Robust Simple Adaptive Control with Relaxed Passivity and PID control of a Helicopter Benchmark *

D. Peaucelle *,** **B. Andrievsky** ***,**** **V. Mahout** *,**
A. Fradkov ***,****

* CNRS; LAAS; 7, avenue du Colonel Roche, F-31077 Toulouse, France.
(email: peaucelle@laas.fr)
** Université de Toulouse; UPS, INSA, INP, ISAE ; LAAS ; F-31077
Toulouse, France.
*** Institute for Problems of Mechanical Engineering of RAS, 61 Bolshoy av.
V.O., St Petersburg, 199178, Russia
**** Saint Petersburg State University of IT, Mechanics and Optics, Saint
Petersburg, Russia

Abstract: The paper is dedicated to experimenting a simple LMI procedure to design simple adaptive control laws for 'almost stable' systems. The procedure initially conceived for regulation problems around zero equilibrium point is extended to PID like tracking control problem. LMI results gives conditions to guarantee robustness of the simple adaptive control law with respect to parametric uncertainties. Experiments are done on a 3D helicopter benchmark model.

Keywords: Adaptive Control, Robust Control, LMI, PID

1. INTRODUCTION

Adaptive control is a control strategy with online modifications of the control law to better reject inevitable uncertainties and perturbations. One such adaptive scheme ([Ioannou and Sun, 1996, Åström and Wittenmark, 1989]) relies on estimating on-line uncertainties and perturbations and with that knowledge to tune the control law. Another adaptation scheme ([Fradkov, 1974, Kaufman et al., 1994]) proposes to directly tune the control gains with the sole output measurements. Often called direct adaptive control it has the advantage of simplicity compared to estimation/gain-scheduling and other more complex schemes. On the other hand it has the disadvantage to require strong assumptions on the system characteristics in order to prove closed-loop stability. One such direct adaptive control scheme is the passification-based strategy (also called simple adaptive control in [Kaufman et al., 1994]). The results presented in this paper enter this framework.

The passivity-based adaptive control framework was first restricted to square systems having same number of inputs and outputs ([Fradkov, 1974]). Moreover, the systems were assumed to have open-loop hyper-minimum phase properties or equivalently in [Kaufman et al., 1994] to be 'almost strictly passive' (*i.e.* being closed-loop passive with some static output feedback gain). The conditions have been mitigated in [Barkana et al., 2006] and relaxed in [Fradkov, 2003, 2008] to non square systems at the expense of finding a linear combination of outputs Gy that makes the new weighted system 'almost strictly passive'. Further extension is proposed in [Peaucelle and Fradkov, 2008] by the introduction of a parallel feedforward gain (also known as a *shunt*, see [Kaufman et al., 1994]). These extensions conclude that a necessary condition for passivity-based adaptive control is to have a static output feedback $u = F_0 y$ that renders the closed-loop strictly passive with respect to some output $Gy + Du$. Unfortunately, finding these F_0, G, D matrices appeared in that paper as a hard problem and does not even guarantee the validity of the adaptive control (the F_0, G, D matrices have to satisfy additional LMI conditions). Finally, [Peaucelle et al., 2011] gives a new result proving that as soon as a system is 'almost stabilizable' (*i.e.* closed-loop stable for some static output feedback gain F_0) then a direct adaptive control can be built. This last result is in extended in the present paper to the robust case.

Moreover, the paper proposes a special two term formulation of the adaptive control law. The direct adaptive scheme we study involves, as usual in this framework, a feedback control $u(t) = K(t)y(t)$ where the time-varying gain $K(t)$ is adapted according to a differential equation. This differential equation includes classically a gradient type term $-Gyy^T$ that drives the feedback gain to stabilizing values. We explore a modification of this structure into a two term control $u(t) = K_y(t)y(t) + K_\eta(t)\eta(t)$ in which the adaptation differential equation does not involve any squared term of the η signal. This modification is proved to be of interest for adaptive PID control. It is then applied to the non-linear model of a helicopter benchmark.

The outline of the paper is as follows. First a section is devoted to the theoretical results about the two term adaptive control and its stability as soon as the system is 'almost stable'. Section 3 extends these results to the robust case. Section 4 exposes

the helicopter benchmark control problem which is solved in Section 5. Finally some conclusions are driven.

Notations

$\mathsf{R}^{m \times p}$ is the set of real m by p matrices. A^T is the transpose of the matrix A. $\mathrm{Tr}(A)$ is the trace of A. $\mathbf{1}$ and $\mathbf{0}$ are respectively the identity and the zero matrices of appropriate dimensions. For symmetric matrices, $A > (\geq) B$ means that $A - B$ is positive (semi) definite. For a non symmetric matrix $\{A\}^{\mathcal{S}} = A + A^T$. $\|A\|_D = \sqrt{\mathrm{Tr}(A^T D A)}$ is the weighted Frobenius norm of A with weight $D > \mathbf{0}$.

$\phi_{D,\alpha}(\mathsf{R}^{m \times p} \to \mathsf{R}^{m \times p})$ is a dead-zone function defined by

$$\phi_{D,\alpha}(K) = \psi_\alpha(\|K\|_D^2) \cdot DK$$

where $\psi_\alpha([0 \ \alpha\beta[\to [0 \ +\infty[)$ is such that

$$\psi(0 \leq k \leq \alpha) = 0, \ \ \psi(\alpha \leq k < \alpha\beta) = \frac{k-\alpha}{\alpha\beta-k}.$$

2. MAIN RESULTS

Linear MIMO systems described in state-space are considered

$$\dot{x} = Ax + Bu, \ \ y = C_y x, \ \ \eta = C_\eta x$$

where $x \in \mathsf{R}^n$ is the state, $u \in \mathsf{R}^m$ is the control input, $y \in \mathsf{R}^p$, $\eta \in \mathsf{R}^q$ are measured outputs. We denote $C = \begin{bmatrix} C_y^T & C_\eta^T \end{bmatrix}^T$.

It is assumed that the system may be stabilized by a static output feedback

$$u = F_0 \begin{pmatrix} y \\ \eta \end{pmatrix} = F_{0y} y + F_{0\eta} \eta$$

and one such gain is assumed to be known. Our goal is to provide design conditions for the simple adaptive control:

$$u(t) = K_y(t) y(t) + K_\eta(t) \eta(t)$$

with the following type of adaptation:

$$\dot{K}_y(t) = -G_y \begin{pmatrix} y(t) \\ \eta(t) \end{pmatrix} y^T(t) \Gamma_y - \phi_{D_y,\alpha_y}(K_y(t) - F_{0y}) \Gamma_y,$$

$$\dot{K}_\eta(t) = -G_\eta y(t) \eta^T(t) \Gamma_\eta - \phi_{D_\eta,\alpha_\eta}(K_\eta(t) - F_{0\eta}) \Gamma_\eta.$$

No assumption is made about the closed-loop passivity but the following proposition gives a way to define passive outputs for that closed-loop system.

Proposition 1. If $A(F_0) = A + BF_0 C$ is asymptotically stable then there exists $(P, G_y, D_y, G_\eta, D_\eta)$ solution to the following LMI problem: $P > \mathbf{0}$,

$$\begin{bmatrix} \{PA(F_0)\}^{\mathcal{S}} & PB - C^T G_y^T & PB - C_\eta^T G_\eta^T \\ B^T P - G_y C & -D_y & \mathbf{0} \\ B^T P - G_\eta C_y & \mathbf{0} & -D_\eta \end{bmatrix} < \mathbf{0}. \quad (1)$$

The pairs (G_y, D_y) and (G_η, D_η) are then such that the two following systems are passive

$$\begin{aligned} \dot{x} &= A(F_0)x + Bw \ , \ z_y = G_y Cx + 1/2 D_y w \\ \dot{x} &= A(F_0)x + Bw \ , \ z_\eta = G_\eta y + 1/2 D_\eta w. \end{aligned} \quad (2)$$

The proof is quite simple and is a direct extension of the one in [Peaucelle et al., 2011].

Having obtained some passivity property of the system, the following theorem allows to prove closed-loop stability with a simple adaptive control.

Theorem 2. Let $\beta > 1$ be a given scalar. If $(F_0, G_y, D_y, G_\eta, D_\eta)$ are feasible values for LMIs (1), then there exists $(Q > \mathbf{0},$

$R_y, R_\eta, T_y, T_\eta, F_y, F_\eta, \alpha_y, \alpha_\eta)$ solution to the following LMI problem

$$\begin{bmatrix} R_y & QB - C^T G_y^T \\ B^T Q - G_y C & D_y \end{bmatrix} \geq \mathbf{0}, \quad (3)$$

$$\begin{bmatrix} R_\eta & QB - C_\eta^T G_\eta^T \\ B^T Q - G_\eta C_y & D_\eta \end{bmatrix} \geq \mathbf{0}, \quad (4)$$

$$\begin{bmatrix} T_y & (F_y - F_{0y})^T \\ (F_y - F_{0y}) & D_y^{-1} \end{bmatrix} \geq \mathbf{0}, \ \ \mathrm{Tr}(T_y) \leq \alpha_y, \quad (5)$$

$$\begin{bmatrix} T_\eta & (F_\eta - F_{0\eta})^T \\ (F_\eta - F_{0\eta}) & D_\eta^{-1} \end{bmatrix} \geq \mathbf{0}, \ \ \mathrm{Tr}(T_\eta) \leq \alpha_\eta, \quad (6)$$

$$\begin{aligned} \{QA(F_0)\}^{\mathcal{S}} \\ + \alpha_y \beta C_y^T C_y + R_y + \{C^T G_y^T (F_y - F_{0y}) C_y\}^{\mathcal{S}} \\ + \alpha_\eta \beta C_\eta^T C_\eta + R_\eta + \{C_y^T G_\eta^T (F_\eta - F_{0\eta}) C_\eta\}^{\mathcal{S}} < \mathbf{0}. \end{aligned} \quad (7)$$

The solution is such that the adaptive controller stabilizes the system.

The proof follows the lines of the proof in [Peaucelle et al., 2011]. The Lyapunov function used for the proof is as follows:

$$x^T Q x + \|(K_y - F_y)^T\|_{\Gamma_y^{-1}} + \|(K_\eta - F_\eta)^T\|_{\Gamma_\eta^{-1}}.$$

The complete proof is not produced here to give sufficiently space for discussing its application on an example.

At this stage the results can seem confusing. Theorem 2 gives, at the knowledge of a stabilizing SOF gain F_0, an alternative adaptive controller, more complex than the SOF control. Its benefit is expected in terms of higher robustness properties than the original SOF and this issue is discussed in the following section.

Remark 1. The adaptive control law is such that the gain K varies around the original stabilizing gain F_0. It cannot escape from the bounded set defined by

$$\|K_y(t) - F_{0y}\|_{D_y}^2 \leq \alpha_y \beta, \ \ \|K_\eta(t) - F_{0\eta}\|_{D_\eta}^2 \leq \alpha_\eta \beta$$

even in the presence of bounded noise (see [Peaucelle et al., 2011] and [Peaucelle et al., 2008]) and as the system stabilizes it converges to the bounded set defined by

$$\|K_y(\infty) - F_{0y}\|_{D_y}^2 \leq \alpha_y, \ \ \|K_\eta(\infty) - F_{0\eta}\|_{D_\eta}^2 \leq \alpha_\eta.$$

Trivially one cannot expect any improvement from the adaptive control if these sets are small. One should therefore seek for solutions maximizing α_y and α_η (when solving LMIs of Theorem 2) while minimizing D_y and D_η (when solving LMIs of Proposition 1).

3. GUARANTEED ROBUSTNESS

As in [Peaucelle and Fradkov, 2008] we shall consider constant polytopic uncertainties. Results may as well be extended following the lines of [Peaucelle et al., 2009] to the case of time-varying uncertainties. It is not done here for space limitation reasons.

The uncertain system is assumed to have uncertainties only on the differential equation

$$\dot{x} = A(\xi)x + B(\xi)u, \ \ y = C_y x, \ \ \eta = C_\eta x.$$

The uncertainties are assumed affine

$$\begin{bmatrix} A(\xi) & B(\xi) \end{bmatrix} = \sum_{v=1}^{\bar{v}} \xi_v \begin{bmatrix} A^{[v]} & B^{[v]} \end{bmatrix}$$

of polytopic type $\xi \in \Xi = \{ \xi_v \geq 0 \ , \ \sum_{i=1}^{\bar{v}} \xi_v = 1 \}$.

Denote $A^{[v]}(F_0) = A^{[v]} + B^{[v]}F_0C$. The following Theorem gives conditions for robust stability with the adaptive control.

Theorem 3. Let $(\beta > 1, F_0, G_y, D_y, G_\eta, D_\eta)$ be given, if there exists $(Q^{[v]} > \mathbf{0}, R^{[v]}, H, H_R \, T_y^{[v]}, T_\eta^{[v]}, F_y^{[v]}, F_\eta^{[v]}, \alpha_y, \alpha_\eta)$ solutions for all $v = 1 \ldots \bar{v}$ to the following LMIs

$$
\begin{bmatrix}
R^{[v]} & -C^T G_y^T & -C_y^T G_\eta^T & Q^{[v]} \\
-G_y C & D_y & \mathbf{0} & \mathbf{0} \\
-G_\eta C_y & \mathbf{0} & D_\eta & \mathbf{0} \\
Q^{[v]} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{bmatrix} \\
\geq \left\{ H_R \begin{bmatrix} \mathbf{0} & B^{[v]} & B^{[v]} & -\mathbf{1} \end{bmatrix} \right\}^{\mathcal{S}}, \quad (8)
$$

$$
\begin{bmatrix}
T_y^{[v]} & (F_y^{[v]} - F_{0y})^T \\
(F_y^{[v]} - F_{0y}) & D_y^{-1}
\end{bmatrix} \geq \mathbf{0}, \ \ \mathrm{Tr}(T_y^{[v]}) \leq \alpha_y, \quad (9)
$$

$$
\begin{bmatrix}
T_\eta^{[v]} & (F_\eta^{[v]} - F_{0\eta})^T \\
(F_\eta^{[v]} - F_{0\eta}) & D_\eta^{-1}
\end{bmatrix} \geq \mathbf{0}, \ \ \mathrm{Tr}(T_\eta^{[v]}) \leq \alpha_\eta, \quad (10)
$$

$$
\begin{bmatrix} L^{[v]} & Q^{[v]} \\ Q^{[v]} & \mathbf{0} \end{bmatrix} < \left\{ H \begin{bmatrix} A^{[v]}(F_0) & -\mathbf{1} \end{bmatrix} \right\}^{\mathcal{S}}
$$

$$
L^{[v]} = R^{[v]} + \alpha_y \beta C_y^T C_y + \left\{ C^T G_y^T (F_y^{[v]} - F_{0y}) C_y \right\}^{\mathcal{S}}
$$
$$
+ \alpha_\eta \beta C_\eta^T C_\eta + \left\{ C_y^T G_\eta^T (F_\eta^{[v]} - F_{0\eta}) C_\eta \right\}^{\mathcal{S}}.
$$
$$(11)$$

then the adaptive controller stabilizes robustly the system.

Proof Theorem 3 is obtained by classical robustness arguments for affine polytopic systems. See for example [Peaucelle and Fradkov, 2008]. ∎

It should be noted that while Theorem 2 is defined assuming F_0 is a stabilizing SOF, no such assumption is made in Theorem 3. In Theorem 2 the assumption is used for guaranteeing the existence of a solution, but solutions may exist even if $A(F_0)$ is unstable. For initialization of the data $(\beta > 1, F_0, G_y, D_y, G_\eta, D_\eta)$ Proposition 1 can be used for the nominal system (center of the polytope) and F_0 needs to stabilize this nominal system but may not be a robust control. No assumption is made in Theorem 3 about the robust stabilization with a unique, parameter-independent controller. Studying the proof provided in [Peaucelle et al., 2011] one can nevertheless see that some robustly stabilizing parameter-dependent static output-feedback should exist for the theorem to hold. In the present case this feedback gain is

$$
F(\xi) = \sum_{v=1}^{\bar{v}} \xi_v \begin{bmatrix} F_y^{[v]} & F_\eta^{[v]} \end{bmatrix}
$$

but cannot be implemented because the parameters are not measured.

4. THE PID CASE

Let a linear plant

$$
\dot{\mathsf{x}} = \mathsf{A}\mathsf{x} + \mathsf{B}u + d, \ \ \mathsf{y} = \mathsf{C}\mathsf{x}
$$

for which we aim at designing an adaptive PID controller:

$$
u(t) = K_P(t)(\mathsf{y}(t) - \mathsf{y}^*) + K_I(t) \int_0^t (\mathsf{y}(\tau) - \mathsf{y}^*)d\tau + K_D(t)\hat{\mathsf{y}}(t)
$$

capable of driving the closed-loop system outputs to reference constant values y^* whatever bounded steady-state disturbance

$d(t) \overset{t \to \infty}{\Longrightarrow} d^*$. The derivative of the output is either assumed to be measured $\hat{\mathsf{y}} = \dot{\mathsf{y}}$ or obtained through a linear filter

$$
\dot{x}_f = A_f x_f + B_f \mathsf{y}, \ \ \hat{\mathsf{y}} = C_f x_f + D_f \mathsf{y}.
$$

The integral component of the control guarantees, if the closed-loop is stable, the convergence of y to y^*. To design the parameters of the adaptive PID the problem is converted into the stabilization problem of the following model

$$
\dot{x} = \begin{bmatrix} \mathsf{A} & 0 & 0 \\ \mathsf{C} & 0 & 0 \\ B_f \mathsf{C} & 0 & A_f \end{bmatrix} x + \begin{bmatrix} \mathsf{B} \\ 0 \\ 0 \end{bmatrix} u - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mathsf{y}^*
$$
$$
y = \begin{bmatrix} \mathsf{C} & 0 & 0 \\ D_f \mathsf{C} & 0 & C_f \end{bmatrix} x - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathsf{y}^*, \ \ \eta = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix} x
$$
$$(12)$$

where $x^T = \begin{pmatrix} \mathsf{x}^T & \int(\mathsf{y} - \mathsf{y}^*)^T & x_f^T \end{pmatrix}$, $y^T = \begin{pmatrix} (\mathsf{y} - \mathsf{y}^*)^T & \hat{\mathsf{y}}^T \end{pmatrix}$ and $\eta = \int \mathsf{y} - \mathsf{y}^*$.

The problem is formalized appropriately to apply results of the previous sections. The important issue here is the choice of the two outputs y and η. This is done to make possible the convergence of the gains. Given a non-zero reference $\mathsf{y}^* \neq 0$, if the system is asymptotically stable, $\mathsf{y} - \mathsf{y}^*$ and the 'derivative' $\hat{\mathsf{y}}$ will converge to zero while the integral $\int_0^t(\mathsf{y}(\tau) - \mathsf{y}^*)d\tau$ will converge to a constant value. No adaptive scheme is therefore possible based on a adaptation law depending of the square of this signal. And indeed with the proposed structure of adaptation there is no such term. The adaptation will stop as soon as the system has converged to the reference.

5. THE "HELICOPTER" BENCHMARK

The 3DOF helicopter (Figure 1) is manufactured by *Quanser Consulting Inc.*, www.quanser.com. This setup was modified under demand of LAAS-CNRS for implementation and testing of robust control laws. It consists of a base on which a long arm is mounted. The arm carries the helicopter body on one end and a counterweight on the other end. The arm can tilt on an elevation axis as well as swivel on a vertical (travel) axis. Quadrature optical encoders mounted on these axes measure the elevation and travel of the arm. The *helicopter body*, which is mounted at the end of the arm, is free to pitch about the *pitch axis*. The pitch angle is measured via a third encoder. Two motors with propellers mounted on the helicopter body can generate a force proportional to the voltage applied to them. The force, generated by the propellers, causes the helicopter body to lift off the ground and/or to rotate about the pitch axis. The system is also equipped with a motorized *lead screw* that can drive a mass along the main arm in order to impose known controllable disturbances (the so-called *Active Disturbance Option*, ADO).

The following notation is used (see Fig. 1): θ is the pitch angle; ε is the elevation angle; λ is the travel angle; v_f, v_r is control voltages of the front and the rear motors. Denote $u = v_f - v_r$ and $v = v_f + v_r$. The control voltages v_f and v_r, applied to the front and rear motors are calculated based on the command signals u, v as follows:

$$
v_f = 0.5(v + u), \quad v_r = 0.5(v - u). \quad (13)
$$

We make the following simplifying assumptions: the 'Helicopter' is considered as a rigid body, i.e. it is assumed that the long arm of the 'Helicopter' rotates about the long axis together with the crossbar, and bending of structural components is neglected; the gyroscopic torques, developed by motor/propeller pairs are neglected; dependence of motor/propeller force gain
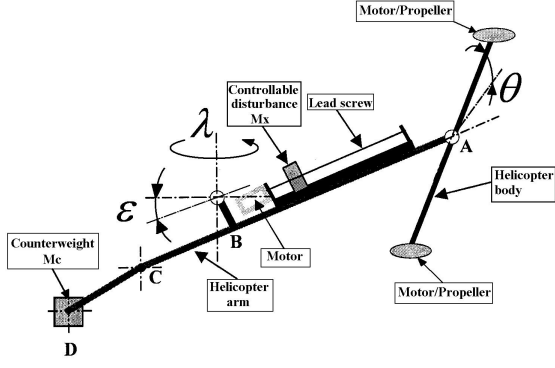
Fig. 1. Lay-out of the 'Helicopter'.

on the 'Helicopter' airspeed is neglected; influence of aerodynamical pressure forces on the 'Helicopter' body is neglected; dry friction in the pivots is neglected. Based on the given assumptions, the following simplified model is obtained (see also [Kiefer et al., 2006, 2004, Peaucelle et al., 2007]):

$$\begin{cases} \ddot{\varepsilon} = -a_{1\varepsilon}\dot{\varepsilon} - a_{0\varepsilon}\sin(\varepsilon - \varepsilon_0) + b_\varepsilon \cos\theta \cdot v \\ \ddot{\theta} = -a_{1\theta}\dot{\theta} - a_{0\theta}\sin(\theta - \theta_0) + b_\theta \cos\varepsilon \cdot u \\ \ddot{\lambda} = -a_{1\lambda}\dot{\lambda} + b_\lambda \sin\theta \cdot v \end{cases} \quad (14)$$

where $a_\star, b_\star, \theta_0$ and ϵ_0 are uncertain plant model parameters for which identification experiments allow to give upper and lower bounds (see [Peaucelle et al., 2007]). The constants θ_0 and ε_0 denote pitch and elevation balance angles.

We assume in the following that only positive values of the normal force command signal $v(t)$ are applied, $v \in [0.5, 10]$. This means that we are interested only in *positive* values of the desired elevation angle. The elevation angle has physical limitations $\epsilon \in [-5 \quad 20]$ deg. Moreover the pitch angle has mechanical restrictions as $|\theta| \leq 90$ deg but in the following we assume that $\theta(t)$ is kept within the working area $|\theta| \leq 60$ deg.

With all these assumptions and uncertainties, the system also writes as the following time-varying linear system

$$\begin{cases} \ddot{\varepsilon} = -a_{1\varepsilon}\dot{\varepsilon}(t) - a_{0\varepsilon}(t)\varepsilon(t) + b_\varepsilon(t)v(t) + d_\varepsilon(t) \\ \ddot{\theta} = -a_{1\theta}\dot{\theta} - a_{0\theta}(t)\theta(t) + b_\theta(t)u(t) + d_\theta(t) \\ \ddot{\lambda} = -a_{1\lambda}\dot{\lambda} + b_\lambda(t)\theta(t) \end{cases} \quad (15)$$

where the constant and time-varying parameters are defined as:

$a_{1\epsilon} = a_{1\epsilon}$	$\in [0.02 \quad 0.15]$
$a_{0\varepsilon}(t) = a_{0\varepsilon}\cos\varepsilon_0\sin\varepsilon(t)/\varepsilon(t)$	$\in [3 \quad 5]$
$b_\varepsilon(t) = b_\varepsilon\cos\theta(t)$	$\in [0.2 \quad 0.4]$
$d_\varepsilon(t) = a_{0\varepsilon}\sin\varepsilon_0\cos\varepsilon(t)$	$\in [-1.3 \quad 0]$
$a_{1\theta} = a_{1\theta}$	$\in [0.02 \quad 0.15]$
$a_{0\theta}(t) = a_{0\theta}\cos\theta_0\sin\theta(t)/\theta(t)$	$\in [0.3 \quad 0.7]$
$b_\theta(t) = b_\theta\cos\varepsilon(t)$	$\in [0.2 \quad 0.4]$
$d_\theta(t) = a_{0\theta}\sin\theta_0\cos\theta(t)$	$\in [-0.14 \quad 0.14]$
$a_{1\lambda} = a_{1\lambda}$	$\in [0.02 \quad 0.05]$
$b_\lambda(t) = b_\lambda v(t)\sin\theta(t)/\theta(t)$	$\in [0.05 \quad 0.1]$.

Written in this way, the model exhibits a decoupling of the elevation dynamics actuated by v on one side and the pitch-travel dynamics actuated by u on the other. A decoupled PID-type control is therefore considered:

$$v = K_{P\varepsilon}(\epsilon - \epsilon^*) + K_{I\varepsilon}\int(\varepsilon - \varepsilon^*)d\tau + K_{D\varepsilon}\dot{\varepsilon}$$

$$u = K_{P\lambda}(\lambda - \lambda^*) + K_{I\lambda}\int(\lambda - \lambda^*)d\lambda + K_{D\lambda}\dot{\lambda} \quad (16)$$

$$+ K_{P\theta}\theta + K_{D\theta}\dot{\theta}.$$

6. STATIC AND ADAPTIVE CONTROL OF THE "HELICOPTER"

To apply the results of the previous sections it is first needed to conceive stabilizing static feedback gains:

$$F_{0v} = [\, F_{0P\varepsilon} \quad F_{0I\varepsilon} \quad F_{0D\varepsilon} \,]$$
$$F_{0u} = [\, F_{0P\lambda} \quad F_{0I\lambda} \quad F_{0D\lambda} \quad F_{0P\theta} \quad F_{0D\theta} \,]$$

for both PIDs. In this 'Helicopter' example, both positions $\epsilon, \lambda, \theta$ and rotation speeds $\dot{\epsilon}, \dot{\lambda}, \dot{\theta}$ are measured. Thus, the design of these gains corresponds to state-feedback design problems for the augmented systems of the type (12):

$$\dot{x}_\varepsilon = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ -a_{0\varepsilon} & 0 & -a_{1\varepsilon} \end{bmatrix} x_\varepsilon + \begin{bmatrix} 0 \\ 0 \\ b_\varepsilon \end{bmatrix} v \quad (17)$$

$$\dot{x}_\lambda = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_{1\lambda} & b_\lambda & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -a_{0\theta} & -a_{1\theta} \end{bmatrix} x_\lambda + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ b_\theta \end{bmatrix} u \quad (18)$$

with $x_\varepsilon = \begin{pmatrix} \epsilon & \int\varepsilon & \dot{\varepsilon} \end{pmatrix}^T$ and $x_\lambda = \begin{pmatrix} \lambda & \int\lambda & \dot{\lambda} & \theta & \dot{\theta} \end{pmatrix}^T$.

Such state-feedback gains may be designed using, for example, the RoMulOC toolbox ([Peaucelle, 2005],[Peaucelle and Arzelier, 2006]). For space limitations reasons we do not detail this static feedback design step. For both gains specifications were in terms of pole location plus, in the travel-pitch case, a specification to minimize the pitch peak in response to steps on the travel reference. The obtained gains are

$$F_{0v} = -[\, 44.089 \quad 45.750 \quad 26.518 \,]$$
$$F_{0u} = -[\, 174.834 \quad 9.846 \quad 1026.224 \quad 172.897 \quad 47.646 \,].$$

Inequalities of Proposition 1 are solved assuming parameters values in the middle of intervals. For the elevation model and using F_{0v} as a stabilizing gain one finds

$$G_{yv} = [\, 6.210 \quad 4.890 \quad 0.098 \,], \quad D_{yv} = 0.0010$$
$$G_{\eta v} = [\, 6.328 \quad 4.671 \,], \quad D_{\eta v} = 0.0011.$$

Solving the inequalities this time for the travel-pitch models using F_{0u} as a stabilizing gain gives

$$G_{yu} = [\, 6.642 \quad 45.429 \quad 1.460 \quad 4.304 \quad 0.003 \,],$$
$$G_{\eta u} = [\, 6.665 \quad 45.096 \quad 1.449 \quad 4.302 \,],$$
$$D_{yu} = 3.245 \cdot 10^{-4}, \quad D_{\eta u} = 3.257 \cdot 10^{-4}.$$

The final step is to compute the scalars α_v and α_u that define the domains in which the adaptation gains evolve while preserving stability. These scalars are computed using Theorem 3. The obtained values are

$$\alpha_{yv} = 1012.6, \quad \alpha_{\eta v} = 0.343,$$
$$\alpha_{yu} = 86.803, \quad \alpha_{\eta u} = 8.210 \cdot 10^{-4}$$

All LMIs were coded using YALMIP of [Löfberg, 2004] and solved with SeDuMi solver of [Sturm, 1999]. Computation time for the elevation model (polytope with 8 vertices, system of order 3) is 1.9 seconds on Mac OSX computer with 2.66 GHz Inter Core 2 Duo processor. For the travel-pitch model (polytope with 32 vertices, system of order 5) the computation time was 4.3 seconds. The proposed solutions are not any feasible solutions but those that minimize $D_y + D_\eta$ and maximize $\alpha_y + \alpha_\eta$. In this manner one can expect to get larger adaptation regions (see Remark 1).

If implemented as is, the gains K_{yv} are allowed to vary too far away from the stabilizing value F_{0v}. In practice the adaptation

then drives the gains to values reducing the time response but this is done at the expense of larger values of the control input u. In practice this input is saturated (limited capacity of actuators) and the adaptation thus drives the gains to destabilizing situations. To avoid this, we chose to limit the adaptation and took $\alpha_{yv} = 0.1$. The gains K_{yv} are therefore kept close to the static ones.

Simulations are done on the non-linear model with the following choice of parameters

$$a_{1\epsilon} = 0.085, \quad a_{0\epsilon} = 4.5, \quad b_\epsilon = 0.3, \quad \varepsilon_0 = 0,$$
$$a_{1\theta} = 0.085, \quad a_{0\theta} = 0.5, \quad b_\theta = 0.3, \quad \theta_0 = 0,$$
$$a_{1\lambda} = 0.035, \quad\quad\quad\quad\quad b_\lambda = 0.075.$$

Two square repeating sequences are applied as references on the elevation and the travel. The elevation reference switches between $1°$ and $10°$ every 10 seconds. The travel reference switches between 0 and $9°$ every 70 seconds.

Figure 2, 3 and 4 show the time histories of the travel, pitch and elevation angles respectively. The dotted lines are the square reference signals. The dashed lines are the responses with static feedback gains F_{0u}, F_{0v}. The solid lines are the responses with adaptive PID control. The elevation plot being hardly readable Figure 5 gives a zoom of this plot.
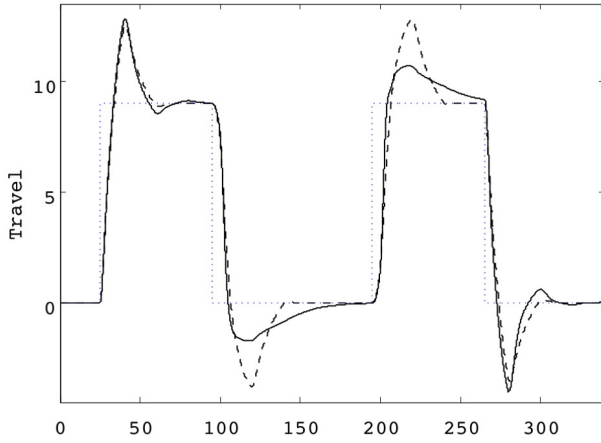

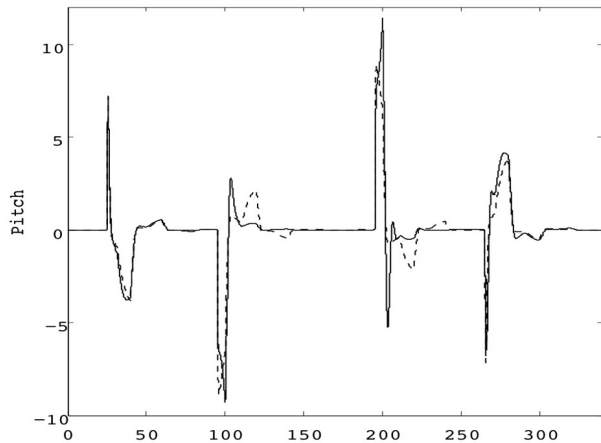
Fig. 2. Times histories of travel angle $\lambda(t)$

Fig. 3. Times histories of pitch angle $\theta(t)$

Some explanations about the simulations: Firstly, notice the answer of the travel axis is not identical in all situations. It very much depends on wether the helicopter is at high of low elevations. In the former case (first travel rise) the force of the
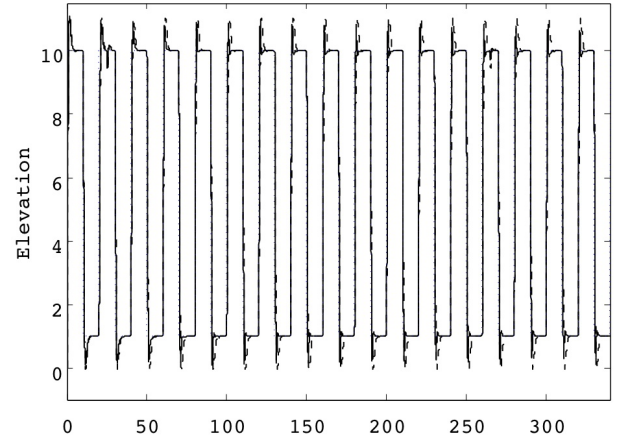

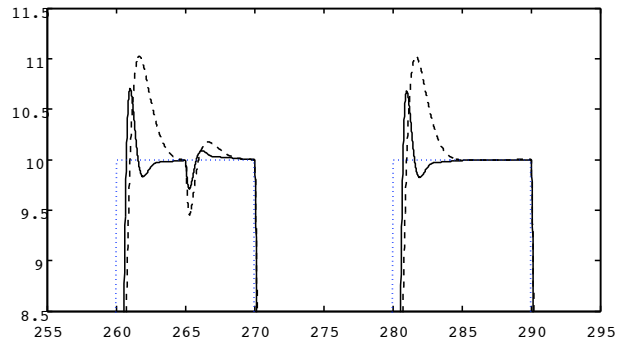Fig. 4. Times histories of elevation angle $\epsilon(t)$

Fig. 5. Zoom on times histories of elevation angle $\epsilon(t)$

propellers (v) is large at the instant of the step, it generates a strong torque for travel and a rapid response. In the latter case (second travel rise) the force of the propellers is small and has less effect. Secondly, notice in Figure 5 disturbance at time 265. This disturbance is due to the coupling between the axes. At that instant the step on the travel angle generates a strong pitch angle θ. The vertical component of the propellers force is $\cos\theta \cdot v$, it suddenly decreases thus creating this dropping of the helicopter. These two behaviors of the non-linear model are coherent with what is observed on the real plant.

Comparisons of the static PID and the adaptive control law do not give any decisive conclusion on the superiority of one or the other. Experiments on the real plant will be done in the future and may give some more insight on the respective properties. We expect to recover properties we had reported in a former paper ([Andrievsky et al., 2007]). Meanwhile, experiments have been conducted including noise on the measurements. No major difference is noted compared to the given figures.

Figures 6 and 7 give the time histories of the gains K_v (adaptive PID for the elevation axis) and K_u (adaptive PID for the travel-pitch angles). It shows that the domains in which the gains are bounded do allow quite large variations. When the limits of this region are reached (as in Figure 6) the barrier functions do constrain the gains to keep in the region.

CONCLUSION

A new theoretical result is provided for designing robust adaptive controllers. The method only needs the knowledge of a stabilizing static gain thus relaxing passivity assumptions. Di-
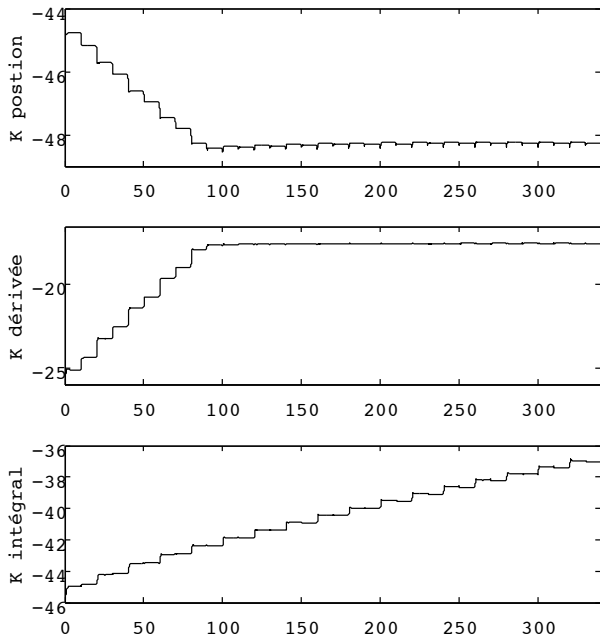
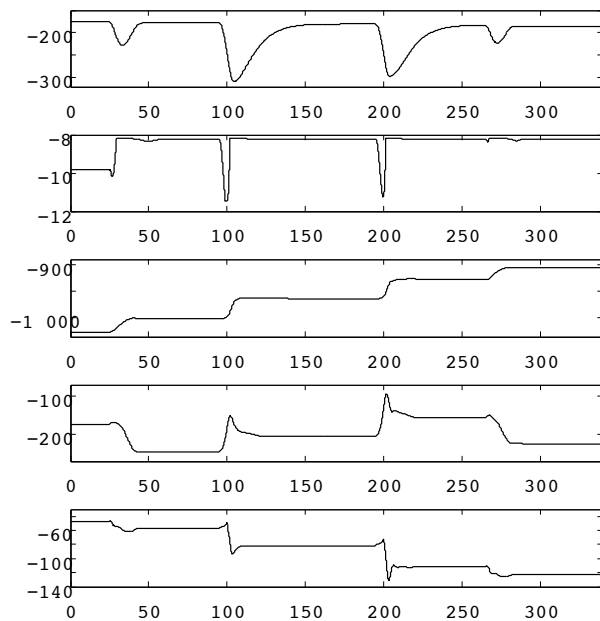Fig. 6. Times histories of adaptive PID $K_v(t)$ for elevation angle

Fig. 7. Times histories of adaptive PID $K_u(t)$ for travel-pitch angles

viding the adaptive law in two parts involving different signals the method also allows to build adaptive PID controllers. The methodology is satisfactorily applied on the model of a helicopter benchmark. It validates the design procedure. The advantages of the adaptive control are not straightforward on the simulations. Experiments on the benchmark will be done in the close future and may give more insight with respect to this comparison.

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