that the saturation nonlinearity can result in co-existence of multiple steady-state solutions. That phenomenon occurs if the disturbances present in the system are not negligible. To overcome this difficulty we present an observer-based approach that makes the closed loop system convergent: for any bounded disturbance there is only one bounded and stable steady-state mode. Therefore the system can be defined as a (Lipschitz) continuous operator from the input to states in a functional space. It makes it possible to study the steady-state performance as properties of this operator.

Once a control algorithm for a single manufacturing machine is known, a natural question arise: To study control problems for manufacturing networks of more complex topology. In this paper we address a solution to this problem in both continuous- and discrete-time settings. The approach presented in the paper is based on a surplus-based control policy, while the system performance is studied in terms of deterministic inequalities describing the worst case scenario. Those inequalities come from the direct Lyapunov method based on particular choice of the Lyapunov functions. In the surplus approach, control decisions are made based on the difference between the cumulative demand and the cumulative output, respecting buffer constraints and penalizing the system for deviations of the buffer content from some desired safe level. Some examples of similar ideas can be found in (Bielecki and Kumar, 1988; Bonvik et al., 1997; Lefebvre, 1999; Gershwin, 2000; Quintana, 2002; Kogan and Perkins, 2003; Boukas, 2006; Subramaniam et al., 2009; Savkin and Somlo, 2009; Nilakantan, 2010) and references therein.

It is common practice to use a computer simulation to validate theoretical results on control of manufacturing networks. In this paper, we also briefly mention an alternative experimental approach based on a water emulator of manufacturing networks. This emulator is designed to visualize the results of different control policies for various manufacturing networks. The experimental setup is relatively small in size, which makes it possible to use it as a demonstration tool for education purposes.

The paper is organized as follows. Section 2 presents a simple model for a manufacturing machine with a saturation-like nonlinearity. Some nonlinear control problems caused by this nonlinearity are discussed and a possible solution is presented. Section 3 extends the approach towards a tandem lines via surplus control policy in continuous time. Section 4 presents some results on control of manufacturing systems under surplus based policy in discrete-time. Section 5 is devoted to experimental validation of the derived results via an experimental setup. An utilization of this setup for educational purposes is discussed.

2. CONTINUOUS-TIME MODEL OF ONE MANUFACTURING MACHINE

Let us use a continuous approximation of a single discrete-event manufacturing machine. Namely, consider a manufacturing machine that produces items with a *production rate* $u_p(t) \in \mathbb{R}$, $t \in \mathbb{R}$. Assume that there is always sufficient raw material to feed the machine. The total amount of items produced by the machine is denoted by $y(t) \in \mathbb{R}$ and is related to production rate $u_p(t)$ by the following equation

$$\dot{y}(t) = u_p(t) + f(t), \tag{1}$$

where $f(t) \in \mathbb{R}$ stands for an unknown *external disturbance*. This term may describe manufacturing losses, or variations of the machine capacity, for example. The production rate u_p is considered to be positively valued and has a certain upper bound $u_{p,\max}$, caused by the machine capacity limitation. Therefore, the following bounds are valid for the production rate:

$$0 \leq u_p \leq u_{p,\max}. \tag{2}$$

Assuming that the production capacity of the machine is limited, inequality (2) introduces a *saturation* in the control loop.

We pose the control goal as to track a non-decreasing cumulative demand. In what follows, we assume that $y_d(t)$ may be modeled as

$$y_d(t) = y_{d,0} + v_d\, t + \varphi(t), \tag{3}$$

where $y_{d,0}$ denotes the desired production at $t = 0$, v_d is a constant that represents the average desired production rate, $\varphi(t)$ is a bounded function, describing fluctuation of the desired production from the linearly increasing time-varying demand, caused by market (e.g. seasonal) fluctuations. It is natural to suppose that the following inequalities (also known as capacity condition) are satisfied

$$0 \leq v_d \leq u_{p,\max}. \tag{4}$$

Thus the production rate is nonnegative and, on average, the machine is capable to satisfy the product demand. It may be also assumed that $\varphi(t)$ has a "zero mean" in some sense, because its averaged value may be considered as part of $y_{d,0}$. To derive a controller that solves the control problem, let us neglect for a moment constraint (2) assuming that both $f(t)$ and $\varphi(t)$ are zero. In this case we get a linear system (the integrator - (1)) that should follow a linearly growing demand. By the Final Value Theorem it is straightforward to see that there is no *static* controller capable to solve the control goal asymptotically, i.e. $|y(t) - y_d(t)| \to 0$ as $t \to \infty$. Particularly, any static feedback will cause a static mismatch between y and y_d and this mismatch can be made small either by a feedforward compensation or by a sufficiently large control gain. For the feedforward compensation, the rate v_d should be known in advance – an assumption that is not realistic in many practical situations. To increase the static gain is not a good option in some cases too: In this case all the disturbances present in the system will be amplified by that gain and hence, the overall performance can degrade. From the Final Value Theorem one concludes that, in order to achieve the control objective asymptotically, the controller should contain an integrator to eliminate the static error. The simplest way to accommodate this is by means of a PI controller: the production rate should be proportional to the mismatch between $y(t)$ and $y_d(t)$ as well as to the integral of this mismatch from the initial time to the current time.

Let us investigate how this controller performs in the presence of saturation *and* disturbances $f(t), \varphi(t)$: to this end let us consider a dynamical system that represents an integrator controlled by a proportional-integrator controller subject to saturation in control loop and external disturbances. A qualitative mathematical model of such a system can be represented by a simple second-order differential equation:

$$\ddot{x} = \mathrm{sat}(-\dot{x} - x + w(t)) \tag{5}$$

where $x(t)$ stands for the mismatch between y and y_d, $w(t)$ represents a cumulative disturbance and $\mathrm{sat}(\cdot)$ stands for the saturation function, taken for example as $\mathrm{sat}\,\xi = \mathrm{sign}(\xi)\min\{1, |\xi|\}$. In order to get the description (5) one has to assume that the nominal demand rate is one half of the maximal production rate, that corresponds to an average utilization of 50% of the manufacturing machine. The controller gains are chosen to be 1's. One may argue that both assumptions are impractical, however to capture the effect of saturation in the presence of disturbances, such simple settings suffice.

2.1 Dynamics of system (5)

Let us summarize some facts about system (5). First of all, it is straightforward to prove that in the absence of disturbances ($w(t) \equiv 0$), the system is globally asymptotically stable. The Lyapunov function that proves this claim is not quadratic, yet simple: a weighted sum of x^2, $\mathrm{sat}^2(\dot{x} + x)$ and $\int_0^{\dot{x}+x} \mathrm{sat}(\xi)d\xi$ is a good candidate for the Lyapunov function.

In the presence of disturbances, however, things become more complicated: it is known that system (5) is not L_p-stable from w to $(x, \dot{x})^\top$ and the question if there is a finite L_2-gain from w to $\ddot{x}$ is still open (Rantzer, 1999).

A simple numerical experiment illustrates effects caused by the disturbances, as is depicted in Fig. 1. Specifically, for a harmonic input $w(t) = b \sin \omega t$ let us increase the amplitude b of the excitation from zero so slowly that the transient behavior becomes negligible. In doing so, we see that the amplitude of the argument $-\dot{x}-x+w$ of the saturation nonlinearity increases gradually until a jump occurs. Surprisingly, if one then starts to slowly decrease b, the amplitude of ξ demonstrates a hysteresis-like behavior clearly displayed in Fig. 1 for $\omega = 0.75$. It follows that the same input signal gives rise to *multiple steady-state periodic solutions* for b's beneath the hysteresis loop (approximately for $b \in (1.6, 1.8)$), which is incompatible with global stability of any of them. With practical relevance in mind, it would be interesting to obtain conditions on the input signal that guarantee global stability of the steady-state forced oscillations in systems with saturation in feedback. Such conditions are derived in Pogromsky and Matveev (2013) and Pogromsky et al. (2013). An approach pursued therein is based on the concept of the averaging functions – a useful tool based on the ideas originated in studies on estimation of dimensions (Hausdorff, fractal, Lyapunov) of attractors of dynamical systems Leonov (2008); Boichenko et al. (2005). In particular, it is possible to prove that if the input $w(t)$ is bounded on $\mathbb{R}$ and satisfies the following condition:

$$\limsup_{T\to\infty} \sup_{t_0} \frac{1}{T} \int_{t_0}^{t_0+T} \dot{w}^2(s)ds < \frac{1}{2}, \tag{6}$$

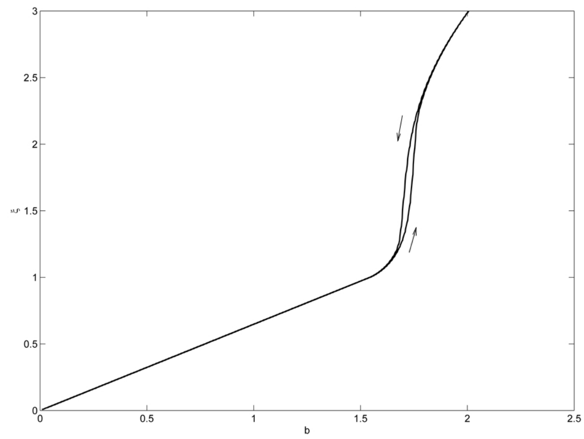

Fig. 1. Amplitude of $-\dot{x} - x + w$ versus the amplitude b of the excitation signal $w(t) = b \sin \omega t$ with $\omega = 0.75$.

then system (5) possesses a unique solution $\bar{x}$, bounded on $\mathbb{R}$, and this solution is uniformly globally asymptotically stable. Though condition (6) is a sufficient condition for stability, numerical experiments, similar to the one explained above, show that it is relatively close to necessary at least for some parameters of the harmonic input signal. Those experiments reveal that for the same harmonic input $w(t)$, the system can have two *stable* steady-state solutions. At one of those steady states, the system has approximately the same performance as the linear one (e.g. neglecting saturation), while at the second steady-state mode the performance of the system is significantly worsened and corresponds to a bang-bang mode of the saturation nonlinearity. This degradation is caused by the so-called "integrator windup". A terrifying illustration of this detrimental effect is given by the pilot-induced oscillations that entailed the YF-22 crash in April 1992 Dornheim (1992) and Gripen crash in August 1993 Shifrin (1993). The aforementioned bang-bang mode is clearly seen on the video

`http://www.youtube.com/watch?v=faB5bIdksi8`

capturing the accident with YF-22. To prevent such performance degradation, various "anti-windup" (AW) techniques were proposed; for excellent survey, we refer the reader to (Tarbouriech and Turner, 2009; Zaccarian and Teel, 2011), see also (Leonov et al., 2012; Andrievsky et al., 2012b).

Conditions similar to (6) can be derived also for dynamical systems more complex than (5), see the forthcoming publication Pogromsky et al. (2013), though its particular form is not relevant for this presentation. We conclude this section by the lesson one can learn from it: the PI controller is the simplest possible linear controller capable to achieve the control objective asymptotically if one neglects saturation and disturbances $f(t), \varphi(t)$. At the same time in the presence of saturation and disturbances it can result in very poor performance. In particular, for "large disturbances", violating conditions similar to (6) the manufacturing machine can operate in a bang-bang mode – an undesired behavior when the machine is idle for some relatively long period and working at full speed during another relatively long period.

2.2 An observer-based approach to the AW compensation

As is mentioned above, the main reason to introduce the integrator in the feedback loop is to cope with an unknown demand rate v_d. At the same time the approach based on the integrator in feedback has severe limitations caused by the saturation and disturbances. A possible way to overcome the problem is to introduce an anti-windup (AW) compensation to prevent the system from performance degradation caused by the saturation in the control loop.

In this section we briefly outline an approach to design the observer-based AW compensation (Hippe, 2006; Andrievsky et al., 2009; Pogromsky et al., 2010; Tarbouriech et al., 2011; Andrievsky et al., 2012a). To expose this idea, assume for a moment that the demand rate v_d is known, the disturbance $f(t)$ is measured and the fluctuation $\varphi(t)$ is zero. The controller has to update the production speed u_p based on the signal $e := y_d(t) - y(t)$ that is also assumed to be measured. In this case one can attempt to design the following controller:

$$u_p(t) = \mathrm{sat}_{[0,u_{p,\max}]}\big(k_p e(t) + r(t)\big), \tag{7}$$

where $r(t) := v_d - f(t)$, k_p is the controller parameter (a *proportional gain*), to ease the presentation, take $k_p = 1$, $\mathrm{sat}_{[a,b]}(z)$ denotes the shifted *saturation function*,

$$\mathrm{sat}_{[a,b]}(z) = \begin{cases} b & \text{if } z \geq b, \\ a & \text{if } z \leq a, \\ z & \text{otherwise}, \end{cases} \qquad (b > a). \tag{8}$$

Equations (1), (7) describe the closed-loop manufacturing system model for time-varying demand $y_d(t)$ given by (3). The analysis for the case of linear time-varying demand under the valid assumption $0 < r(t) < u_{p,\max}$ for all t shows that the closed-loop system, expressed in terms of demand mismatch,

$$\dot e(t) = -\,\mathrm{sat}_{[0,u_{p,\max}]}\big[e(t) + r(t)\big] + r(t), \tag{9}$$

presents an asymptotically stable behavior in the following sense: There is a unique solution $\bar e(t)$, bounded on $\mathbb{R}$, and this solution is globally uniformly asymptotically stable. At this moment we emphasize that all the solutions are bounded on the semi- axis $[t_0, \infty)$, yet there is only one solution bounded for both negative and positive times. This mathematical abstraction allows to give a rigorous definition of what we call the "steady-state" mode. Systems that have a unique asymptotically stable mode are usually referred to as *convergent systems*, see Pavlov et al. (2004) for related results.

The control algorithm (7) contains the feedforward term $r(t)$ and, in practically relevant cases, it is not available for measurements. In this case a practical solution is to estimate r via the measured signal e and to use this estimate in (7) instead of real value $r(t)$. This is a typical observer design problem for an *unknown input*. To design an observer for the unknown input one has to augment the system dynamics with extra dynamics that models the input behavior. The simplest model in this case is to assume that r is constant and to build an observer for that constant. Then, if the dynamics of the observer are fast enough the observer will be able to update the estimate of r even if this signal is varying with time. In this case the reduced-order observer (Luenberger, 1971) is given by the following equations

$$\begin{cases} \dot\sigma(t) = -\lambda\sigma(t) - \lambda^2 e(t) + \lambda u_p(t), \\ \hat r(t) = \sigma(t) + \lambda e(t), \end{cases} \tag{10}$$

where $\hat r(t)$ denotes the estimate of the signal $r(t)$, produced by observer (10), $\lambda > 0$ is the observer parameter (observer gain), setting the transient time for the estimation procedure.

An observer of full order, estimating the constant r, has the following form (Kwakernaak and Sivan, 1972; Andrievsky et al., 2012c):

$$\begin{cases} \dot\sigma_1(t) = -l_1\sigma_1(t) + \sigma_2(t) - u_p(t) + l_1 e(t), \\ \dot\sigma_2(t) = -l_2\sigma_1(t) + l_2 e(t), \end{cases} \tag{11}$$

$$\hat r(t) = \sigma_2(t), \tag{12}$$

where $L = [\lambda_1, \lambda_2]^\top \in \mathbb{R}^2$ is the vector of observer (12) design parameters. It is worth mentioning that the full order observer (12) is less sentitive to measurement errors than the reduced-order observer (10), but the last one produces the estimate in a faster fashion. Since the measurement errors are not significant for the considered manufacturing control problem, in what follows the reduced-order observer (10) is employed. If the estimate $\hat r(t)$ is used in control law (7), instead of $v_d(t) - f(t) = r(t)$, then the control signal $u_p(t)$ takes the form:

$$u_p(t) = \mathrm{sat}_{[0,u_{p,\max}]}\big(e(t) + \hat r(t)\big), \tag{13}$$

where $e(t) = y_d(t) - y(t)$, $\hat r(t)$ is governed by (10).

Equations (10), (13) describe the first-order feedback controller. The control signal $u(t) = e(t) + \hat r(t)$ is calculated based on the measurement error $e(t)$ only. The gain $\lambda > 0$ is a design parameter. The performance of the closed-loop system (1), (10), (13) differs from that of the system with controller (7) due to presence of the estimation error $\varepsilon_r(t) = r(t) - \hat r(t)$. This error is caused by the difference in the initial conditions of the external and estimated signals, fluctuation $\varphi(t)$ of $y_d(t)$, inconstancy of $f(t)$ and the measurement errors.

To find the estimation error let us write down the plant–observer model taking into account representation (3) for the reference signal $y_d(t)$. This leads to the following equations:

$$\begin{cases} \dot y(t) = u_p(t) + f(t), \\ e(t) = y_{d,0} + v_d t + \varphi(t) - y(t), \\ \hat r(t) = \sigma(t) + \lambda e(t), \\ \dot\sigma(t) = -\lambda\sigma(t) - \lambda^2 e(t) + \lambda u_p(t). \end{cases} \tag{14}$$

where $u_p(t)$, $f(t)$, $y_{d,0}$, v_d, $\varphi(t)$ are external inputs. After the simple algebra we obtain from (14) that

$$\begin{aligned} &\varepsilon_r(t) = \mu(t) - \xi(t), \\ &\dot\mu(t) + \lambda\mu(t) = \lambda\xi(t), \;\; \mu(0) = \mu_0, \end{aligned} \tag{15}$$

where $\xi(t) = f(t) + \lambda\varphi(t)$. We see that the error $\varepsilon_r(t)$ is independent of $u_p(t)$, $y_{d,0}$, v_d in (14). Taking into account the estimation error $\varepsilon_r(t)$, the control law (7) reads as

$$u_p(t) = \mathrm{sat}_{[0,u_{p,\max}]}\big[e(t) + v_d - f(t) - \varepsilon_r(t)\big], \tag{16}$$

and the closed-loop system dynamics is described by (1), (15), (16). If there exists a fluctuation of the desired production $\varphi(t) \neq \mathrm{const}$, convergence of the tracking error $e(t)$ to zero can not be attained. In such a case it is advisable, both for simplification of the analysis and for industrial applicability of the system, to assure convergence of the system trajectories to each other for different initial conditions. This property is represented by the notion of the *incremental stability* (Angeli, 2002). To verify incremental stability of the considered system, let us rewrite (1), (7) in transformed coordinates. Introduce a new variable z as $z(t) = y(t) - y_{d,0} - v_d t$. Differentiating $z(t)$ wrt t and taking into account (1), we obtain that $\dot z(t) = u_p(t) - v_d + f(t)$. Then the tracking error $e(t)$ reads as

$$e(t) = y_d(t) - y(t) = y_{d,0} + v_d t + \varphi(t) - y(t) = \varphi(t) - z(t).$$

Let us rewrite expression (7) for control signal u_p using the standard (symmetric) saturation function $\mathrm{sat}_m(x) = \min\big(m, \max(-m, x)\big)$. Introducing the "unsaturated" control signal as $u(t) = e(t) + v_d - f(t)$ we obtain that $u_p =$

$\mathrm{sat}_{[0,u_{p,\max}]}(u) = \mathrm{sat}_m(e + v_d - f - m) + m$. This leads to the following closed-loop system model in the transformed coordinates:

$$\dot{z}(t) = \bar{u}(t) + \eta(t), \tag{17}$$

$$\begin{cases} \bar{u}(t) = \mathrm{sat}_m \big[e(t) + \hat{r}(t) - m\big], \\ e(t) = \varphi(t) - z(t), \end{cases} \tag{18}$$

$$\begin{cases} \hat{r}(t) = \sigma(t) + \lambda e(t), \\ \dot{\sigma}(t) = -\lambda\sigma(t) - \lambda^2 e(t) + \lambda(\bar{u}(t) + m), \end{cases} \tag{19}$$

where $\eta(t) = f(t) + m - v_d$, $m = u_{p,\max}/2$.

Recall that *convergent systems* being excited by a bounded input have a unique bounded globally asymptotically stable steady-state solution (Pavlov et al., 2004, 2007). Therefore, for any given input the solutions of the convergent system independently of the initial conditions converge to the uniquely defined limit solution. Let us check if this property is valid for system (17)–(19), forced by the reference signal $\varphi(t)$. Let us apply the general results presented in van den Berg (2008). For doing that, rewrite the closed-loop system model in the following state-space form:

$$\begin{aligned} &\dot{x}(t) = Ax(t) + B\,\mathrm{sat}_m(u) + Fw(t), \\ &u(t) = Cx(t) + Dw(t), \;\; z(t) = Hx(t). \end{aligned} \tag{20}$$

To this end, introduce the variables $\psi(t) = \sigma(t) - m$ and $\rho(t) = \lambda e(t) + \sigma(t) - m$. In this new notation, (17)–(19) read as:

$$\begin{aligned} &\dot{z}(t) = \bar{u}(t) + \eta(t), \\ &\bar{u}(t) = \mathrm{sat}_m \big[e(t) + \rho(t)\big], \\ &e(t) = \varphi(t) - z(t), \\ &\rho(t) = \psi(t) + \lambda e(t), \\ &\dot{\psi}(t) = -\lambda\psi(t) - \lambda^2 e(t) + \lambda\bar{u}(t), \\ &\eta(t) = f(t) + m - v_d. \end{aligned} \tag{21}$$

Introducing the state-space vector $x(t) \in \mathbb{R}^2$ as $x = [x_1, x_2]^\top$ where $x_1(t) = z(t)$, $x_2(t) = \psi(t)$ and the external input vector $w(t) \in \mathbb{R}^2$ as $w = [w_1, w_2]^\top$, where $w_1(t) = \varphi(t)$, $w_2(t) = \eta(t)$ we obtain the system model in the form (20) with the following matrices:

$$\begin{aligned} &A = \begin{bmatrix} 0 & 0 \\ \lambda^2 & -\lambda \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ \lambda \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 1 \\ -\lambda^2 & 0 \end{bmatrix}, \\ &C = \big[-(1+\lambda), 1\big], \quad D = \big[1+\lambda, 0\big], \quad H = [1, 0]. \end{aligned} \tag{22}$$

From now on the stability of solutions can be analyzed with quadratic Lyapunov functions: from the representation (22) one can conclude that, if $\lambda > 0$, then there is a positive definite matrix P so that for any *two* different solutions $x^1(t), x^2(t)$ it follows that the quadratic function $V(x^1, x^2) = (x^1 - x^2)^\top P(x^1 - x^2)$ satisfies this inequality $\dot{V} \le 0$.

It is very important to emphasize that stability is a very useful property: once it has been proven, computer simulation becomes a rigorous tool to analyze the system performance for different disturbances. Unlike the situation discussed in Sec. 2.1 the system "forgets" its initial conditions and eventually its performance is solely determined by the input rather than by initial conditions.

3. CONTROL OF A TANDEM LINE OF MANUFACTURING MACHINES

In the previous section a control problem for a sinlle manufacturing machine was posed as a tracking problem. The control objective is stated as to track an unknown cumulative demand in the presence of disturbances. It is shown that due to a possible integrator windup, the saturation in the control loop can not be neglected during the design procedure and a possible way to overcome this difficulty is to employ an observer-based design. In this section this idea is further developed on for a tandem line of manufacturing machines.

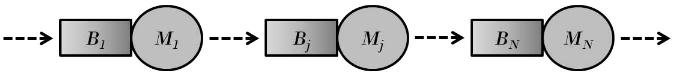

Fig. 2. Manufacturing Line.

Consider a line of N manufacturing machines M_1, M_2, $\ldots M_N$, which are separated by buffers B_j, $j = 1, \ldots, N$ with infinite capacity (see Fig. 2). The first machine M_1 is supplied by raw material, the Nth machine M_N produces finished product. Each machine M_j takes out a raw product from the corresponding input buffer B_j and puts a processed product in the output buffer B_{j+1}. In what follows, suppose that there is always sufficient raw material to feed the first machine, i.e. buffer B_1 is never exhausted.

Assume that a manufacturing machine produces items continuously in time $t \in \mathbb{R}$ with a certain *production rate* $u_j(t) \in \mathbb{R}$, where $j = 1, \ldots N$ is a number of the machine. The total amount of items produced by jth machine is described by a continuous variable $y_j(t) \in \mathbb{R}$. Interaction between the machines is described by the *buffer content* variables $w_j(t) = \max(y_{j-1} - y_j, 0)$, $j = 2, \ldots, N$. The case of $w_j(t) = 0$ means absence of the raw material in the input buffer of jth machine and, therefore, the machine M_j works at the rate of its raw material inflow. The above reasons lead to the following continuous model of the manufacturing machine:

$$\dot{y}_j(t) = \begin{cases} u_j(t) + f_j(t), & \text{if } w_j(t) > 0, \\ 0, & \text{otherwise}, \end{cases} \tag{23}$$

where $t \in \mathbb{R}$ stands for continuous time argument; $j = 1, \ldots, N$ is a machine number; variables $f_j(t) \in \mathbb{R}$ denote *external disturbances*. For example, f_j may describe manufacturing losses, or variations of the machine capacity. The production rates u_j are bounded by $u_{\max}$ due to machine capacity limitation. In the sequel we assume, without loss of generality, that all the machine capacities in the line have the same upper bound $u_{\max}$. Since the production rates u_j can not be negative, the following bounds are valid for $u_j(t)$:

$$0 \le u_j(t) \le u_{\max}, \; j = 1, \ldots, N, \; \forall t \ge 0. \tag{24}$$

Inequalities (24) lead to a *saturation effect* in the system. This effect restricts the production rate, and complicates the design of the controller and the system performance analysis.

Summarizing, we obtain the following manufacturing line model:

$$\begin{cases} \dot{y}_1(t) = u_1(t) + f_1(t), \\ \dot{y}_2(t) = \big(u_2(t) + f_2(t)\big) \cdot \mathrm{sign}(w_2(t)), \\ \ldots\ldots \\ \dot{y}_N(t) = \big(u_N(t) + f_N(t)\big) \cdot \mathrm{sign}(w_N(t)), \end{cases} \tag{25}$$

where $\mathrm{sign}(\theta) = \big(1, \text{ if } \theta > 0 \mid 0, \text{ otherwise}\big)$.

Let us extend the control strategy for a single-machine (10), (13) to the control of a manufacturing line. Direct usage of (10), (13) for each machine is unsatisfactory, because it does not take into account the buffer contents, which leads to exhaustion of some buffers or, alternatively, to stacking in buffers an extra amount of material. Besides, for implementational reasons, it is desirable to organize interactions between neighboring machines only and to avoid transferring the reference signal to each machine. Due to these reasons, in this section, we propose a modification of the control strategy (10), (13), intended to control of a manufacturing line.

Firstly, introduce the desired constant buffer content level $w_d > 0$. Add the "penalty" term $k_w\big(w_d - w_{j+1}(t)\big)$ (used to slow down the production in case the buffer content is bigger than the desired one), where $k_w > 0$ is a certain gain (design parameter) to jth control action $u_j(t)$. Secondly, change the demand signal for jth machine to ensure equality $y_{j-1}(t) = y_j(t) + w_d$ in the steady-state nominal regime. Starting from these reasons, the following control strategy for a line of N machines is obtained. This strategy is described below in recursive form.

Take the control law for Nth machine in the form (10), (13), namely let the control signal $u_N(t)$ be calculated as:

$$\begin{cases} u_N = \mathrm{sat}_{[0,u_{\max}]}\left(k_p\varepsilon_N + \hat{r}_N\right), \\ \hat{r}_N(t) = \sigma_N(t) + \lambda e(t), \\ \dot{\sigma}_N(t) = -\lambda\sigma_N(t) - \lambda^2 e(t) + \lambda u_N(t), \end{cases} \tag{26}$$

where $\varepsilon_N(t) \equiv e(t) = y_d(t) - y_N(t)$ is the reference error. Take the control law for machine M_j, $j = 1,\dots, N-1$ in the following form:

$$\begin{cases} u_j = \mathrm{sat}_{[0,u_{\max}]}\big(k_p\varepsilon_j + \hat{r}_j + k_w(w_d - w_{j+1})\big), \\ \varepsilon_j(t) = w_d + \varepsilon_{j+1}(t) - w_{j+1}(t) \\ \hat{r}_j(t) = \sigma_j(t) + \lambda\varepsilon_j(t), \\ \dot{\sigma}_j(t) = -\lambda\sigma_j(t) - \lambda^2\varepsilon_j(t) + \lambda u_j(t), \end{cases} \tag{27}$$

where $w_{j+1}(t) = y_j(t) - y_{j+1}(t)$; w_d is the buffer contents demand. Formulas (26), (27) recursively specify the distributed controller for a line of $N \geq 2$ manufacturing machines.

Performance analysis of the introduced model has been performed in (Andrievsky et al., 2012a, 2009). This analysis benefits from the representation of the system equations as a cascade of convergent systems. Such a representation makes it possible to analyze the bull-whip effect in the tandem line. It is worth mentioning that the analysis performed in our previous publications assumes that the desired buffer content as a controller parameter ensures that the intermediate buffers are never empty. A more detailed analysis is the subject of forthcoming research.

4. SURPLUS-BASED CONTROL OF MANUFACTURING NETWORKS IN DISCRETE-TIME

The models of manufacturing networks discussed so far are based on a continuous-time approximation. Moreover, it was assumed that the production speed can change continuously within a certain admissible interval. In some situations the production speed can take only fixed discrete values, that represent, for example, the status of the manufacturing machine: either busy, or idle. In this section it is shown how to analyze the system performance in this case.

The main ideas exploited in the previous section can be utilized to some extent to the discrete-time case. For example, a manufacturing process can be modeled as follows:

$$y(k+1) = y(k) + u(k) + f(k),$$

where $y(k)$ is the cumulative output, $u(k)$ is the production speed, and $f(k)$ stands for the disturbance that should satisfy the constraint $u(k) + f(k) \geq 0$. As before, the control objective can be posed as a tracking problem for the cumulative demand

$$y_d(k) = y_{d0} + v_d k + \varphi(k) \tag{28}$$

with some unknown nominal demand speed v_d and disturbance $\varphi(k)$. Let us assume that the production speed can take only two values: the machine can either work at full speed or be idle. In this case, if one seeks for an optimal controller that minimizes the mismatch between the cumulative output and cumulative demand, the on-off static controller is a natural candidate. This is indeed the case at least for a single manufacturing machine, see Starkov et al. (2012a). Motivated by this study one can attempt to utilize a simple static on-off controller in a more sophisticated case of multiple manufacturing machines.

Keeping in mind possibly different production speeds for each machine and the constraints imposed by buffers between the machines, one can derive the following model of line of manufacturing machines:

$$y_1(k+1) = y_1(k) + \beta_1(k)u_1(k), \tag{29}$$

$$\begin{aligned} y_j(k+1) = {} & y_j(k) \\ & + \beta_j(k)u_j(k)\mathrm{sign}_{\mathit{Buff}}(w_j(k) - \beta_j(k)), \\ & \forall j = 2,\dots,N, \end{aligned} \tag{30}$$

where all the events within the model occur at given time instances and k represents the current time, so that the time step between all the events is constant. Here $y_j(k)$ is the cumulative output of machine M_j at time k, $w_j(k) = y_{j-1}(k) - y_j(k)$ is the buffer content of buffer B_j, $\beta_j(k) = \mu_j + f_j(k)$, $\forall j = 1,\dots,N$, μ_j is the processing speed of machine j, f_j is the external disturbance affecting machine M_j (e.g. production speed variations, delay), u_j is the control input of machine M_j and $\mathrm{sign}_{\mathit{Buff}}(w_j(k) - \beta_j(k)) = 1$, if $w_j(k) - \beta_j(k) \geq 0$ and 0, otherwise. This last function describes the minimal buffer level that is needed in order for machine M_j to start its production. The equation (29) and (30) present a general model that can describe a product flow for a wide range of manufacturing lines. Further specific assumptions on system (29) and (30) are given later on.

The on-off controller for the tandem line is given by:

$$\begin{aligned} u_j(k) &= \mathrm{sign}_+(\varepsilon_{j+1}(k) + (w_{d_{j+1}} - w_{j+1}(k))), \\ & \qquad \forall j = 1,\dots,N-1, \end{aligned} \tag{31}$$

$$u_N(k) = \mathrm{sign}_+(y_d(k) - y_N(k)), \tag{32}$$

where $w_{d_{j+1}}$ is the desired buffer level (base stock) of buffer B_{j+1}, ε_{j+1} is the demand tracking error of machine M_{j+1} and $y_d(k)$ is the cumulative production demand given by (28). The function sign_+ takes value 1 for positive arguments and 0 for negative, its value for the zero argument can be arbitrary between 0 and 1. The demand tracking error of each machine is given by:

$$\begin{aligned} \varepsilon_j(k) &= \varepsilon_{j+1}(k) + (w_{d_{j+1}} - w_{j+1}(k)), \\ & \qquad \forall j = 1,\dots,N-2, \end{aligned} \tag{33}$$

$$\varepsilon_{N-1}(k) = \varepsilon_N(k) + (w_{d_N} - w_N(k)), \tag{34}$$

$$\varepsilon_N(k) = y_d(k) - y_N(k). \tag{35}$$

Thus, the demand tracking error $\varepsilon_j(k)$ depends on the number of produced products $y_j(k)$ with respect to current demand $y_d(k)$ and desired buffer content $w_{d_{j+1}}$ of each downstream buffer. This means that every upstream machine needs to supply $w_{d_{j+1}}$ lots more than the downstream one. The constant parameter w_d is introduced in order to prevent downstream machines from lot starvation, e.g. in case of a sudden growth of the product demand.

Basically, model (29), (30) describes the product flow through the line of N manufacturing machines. The first machine described by (29) is considered to have always access to raw material and there is always sufficient raw material. The provision of this raw material to machine M_1 is decided by the control input (31). Here we consider that our control input is acting as an authorizing switch, which turns on M_1 iff its demand tracking error (33) is positive and turns M_1 off if its demand tracking error is negative or zero. Tracking error $\varepsilon_1(k)$, see (33), consists of the difference between what has been done ($w_2(k)$), what has to be done ($\varepsilon_2(k)$) and what has to be always in the buffer (w_{d_2}). It can be seen from (33), (34) and (31) that the same demand tracking error and control logic are applied to the rest of the machines till machine M_{N-1}. As for the last machine in the line, which is machine M_N, the control action is still based on the authorizing switch. However, the difference consists in the logic that triggers this switch. We expect that on the output of machine M_N the cumulative product demand is followed by the cumulative production of this machine. The control switch activates or deactivates machine M_N based directly on the production demand status (35). This control logic is the same as in the single machine case, presented in the previous section. The difference in models for the other machines can be seen through the general flow model (30). Here, for each machine in the line we introduce an extra restriction on the buffer content of each upstream buffer. Function $\text{sign}_{\mathit{Buff}}(w_j(k) - \beta_j(k))$ is acting as an extra authorization together with the control input. Thus, any machine M_j, with $j = 1, \dots, N-1$, is activated only if two authorizations are given. The first authorization comes from the control input ($u_j(k)$) of the machine, which is based on the current demand tracking error status of this machine ($\varepsilon_j(k)$). The second authorization comes from the buffer content restriction which is granted if the buffer contains at least the minimal number of products required ($\beta_j(k)$) in order for the machine M_j to start its work. By $\text{sign}_{\mathit{Buff}}(w_j(k) - \beta_j(k))$ we imply that in order to start producing machine j is restricted to take a certain nonzero amount of products from its upstream buffer. This amount is defined by the processing speed of the machine from time k till time $k+1$, which for this authorizing action is assumed to be known in advance. In reality the interpretation of the function $\text{sign}_{\mathit{Buff}}(w_j(k)-\beta_j(k))$ may vary depending on the physical system and its limitations. For example the restrictive action, given by this function, may already be implicit in the system, i.e. the system can be stopped if no product is detected on its input, or the system can be stopped unless some fixed quantity of material, e.g., a batch of products, is present on its input. In its essence the function $\text{sign}_{\mathit{Buff}}(w_j(k) - \beta_j(k))$ is imposed so to prevent the model of the network from describing active manufacturing processes while having insufficient product content in its buffers in order to initiate these processes. Thus, though the interpretation of function $\text{sign}_{\mathit{Buff}}(w_j(k) - \beta_j(k))$ may vary, the essence of its restrictive action is preserved for any buffered manufacturing network. It is also important to take into account that the control actions are decentralized throughout the network. In other words, the control action of each machine in the line depends only on the demand tracking error of its neighboring downstream machine (except for machine M_N, which depends directly on cumulative demand input) and the current buffer content of its upstream buffer.

To analyze the system behavior one has to impose some assumptions on the disturbances and the production speeds. In particular, we assume that there are constants α_1, α_2 and α_3 such that

$$\alpha_1 < \varphi(k+1) - \varphi(k) - f_j(k) < \alpha_2,\ \forall k \in \mathbb{N},\ j = 1, \dots, N,$$

and $f_j(k)$ satisfies

$$f_j(k) \le \alpha_3,\ \forall k \in \mathbb{N},\ j = 1, \dots, N. \tag{36}$$

Moreover, those constant satisfy the following constraints

$$\alpha_2 < \mu_j - v_d,\ \forall j = 1, \dots, N, \tag{37}$$

$$\alpha_1 > -v_d. \tag{38}$$

The previous inequalities are referred to as *capacity condition*. The performance of the closed loop system can be analyzed via the direct Lyapunov method. In this case, one is interested in the worst case bounds for the mismatch between the cumulative output and cumulative demand as time is sufficiently large.

To illustrate the main idea behind this method, suppose that the line has only two manufacturing machines. In this case the Lyapunov function candidate for the performance analysis looks as

$$V^{2M}(\varepsilon_1(k), \varepsilon_2(k)) = V_1(\varepsilon_1(k)) + V_2(\varepsilon_2(k)), \tag{39}$$

where

$$V_1 = \max\left(\left|\varepsilon_1 - v_d - \frac{(\alpha_2+\alpha_1)}{2} + \frac{\mu_1}{2}\right| - \frac{(\alpha_2-\alpha_1)}{2} - \frac{\mu_1}{2}, 0\right),$$

$$V_2 = \frac{1}{n}\max\left(\left|\varepsilon_2 - v_d - \frac{(\alpha_2+\alpha_1)}{2} + \frac{\mu_2}{2}\right| - \frac{(\alpha_2-\alpha_1)}{2} - \frac{\mu_2}{2}, 0\right),$$

with n be a positive constant. To perform a performance analysis of the closed loop system under the worst case scenario one can try to find a constant C so that the level set $\Omega := \{\varepsilon_1, \varepsilon_2 \mid V^{2M} \le C\}$ is positively invariant. In this analysis one can treat the constants w_{d_j} (desired inventory level) as parameters. The performance of the closed loop system indeed depends on such parameters: The larger the buffer contents, the faster the system can react on the disturbances. At the same time, since the disturbances are bounded by known constants, there should be a limit for those w_{d_j}'s after which an increase of the buffer contents will not improve the system performance. To find those limits analytically can be a tricky problem and one can try to solve an easier problem: To find conditions on w_{d_j}'s that correspond to the smallest possible size of Ω for the chosen Lyapunov function. This problem is solved in Starkov et al. (2012c), where it is shown that if the controller parameters satisfy the following inequalities

$$w_{d_j} \ge \mu_j + \mu_{j-1} + \alpha_3 + \alpha_2 - \alpha_1,$$

then the following estimates are valid:

$$\limsup_{k\to\infty} \varepsilon_j(k) \le v_d + \alpha_2, \quad (40)$$

$$\liminf_{k\to\infty} \varepsilon_j(k) \ge v_d + \alpha_1 - \mu_j. \quad (41)$$

This result allows to find a trade-off between the inventory level and accuracy of the tracking given that each base stock w_{d_j} is selected above the predefined level. Furthermore in Starkov et al. (2012a) (see page 12, Theorem 3) a complete trade-off between the inventory levels and accuracy of the demand tracking was obtained regardless the above given condition on w_{d_j}.

For a tandem manufacturing line of 4 machines and 3 buffers Fig. 3 shows a set of comparative graphs between computer simulation and theoretical bounds. It can be observed that theoretical bounds look similar to simulation results.

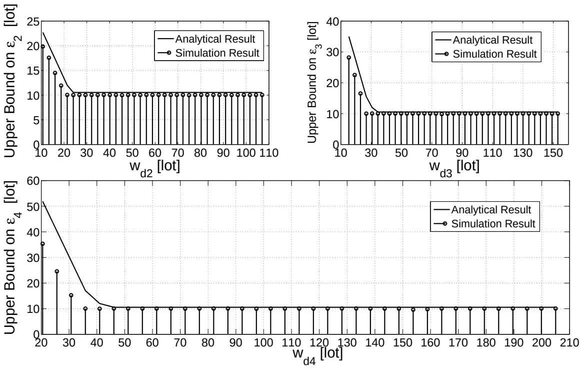

Fig. 3. Demand Tracking Errors vs. Desired Inventory Levels.

The main ideas presented here can be generalized towards more complex manufacturing networks, see Starkov et al. (2013). The general idea remains the same: the direct Lyapunov method allows to find a smallest possible bound on w_{d_j}'s that correspond to the smallest possible size of Ω - the level set of chosen Lyapunov function. Although this idea has proven to be operational, some questions still remain open: A particular interest lies in extension of the ideas towards non-acyclic systems similar to the Kumar-Seidman example (Kumar, 1993).

5. LIQUITROL EXPERIMENTAL PLATFORM

Year round, hundreds of student from Eindhoven University of Technology (TUE) in The Netherlands study multiple subjects on control and performance of production networks offered by the Manufacturing Networks group (MN) of the department of Mechanical Engineering. Though learning these subjects from theory and simulation is sufficient to prepare Bachelor or Master students for their future carrier, the presence of an experimental tool, where all studied theoretical phenomena can be visualized, gives the students an extra assurance of the obtained knowledge as well as facilitates their learning process. Furthermore, hand-on experiments, where students acquire the ability to solve problems using real equipment, form a fundamental part of their education as engineers.

To achieve this goal an experimental setup called Liquitrol (an abbreviation from liquid and control) has been designed. The setup consists of 6 water tanks, and 8 pumps that can be interconnected by flexible pipes. Each tank is equipped with a water level sensor while the water flow rate through the pipes can be measured by flow sensors. All the analogue information is converted to a digital form and processed by a computer under TwinCat 3 software which is compatible with Matlab and Simulink.

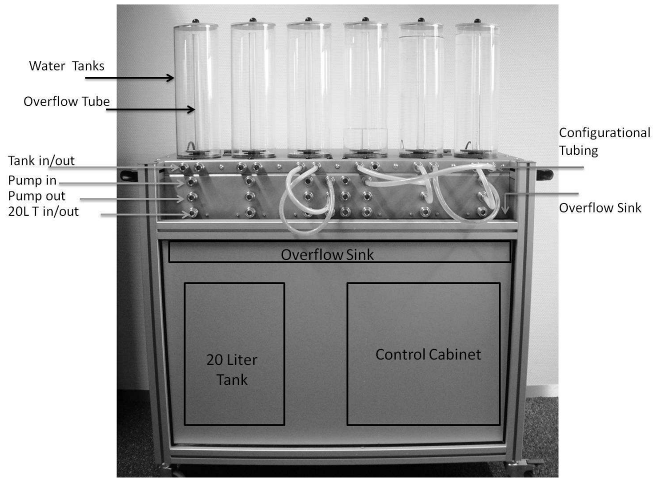

Fig. 4. Liquitrol Platform

Due to its flexibility, the setup can be used to model manufacturing networks of different nature: with one or multiple products including tandem or re-entrant networks. In Starkov et al. (2012b), the setup is described with more technical details and some experiments are outlined. In Starkov (2012), the theoretical results from the previous section are validated by experiments with this setup. The setup has been proven to be a useful platform for both educational and research purposes.

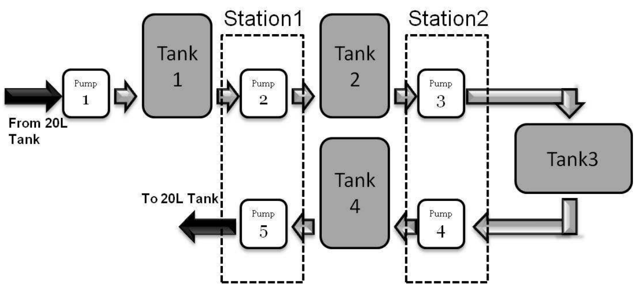

Fig. 5. Kumar-Seidman type re-entrant network

As an example we consider how to model and emulate the instability example due to Kumar and Seidman (Kumar, 1993). This example shows that a particular re-entrant network, that satisfies the capacity condition, results in unstable behavior under a clearing switching policy. In the present example, a re-entrant network (see Figure 5) of 2 stations of 2 stages (pumps) each, is operated under a buffer clearing policy. Thus assuming a constant arrival rate of raw material (water provided by Pump 1) to the first buffer (Tank 1), the goal of each stage of every station is to process all the products of its adjacent buffer, i.e clear its buffer, in one at a time manner. In other words, in each station only one stage can perform its buffer clearing action at a time and only after the buffer of operational stage is empty, another stage is selected to perform its clearing action. In the example the arrival rate is selected in accordance to the *capacity condition*. This condition delimits the product arrival rate to the production speed of the slowest station in the network. From

Figure 5, Pump 2 and 4 operate at 2.7 [liters/min], and Pump 3 and 5 at 1.35 [liters/min]. The arrival rate of liquid to Pump 2 is generated by Pump 1 at 0.8 [liters/min]. In the example

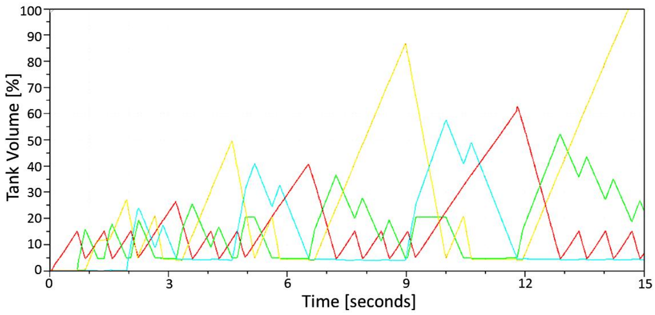

Fig. 6. Kumar-Seidman example on Liquitrol. Volume of tank 1 till 4 is depicted by red, green, yellow and blue lines, respectively.

of Kumar (1993), it was shown that, although the *capacity condition* for the network is satisfied, under a certain selection of production speed values, starvation phenomena occur in the network, which consequently lead the instability of the system. The products are accumulated in some intermediate buffers while in others there is none. Similar to the result in Kumar (1993), the growing oscillatory behavior of the trajectories of 4 buffer contents of the re-entrant network of Figure 5 is shown in Figure 6. More detailed theoretical results on this topic can be found e.g. in Kumar (1993). Note that several control solutions are given to the instability problem of re-entrant networks; see for example "buffer regulator" of Humes (1994), "stream modifier" of Burgess and Passino (1997) and "controlled buffer technique" of Somlo et al. (2004).

From Fig. 6 one can see that a clear indication of instability can be seen after approximately 7-10 minutes of the experiment. So the setup can be used as a demonstration tool during the lectures and seminars.

6. CONCLUSION

In this paper we outlined some nonlinear problems that arise in control of manufacturing networks both in continuous- and discrete-time settings. Some approaches to those problems are mentioned and the related results are cited. A particular attention was drawn to an experimental setup Liquitrol that allows to emulate various manufacturing networks under different control policies. The setup is designed as a demonstration tool for students however it could be used for research purposes as well.

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