Passification Based Synchronization of Nonlinear Systems Under Communication Constraints ⋆

Alexander L. Fradkov *,** **Boris Andrievsky** *,**

* *Institute for Problems of Mechanical Engineering of RAS*
** *National Research University of Information Technologies, Mechanics and Optics, Saint Petersburg, Russia,*
(e-mail: {alexander.fradkov, boris.andrievsky}@gmail.com)

Abstract: A unified exposition of passification-based synchronization of nonlinear systems over the limited-band communication channel is given. A simple adaptive synchronization scheme employing the binary quantizer and the first order coder in the communication channel is presented and analytical bounds for the closed-loop adaptive system performance are found. Relevance of passifiability condition for controlled synchronization of master-slave nonlinear systems for first order and full order coder/decoder pair is also demonstrated. The experimental results for state estimation of nonlinear oscillatory system are presented.

Keywords: Passification method, nonlinear systems, synchronization, state estimation, communication constraints

1. INTRODUCTION

Recently the limitations of control under constraints imposed by a finite capacity of information channel have been investigated in detail in the control literature, see the surveys (Nair et al., 2007; Andrievsky et al., 2010), the monograph (Matveev and Savkin, 2009) and the references therein. It has been shown that stabilization of linear systems under information constraints is possible if and only if the capacity of the information channel exceeds the entropy production of the system at the equilibrium (*Data Rate Theorem*) (Nair and Evans, 2003, 2004; Nair et al., 2004). Results of the previous works on control systems analysis under information constraints do not apply to synchronization systems since in a synchronization problem trajectories in the phase space converge to a set (a manifold) rather than to a point, i.e. the problem cannot be reduced to simple stabilization. Moreover, the Data Rate Theorem is difficult to extend to nonlinear systems.

The first results on synchronization under information constraints were presented in (Fradkov et al., 2006a, 2008a), where the so called observer-based synchronization scheme (Fradkov et al., 2000a) was considered. Fradkov et al. (2009) extended the results of (Fradkov et al., 2006a) to the *output feedback* controlled synchronization of two nonlinear systems assuming that the coupling is implemented via the control signal which is computed based on a measurable innovation (error) signal, transmitted over a limited-band communication channel. Key tools used to solve the problem are quadratic Lyapunov functions and the *Passification method* Fradkov (1974); Fradkov et al. (1999); Fradkov (2003); Andrievskii and Fradkov (2006).

The paper is organized as follows. The Passification method is recalled in Section 2. Different schemes of nonlinear systems synchronization over the limited-band communication channel, employing this method, are outlined in Section 3. The experimental results, demonstrating applicability of the described method for remote state estimation of the nonlinear oscillating system (the multipendulum set-up), are presented in Section 4.

2. PRELIMINARIES. PASSIFICATION METHOD

Consider a linear time invariant (LTI) single-input multiple-output (SIMO) system

$$\dot{x} = Ax + Bu, \quad z = Cx, \tag{1}$$

where $x = x(t) \in \mathbb{R}^n$ is a state vector, $u = u(t) \in \mathbb{R}^1$ is a scalar control variable, $z = z(t) \in \mathbb{R}^l$ is a measured output vector, A, B, C are constant real matrices of sizes $n \times n$, $n \times 1$, $l \times n$ respectively.

Passification problem for the system (1) is understood as finding an $(l \times 1)$-matrix K such that the closed loop system with feedback $u = -K^{\mathrm{T}} z + v$ is strictly passive with respect to an auxiliary output $\sigma = Gz$ (G is $(1 \times l)$-matrix): inequality $\int_0^T (\sigma v - \rho |x|^2)\,\mathrm{d}t \geq 0$ for some $\rho > 0$ and all $T > 0$ holds for all trajectories of (1) starting from $x(0) = 0$. This is equivalent (as follows from Kalman–Yakubovich–Popov Lemma) to finding a matrix K satysfying the *strict positive realness* (SPR) condition: transfer function $W(\lambda) = GC(\lambda I_n - A + BK^{\mathrm{T}} C)^{-1} B$ of the closed-loop system[1] from input v to the output $\sigma = Gz$

⋆ The work was supported by the Russian Foundation for Basic Research (projects 09-08-00803, 11-08-01218), by the Russian Federal Program "Scientific and Scientical-Education Cadres of Innovative Russia" (contract No 16.740.11.0042), and the Program of basic research of OEMPPU RAS No 2 "Control and safety in energy and technical systems". The authors acknowledge help of Alexey Andrievsky for performing the experiments.

[1] I_n denotes $n \times n$ identity matrix.

satisfies the relations $\operatorname{Re} W(i\omega) > 0$ for all $\omega \in \mathbb{R}^1$, $i^2 = -1$ and $\lim_{\omega \to +\infty} \omega^2 \operatorname{Re} W(i\omega) > 0$.

Definition 1. System (1) is called *minimum phase* with respect to the output $\sigma = Gz$, if the polynomial

$$\varphi_0(s) = \det \begin{bmatrix} sI_{\rm n} - A & -B \\ GC & 0 \end{bmatrix} \qquad (2)$$

is Hurwitz; *hyper minimum phase* (HMP), if it is minimum phase and $GCB > 0$.

Theorem 2. (*Passification Theorem*, or *Feedback Kalman–Yakubovich–Popov Lemma*), (Fradkov et al., 1999; Fradkov, 2003).

The following statements are equivalent:

(A1) There exist a positive definite $(n \times n)$-matrix H and an $(l \times 1)$-matrix K such that the relations

$$H(A + BK^{\rm T}C) + (A + BK^{\rm T}C)^{\rm T}H < 0, \quad HB = C^{\rm T}G^{\rm T} \qquad (3)$$

hold.

(B1) The system (1) is hyper minimum phase with respect to the output $\sigma = Gz$.

(C1) There exists a feedback

$$u = K^{\rm T}z + v \qquad (4)$$

rendering the closed-loop system (1), (4) strictly passive with respect to the output $\sigma = Gz$.

Note, that if the condition (B1) is satisfied then the matrix K in (3) can be found in the form $K = -\kappa G^{\rm T}$ where κ is a sufficiently large positive real number.

3. SYNCHRONIZATION AND STATE ESTIMATION OVER THE LIMITED-BAND COMMUNICATION CHANNEL

The present work is focused on the problem of nonlinear systems synchronization over the communication network in presence of information constraints. Although the transmissions delay and transmission channel distortions usually appear in practice, in the present study it is assumed that the coded symbols are available at the receiver side at the same sampling instant, as they are generated by the coder, and transmission channel distortions are absent.

3.1 Coding procedures

Fradkov et al. (2006b) studied the properties of observer-based synchronization for Lurie systems over a limited data rate communication channel with a one-step memory time-varying coder. Under the assumption that a sampling time may be properly chosen, optimality of binary coding in the sense of demanded transmission rate was established, and the relationship between synchronization accuracy and an optimal sampling time was found. Fradkov et al. (2006b) deal with synchronization problem of nonlinear n-th order systems. On the basis of the mentioned results, the present paper deals with a *binary* coding procedure.

Consider the following memoryless (static) binary coder

$$q(y, M) = M \operatorname{sign}(y), \qquad (5)$$

where $\operatorname{sign}(\cdot)$ is the *signum* function: $\operatorname{sign}(y) = 1$, if $y \geq 0$, $\operatorname{sign}(y) = -1$, if $y < 0$; parameter M may be referred to as a *coder range*. Notice that for binary coder each codeword symbol contains one bit of information. The discretized output of the considered coder is found as $\bar{y} = q(y, M)$. It is assumed that the coder and decoder make decisions based on the same information.

The static coder (5) is a part of the time-varying coder with memory (Nair and Evans, 2003; Liberzon, 2003; Tatikonda and Mitter, 2004; Fradkov et al., 2006a; Nair et al., 2007). Two underlying ideas are used for this kind of coders: reducing the coder range M to cover some area around the predicted value for the $(k+1)$th observation $y[k+1]$, $y[k+1] \in \mathcal{Y}[k+1]$ and introducing memory into the coder, which makes possible to predict the $(k+1)$th observation $y[k+1]$, transmitting over the channel only encrypted innovation signal.

Two kinds of coders with memory are considered below: the first-order coder, where the predicted value $y[k+1]$ is taken equal to $y[k]$ and the full-order coder, where the predicted value $y[k+1]$ is calculated based on kth estimate of the state of master system. The master system dynamics are represented in the full-order coder by means of the full-order observer as a part of the coder.

For the *first-order coder* the predicted value $y[k+1]$ is taken equal to $y[k]$ (Tatikonda and Mitter, 2004; Fradkov et al., 2006a). For describing the coder, introduce the sequence of *central numbers* $c[k]$, $k = 0, 1, 2, \ldots$ with initial condition $c[0] = 0$. At step k the coder compares the current measured output $y[k]$ with the number $c[k]$, forming the deviation signal $\partial y[k] = y[k] - c[k]$. Then this signal is discretized with a given $M = M[k]$ according to (5). The output signal

$$\bar{\partial} y[k] = q(\partial y[k], M[k]) \qquad (6)$$

is transmitted over the communication channel to the decoder. Then the central number $c[k+1]$ and the range parameter $M[k]$ are renewed based on the available information about the master system dynamics. Assuming that the master system output y changes at a slow rate, i.e. that $y[k+1] \approx y[k]$. we use the following update algorithms:

$$c[k+1] = c[k] + \bar{\partial} y[k], \quad c[0] = 0, \quad k = 0, 1, \ldots, \qquad (7)$$

$$M[k] = (M_0 - M_\infty)\rho^k + M_\infty, \quad k = 0, 1, \ldots, \qquad (8)$$

where $0 < \rho \leq 1$ is the decay parameter, M_∞ stands for the limit value of $M[k]$. The initial value M_0 should be large enough to capture all the region of possible initial values of y_0. Equations (5), (6), (8) describe the coder algorithm. The same algorithm is realized by the decoder. Namely, the decoder calculates the variables $\tilde{c}[k]$, $\tilde{M}[k]$ based on received codeword flow similarly to $c[k]$, $M[k]$.

The full order coder usually contains 'embedded' asymptotic observer (Tatikonda and Mitter, 2004; De Persis, 2006; Cheng and Savkin, 2007; Andrievsky et al., 2007; Fradkov and Andrievsky, 2009), incorporating the model of the system, see Section 3.3 below.

3.2 Controlled synchronization of passifiable Lurie systems

Fradkov et al. (2008b, 2009) studied the *controlled* synchronization of nonlinear systems over the limited-band communication channel. Two identical dynamical systems modeled in the Lurie form are considered. One of the systems (*the slave system*, or *the follower*) is controlled by a scalar controlling function $u(t)$. Controlled synchronization model is as follows:

$$\dot{x}(t) = Ax(t) + B\psi(y_1),\ y_1(t) = Cx(t), \quad (9)$$

$$\dot{z}(t) = Az(t) + B\psi(y_1) + Bu,\ y_2(t) = Cz(t), \quad (10)$$

where $x(t)$, $z(t)$ are n-dimensional vectors of state variables; $y_1(t)$, $y_2(t)$ are scalar output variables; A is $(n\times n)$-matrix; B is $n \times 1$ matrix; C is $1 \times n$ matrix, $\psi(y)$ is a continuous nonlinearity, acting in span of control. System (9) is called *master* (*leader*) *system.*

Case of transmitting the master system output through a communication channel is studied in (Fradkov et al., 2008b). In this case the observation signal $y_1(t)$ is coded with symbols from a finite alphabet at discrete time instants $t_k = kT$ ($k = 0, 1, 2, \ldots$; T is the sampling period) and transmitted over a digital communication channel.

The goal is to find the control function $\mathcal{U}(\cdot)$ depending on measurable variables such that synchronization error $e(t)$, where $e(t) = x(t) - z(t)$ becomes small as t becomes large. Also the limitations imposed on the synchronization precision by limited transmission rate are studied and the output synchronization error $\varepsilon(t) = y_2(t) - y_1(t) = Ce(t)$ is evaluated.

Let the *zero-order extrapolation* be used to convert the digital sequence $\bar{y}_1[k]$ to the continuous-time input of the response system $\bar{y}_1(t)$. Then the *transmission error* is defined as follows:

$$\delta_y(t) = y_1(t) - \bar{y}_1(t). \quad (11)$$

Controller on the receiver side can use only the signal $\bar{y}_1(t) = y_1(t) + \delta_y(t)$ instead of $y_1(t)$. The control function is taken in the form of static linear feedback

$$u(t) = -K\bar{\varepsilon}(t), \quad (12)$$

where $\bar{\varepsilon}(t) = y_2(t) - \bar{y}_1(t)$, K is a scalar controller gain.

It is assumed in (Fradkov et al., 2008b) that the growth rate of $y_1(t)$ is uniformly bounded. The exact bound L_y for the rate of $y_1(t)$ is $L_y = \sup_{x\in\Omega} |C\dot{x}|$, where $\dot{x}$ is from (9). Based on Theorem 2, the conditions for the existence of a quadratic Lyapunov function $V(e) = e^{\mathrm{T}}Pe$ and controller gain K satisfying inequality $\dot{V}(e) \le -\mu V(e)$ for some $\mu > 0$ for $\delta_y(t) = 0$ and for all ξ satisfying the quadratic inequality $\xi\varepsilon \ge 0$ are found in (Fradkov et al., 2008b). Namely, such V and K exist if and only if the transfer function of the linear part of the system models (9), (10) $W(\lambda) = C(\lambda I - A)^{-1}B$ is HMP. Assuming that the HMP condition holds, matrix P and gain K are chosen properly and the modified Lyapunov inequality $PA_K + A_K^{\mathrm{T}}P \le -\mu P$ is valid for some $\mu > 0$, after simple algebra one obtains $\dot{V} \le -\mu V + |e^{\mathrm{T}}PB(K + L_\psi)\delta_y| \le -\mu V + \sqrt{V}\nu$, where $\nu = \sqrt{V(B)}(|K|+L_\psi)\Delta$. This leads to the following inequality

$$\overline{\lim_{t\to\infty}} \|e(t)\| \le C_e\Delta, \quad (13)$$

where $C_e = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\frac{L_\psi+|K|}{\mu}$. Inequality (13) shows that the total synchronization error is proportional to the upper bound on the transmission error Δ.

Case of transmitting the error signal between the master and the slave systems is considered in (Fradkov et al., 2009). In this case the control signal has the following form:

$$u(t) = -K\bar{\varepsilon}(t), \quad (14)$$

where $\bar{\varepsilon}(t) = \bar{\varepsilon}[k]$ as $t_k < t < t_{k+1}$, $\bar{\varepsilon}[k]$ is the result of transmission of the synchronization error signal $\varepsilon(t) = y_2(t) - y_1(t)$ over the channel, $t_k = kT$, $k = 0, 1, \ldots$.

According to the quantization algorithm (5), the quantized error signal $\bar{\varepsilon}[k]$ becomes

$$\bar{\varepsilon}[k] = M[k]\,\mathrm{sign}(\varepsilon(t_k)), \quad (15)$$

where the range $M[k]$ is defined by (8).

The key point of the approach is application of the so-called *method of continuous models*: analysis of the hybrid nonlinear system via analysis of its continuous-time approximate model (Derevitsky and Fradkov, 1974), see also (Ljung, 1977). In order to analyze the synchronization error two following assumptions are made:

A1. Nonlinearity $\psi(y)$ is Lipschitz continuous:

$$|\psi(y_1) - \psi(y_2)| \le L_\psi|y_1 - y_2| \quad (16)$$

for all y_1, y_2 and some $L_\psi > 0$.

A2. The linear part of (9) is strictly passifiable.

It follows from condition A2 and Theorem 2, that the stability degree η_0 of the polynomial $\beta(\lambda)$ is positive and for any η: $0 < \eta < \eta_0$ there exist a positive definite matrix $P = P^{\mathrm{T}} > 0$ and a number K such that the following matrix relations hold:

$$PA_K + A_K^{\mathrm{T}}P \le -2\eta P,\ PB = C^{\mathrm{T}},\ A_K = A - BKC. \quad (17)$$

Any sufficiently large real number can be chosen as the value of K.

The main result by Fradkov et al. (2009) is formulated as follows.

Theorem 3. Let A1, A2 hold, the controller gain K satisfies passivity relations (17) and the coder parameters ρ, T be chosen to meet the inequalities

$$e^{\eta T}(e^{L_F T} - 1) \le \frac{L_F}{\|C\|(K\|B\| + L_F)},\ e^{-\eta T} < \rho < 1, \quad (18)$$

where $L_F = \|A\| + L_\psi\|B\| \cdot \|C\|$, η is from (17). Let the coder range $M[k]$ be specified as

$$M[k] = M_0\rho^k. \quad (19)$$

Then for all initial conditions $e(0)$ such that $e(0)^{\mathrm{T}}Pe(0) \le M_0^2$ the synchronization error decays exponentially: $|\varepsilon[k]| \le \|e[k]\| \le M_0\rho^k$. In addition, $|\varepsilon(t)| \le |\varepsilon[k]|$ for $t_k \le t < t_{k+1}$.

For practice, it is reasonable to choose the coder range $M[k]$ separated from zero and apply (8) instead of (19) for zooming.

3.3 State estimation of passifiable Lurie systems

Solution to the state estimation problem for passifiable Lurie systems via a limited-capacity communication channel based on the coder of full order straightforwardly follows from Theorem 3 (Fradkov and Andrievsky, 2009). Indeed, consider the plant model (9). Let only the plant output $y(t)$ be measured. The problem is to produce an estimate of the unmeasured state vector $x(t)$ based on available sensor data, taking in a view limitations of communication channel capacity. Introduce the following *nonlinear state estimator of full order*:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\psi(\hat{y}) + L\bar{\varepsilon}(t),\ \hat{y}_1(t) = C\hat{x}(t), \quad (20)$$

where $\hat{x}(t) \in \mathbb{R}^n$ is a vector of estimates; $\hat{y}_1(t)$ is a scalar output variables of the observer; $\varepsilon(t) = y_1(t) - \hat{y}_1(t) = Ce(t)$ is the error between outputs of the plant and observer; $\bar{\varepsilon}(t) = \bar{\varepsilon}[k]$ as $t \in [t_k, t_{k+1}, t_k = kT$, $k = 0, 1, \ldots$; $n \times 1$ (column) matrix L is an observer gain (design parameter). The coded value of $\bar{\varepsilon}[k]$ is transmitted to the receiver side, where the state estimation process is replicated. Namely, the following sate estimation procedure, employing received coded signal $\bar{\varepsilon}[k]$ is realized by the decoder:

$$\dot{\hat{x}}_d(t) = A\hat{x}_d(t) + B\psi(\hat{y}_d) + L\bar{\varepsilon}(t),\ \hat{y}_d(t) = C\hat{x}_d(t), \quad (21)$$

where $\hat{x}_d \in \mathbb{R}^n$ is plant state estimation vector, generated by the decoder, $\hat{x}_d(0) = \hat{x}(0)$. Convergence conditions follow from (Fradkov et al., 2009, Appendix).

3.4 Adaptive synchronization of passifiable Lurie systems

Observer-based adaptive synchronization scheme. Following (Fradkov et al., 2008a), consider a nonlinear uncertain master system described in the state space form:

$$\dot{x} = Ax + \psi_0(y) + B\sum_{i=1}^{m}\theta_i\psi_i(y), \quad y = Cx, \quad (22)$$

where $x \in \mathbb{R}^n$ is the transmitter state vector; $y \in \mathbb{R}^l$ is the vector of output signals (to be transmitted over the communication channel); $\theta = [\theta_1, \ldots, \theta_m]^{\mathrm{T}}$ is the vector of master system parameters. It is assumed that the nonlinearities $\psi_i(\cdot)$, $i = 0, 1, \ldots, m$, the matrices A and C, and the vector B are known, and only the output signal $y(t)$ can be measured by sensors. Vector θ is considered as unknown constant vector of the master system parameters.

To achieve the synchronization between two nonlinear, the adaptive observer approach by Fradkov et al. (2000b) is used. The adaptive observer consists of a *tunable observer block* and an *adaptation block*. In the presence of transmission errors (including measurement errors, distortion in the transmission channel and coding errors, caused by a finite channel capacity) the measured signal $y(t)$ is corrupted and the input signal of the observer can be represented as

$$\bar{y}(t) = y(t) + \delta_y(t), \quad (23)$$

where $\delta_y(t)$ is the total distortion. Then the following regularized version of the adaptive observer may be employed:

$$\dot{\hat{x}} = A\hat{x} + \psi_0(\bar{y}) + B\sum_{i=1}^{m}\hat{\theta}_i\psi_i(\bar{y}) + L(\bar{y} - \hat{y}), \quad \hat{y} = C\hat{x}, \quad (24)$$

$$\dot{\hat{\theta}}_i = -\gamma(\bar{y} - \hat{y})\psi_i(\bar{y}) - \alpha\hat{\theta}_i, \quad i = 1, 2, \ldots, m, \quad (25)$$

where $\gamma > 0$ is *the adaptation gain* and α is *the regularization gain.*

Due to the transmission errors, the *synchronization error* $e(t) = x(t) - \hat{x}(t)$ does not necessary tend to zero and the synchronization goal is specified as $Q \le \Delta_x$, where Δ_x is the prespecified upper bound of the asymptotic error, and

$$Q = \overline{\lim_{t\to\infty}}\|x(t) - \hat{x}(t)\|, \quad (26)$$

is the *limit synchronization error.*

Performance of the adaptive synchronization system (22), (24), (25) is studied in (Fradkov et al., 2008a) under the assumption that no saturation occurs in coder and the total transmission error is uniformly bounded, $\|\delta_y(t)\| \le \Delta$ for all $t \ge 0$. Upper bounds for synchronization error are provided by the following theorem.

Theorem 4. Let the following assumptions hold:

A1. The observer gain matrix L is such that the transfer function $W_L(\lambda) = C(\lambda\mathbf{I} - A + LC)^{-1}B$ is strictly passive.

A2. The system (22) possesses a bounded invariant set $\Omega_\theta \subset \mathbb{R}^n$ for any $\theta \in \Theta \subset \mathbb{R}^m$, where Θ is the set of possible values of uncertain parameters and $x(0) \in \Omega$.

A3. Functions $\psi_i(y)$, $i = 0, 1, \ldots, m$ are bounded and Lipschitz continuous in the closed Δ-vicinity of Ω_θ, i.e. $|\psi_i(y)| \le L_\psi$, $|\psi_i(y') - \psi_i(y)| \le L'_\psi$ for some L_ψ, L'_ψ and for all $y = Cx$, $x \in S_\Delta(\Omega_\theta)$, where $S_\Delta(\Omega_\theta) = \{x : \exists z \in \Omega_\theta : \|x - z\| \le \Delta\}$.

Then there exist constants $C_1 > 0$, $C_2 > 0$ such that for any $\Delta > 0$ the choice of design parameters $\alpha = \Delta^2$, $\gamma = C_2/\Delta^2$ guarantees that the synchronization goal $Q \le \Delta_x$ is achieved for $\Delta_x = C_1\Delta$, i.e. the limit synchronization error Δ_x is proportional to the transmission error Δ.

The proof of Theorem 4 is given in (Fradkov et al., 2008a).

According to the observer version of the passification theorem by Efimov and Fradkov (2006), the vector L satisfying assumption *A1* exists if and only if the transfer function $W(\lambda) = C(\lambda\mathbf{I} - A)^{-1}B$ is HMP. To find vector L satisfying *A1* under the HMP condition it is sufficient to choose L in the form $L = -\kappa C$, where $\kappa > 0$ is large enough.

4. EXAMPLE. STATE ESTIMATIOM OF THE CHAIN OF PENDULUMS

The remote state estimation scheme of Section 3.3 has been implemented and experimentally tested on the *Multipendulum Mechatronic Set-up of the Institute for Problems of Mechanical Engineering of RAS* (MMS IPME) (Fradkov et al., 2010).

4.1 The Multipendulum Mechatronic Set-up

The MMS IPME includes: a modular multi-section mechanical oscillating system; electrical equipment (with the computer interface facilities); the personal computer for experimental data processing and for the real-time representation of the results. The mechanical part of the set-up consists of a number (up to 50) identical pendulums, connected by torsion springs. The foundation of each section is a hollow rectangular body. Inside the body an electrical magnet and electronic controller board are mounted. Additionally, two controlled electric motors may be connected with the first and the last pendulums of the chain via the springs for changing the boundary conditions on the chain. [2] The optical encoders for measuring the angle of the pendulum are mounted on each pendulum section, providing the angular measurements with precision in 2^o.

The Data Exchange System of the setup is aimed at transfer data and control commands from the Control

[2] At present, only the “left-side” motor is in service. The boundary conditions at the “right” end of the chain are free.

Computer to the interface board of the pendulum sections. Each interface board is an intelligent measuring/controlling electronic device, assigned for unloading processor of the Control Computer from forming the control signal and preventing the Control Computer from a wasteful wait state of the sensor replies.

4.2 The chain of pendulums model

Consider the *rotation angle* of the drive shaft, which is connected with the first pendulum, as a control action. No boundary conditions are specified for Nth pendulum. This leads to the following model of the pendulum chain dynamics (Andrievsky and Fradkov, 2009):

$$\begin{cases}\ddot{\varphi}_1 + \varrho\dot{\varphi}_1 + \omega_0^2 \sin\varphi_1 - k(\varphi_2 - 2\varphi_1) = ku(t),\\ \ddot{\varphi}_i + \varrho\dot{\varphi}_i + \omega_0^2 \sin\varphi_i - k(\varphi_{i+1} - 2\varphi_i + \varphi_{i-1}) = 0,\\ \qquad (i = 2, 3, \ldots, N-1),\\ \ddot{\varphi}_N + \varrho\dot{\varphi}_N + \omega_0^2 \sin\varphi_N - k(\varphi_N - \varphi_{N-1}) = 0,\end{cases} \tag{27}$$

where $\varphi_i = \varphi_i(t)$ $(i = 1, 2, \ldots, N)$ are the pendulum deflection angles; $u(t)$ is the control action; ϱ, ω_0, k are the system parameters.

The model (27) was used to design the remote state estimator of Section 3.3. In our experiments, the chain of four pendulum sections and the motor, attached via the spring to pendulum #1 were used. In the course of the experiments, the voltages in the form of harmonic or irregular oscillating signals were applyed to the motor. The rotary angles of the drive shaft and the pendulums were measured with the sampling rate of 500 Hz and 2^o precision by means of the optical sensors, and the obtained data were processed by the binary coding algorithms for transferring over the channel. Since the outside left rotary angle (the angle of the drive shaft) may be reffered to as *exogenous* action, applyed to the plant (the chain of the pendulums), it was coded by means of the one-step memory coder (5)–(8). For coding the rotary angles and the angular velocities estimation of the pendulums, the full-order coding scheme of Section 3.3 was applied to the sensor data flows of the each pendulum. The coding algorithm in each channel includes the nonlinear observer (21) and the time-varying quantizer (5), (8) of the estimation errors $\varepsilon_i(t)$, $i = 1 - 4$.

4.3 Experimental results

The following parameter values were taken in the experiments: the model (27) parameters were preliminary found by means of identification procedure as $\varrho = 0.95$ s^{-1}, $\omega_0 = 5.5$ s^{-1}, $k = 5.8$ s^{-2}; the sampling times for all channels (one motor and four pendulums) were taken identical from the interval $T \in [1.0, 50]$ ms for different data processing operations (then, the overal trasmission rate R_Σ for all five channels was varied from 250 to 1.0k bit/s); the coder parameters $M_{0,m} = 15$ and $M_{0,\varphi} = 2$ were chosen covering the region of the initial values for the motor and pendulums angles, respectively; $M_{\infty,m} = 1.0$ $M_{\infty,\varphi} = 0.1$; parameter ρ in (8) was taken as $\rho = e^{-0.5T}$ for each channel.

The experimental results are reflected in Figs. 1–3. The time histories of the measured signal $\varphi_1(t)$ and its estimate $\hat{\varphi}_1(t)$ for $T = 20$ ms ($R_\Sigma = 250$ bit/s) in the cases of the harmonic and irregular excitation are depicted in Figs. 1, 3. The time history of the correction signal $\bar{\varepsilon}_1(t)$ is plotted in Fig. 2. It is seen that the applied coding procedure ensures the suitable estimation accurasy even for the relatively small data transmision rate. It should be mentioned that the measurements of $\varphi_i(t)$ contain the sensors error in 2^o of magnitude, which imposes limitation on the data transfer accuracy. [3] Normalized limit output estimation error $Q_{\varphi,1}$ vs transmission rate R_Σ is shown in Fig. 4. This result demonstrates that the estimation occurs if the data transmission rate is sufficiently large.

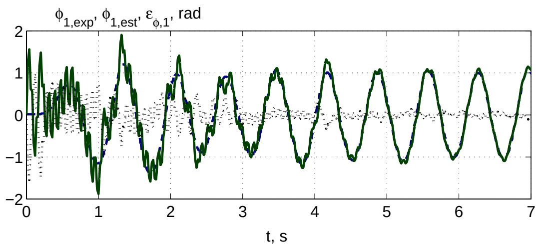

Fig. 1. Time histories in the case of harmonic excitation. $\varphi_1(t)$ – dashed line, $\hat{\varphi}_1(t)$ – solid line, $\varepsilon_1(t)$ – dotted line. $T = 20$ ms, $R_\Sigma = 250$ bit/s.

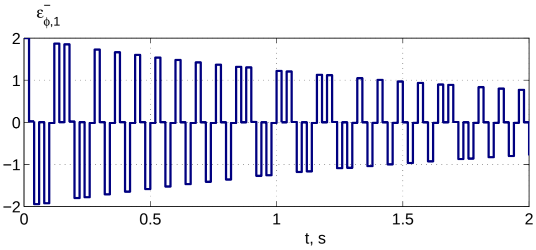

Fig. 2. Correction signal $\bar{\varepsilon}_1(t)$. $T = 20$ ms, $R_\Sigma = 250$ bit/s.

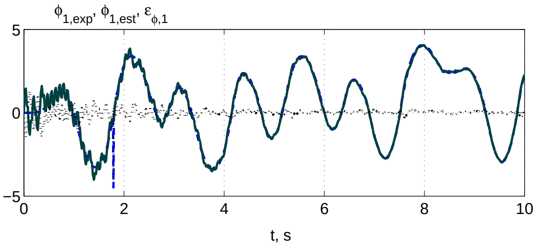

Fig. 3. Time histories in the case of irregular excitation. $\varphi_1(t)$ – dashed line, $\hat{\varphi}_1(t)$ – solid line, $\varepsilon_1(t)$ – dotted line. $T_\varphi = T_m = 20$ ms, $R_\Sigma = 250$ bit/s.

5. CONCLUSIONS

The paper gives a unified exposition of passification-based synchronization of nonlinear systems over the limited-band communication channel. A simple adaptive synchronization scheme employing the binary quantizer and the first order coder in the communication channel is presented and analytical bounds for the closed-loop adaptive system performance are found. Relevance of passifiability condition for controlled synchronization of master-slave nonlinear systems for first order and full order coder/decoder pair is demonstrated and the comprehensive analysis of results on

[3] It this connection, the experimental accuracy evaluation does not rely on the “true” processes $\varphi_i(t)$, but on the measured data only.

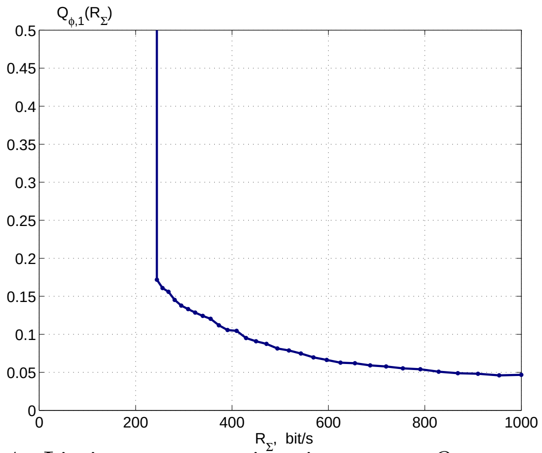

Fig. 4. Limit output estimation error $Q_{\varphi,1}$ v.s. total transmission rate R_Σ.

nonlinear systems synchronization under communication constraints is presented.

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