SPEED-GRADIENT CONTROL OF ENERGY IN SINGULARLY PERTURBED SYSTEMS

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Abstract: Energy speed-gradient control of singulary perturbed Hamiltonian systems is studied both theoretically and by computer simulation. Previous results on stability of speed-gradient control of singularly perturbed systems are extended to the case of partial stability. Quantitative results are obtained for synchronization of two coupled pendulums, taking into account inertia of the coupling link.

Keywords: Energy control, Speed gradient, Singular perturbations, Coupled pendulums

1. INTRODUCTION

Control of system energy is an important problem having different applications to control of mechanical and electromechanical systems, particularly to control of oscillatory modes (Fradkov and Pogromsky, 1998; Åström and Furuta, 2000). A general approach to energy control based on speed-gradient method was proposed in (Fradkov, 1996) and later extended to control of several invariants of a nonlinear systems (Fradkov and Pogromsky, 1998; Shiriaev and Fradkov, 2001).

For control of complex nonlinear systems an important problem is dealing with an unmodeled dynamics, particularly with singularly perturbed systems. It is well known that unmodeled dynamics may not only prevent from achieving the control goal, but also cause unboundedness of control system trajectories (Ioannou and Kokotović, 1983; Kokotović, *et al.*, 1986; Fradkov, *et al.*, 1999). Conditions for stability of singularly perturbed speed-gradient based control systems were proposed in (Fradkov, 1987), see also (Fradkov, *et al.*, 1999). These conditions are well suited for adaptive control systems where stability with respect to only a part of variables may be observed. However, the conditions of (Fradkov, 1987) are not fulfilled for energy control problems, since (A) the energy-based Lyapunov function is not radially unbounded and (B) an unperturbed systems possess weaker stability properties, namely, partial stability with respect to a function rather than stability with respect to a part of variables.

In this paper the results of (Fradkov, 1987) are extended to encompass the problems of speed-gradient based energy control of singularly perturbed Hamiltonian systems. In Section 2 new stability results for singularly perturbed nonlinear systems is presented and applied to energy control of Hamiltonian systems. An example of application to controlled synchronization of two coupled pendulums, taking into account inertia of the coupling link is studied in the Section 3 by computer simulation.

2. SPEED-GRADIENT METHOD FOR SINGULARLY PERTURBED SYSTEMS

Consider the following plant model

$$\dot{x}_1 = f_1(x_1, x_2, u, t) \qquad (1)$$

$$\varepsilon \dot{x}_2 = f_2(x_1, x_2, u, t), \qquad (2)$$

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where u is the control action, $x_1 \in \mathbb{R}^{n_1}$ is the vector of slow variables, $x_2 \in \mathbb{R}^{n_2}$ is the vector of fast variables, and $f_1(\cdot)$, $f_2(\cdot)$ are the vector functions of appropriate dimensions.

Let the control objective be the fulfillment of the relation

$$\lim_{t\to\infty} Q(x_1(t), t) = 0, \tag{3}$$

where $Q(x_1, t)$ is a scalar smooth objective function, $x = \mathrm{col}(x_1, x_2)$.

To design a simplified control law, the initial system (1), (2) is replaced by a reduced-order one obtained by substitution $\varepsilon = 0$ as follows:

$$\dot{x}_1 = \overline{f}(x_1, u, t), \quad \overline{x}_2 = \eta(x_1, u, t), \tag{4}$$

where $\overline{x}_2 = \eta(x_1, u, t)$ is a root of the equation $f_2\ x_1, x_2, u, t) = 0$ (the root is assumed to exist and be unique), $\overline{f}(x_1, u, t) = f_1(x_1, \eta(x_1, u, t), u, t)$.

Then, the speed-gradient control algorithm (see Sec. 2) for the reduced-order system model is designed:

$$u = \ \ \nabla \ \ {}_u\omega(x_1, u, t), \tag{5}$$

where $\ = \ \ ^{\mathrm{T}} > 0$ is a positive definite matrix and

$$\omega(x_1, u, t) = \frac{\partial Q}{\partial t} + (\nabla_x Q)^{\mathrm{T}} \overline{f}(x_1, u, t). \tag{6}$$

The final stage of the design consists in verification of stability properties of the closed-loop system. It is easy to show that, to provide the fulfillment of the control objective $Q \to 0$ as $t \to \infty$ for the reduced-order system (4), (5), it suffices to assume that the function $\omega(\cdot)$ is convex in u and there exists a vector u_* such that the system (4) with substitution $u = u_*$ is exponentially stable with respect to function Q, i.e., $\omega(x_1, u_*, t) \leq \ \ \alpha Q$ for some $\alpha > 0$. However, the fulfillment of the control objective for the reduced-order system does not guarantee the same for the initial one (1), (2), (5), see (Ioannou and Kokotović, 1983). Therefore application of the above design method requires additional conditions assuming, in particular, small value of the parameter ε. These conditions are introduced below.

The known classical results concerning the method of singular perturbations either deal with a finite period of time (Tikhonov's theorem, the first theorem of Bogoliubov) or require uniform asymptotic stability of the reduced-order system (the second theorem of Bogoliubov, the Hoppenstedt theorem).

Fradkov (1987) weakened the uniform asymptotic stability condition and extended the result to the case when $x_1 = (z, \theta)$ and the reduced system is asymptotically stable with respect to x_1, i.e. asymptotically stable with respect to the part of the state variables. Such a case is important for adaptive control where θ is the vector of adjustable parameters. However, in the energy control problems the system (4), (5), in general, exhibits only partial asymptotic stability with respect to some function of state variables $x_1(t)$ and the previous results cannot be applied. Besides, the energy based goal function does not meet standard assumptions of radial unboundedness for Q.

Below we extend the results of (Fradkov, 1987) to the form, suitable for speed-gradient energy control.

For the sake of simplicity consider the case of time-invariant system

$$\dot{x}_1 = f_1(x_1, x_2, u) \tag{7}$$

$$\varepsilon \dot{x}_2 = f_2(x_1, x_2, u) \tag{8}$$

and the time-invariant goal function $Q = Q(x_1)$. In this case the speed-gradient algorithm, derived on the basis of a reduced-order system

$$\dot{x}_1 = \overline{f}(x_1, u), \tag{9}$$

takes the form

$$u = \ \ \nabla \ \ {}_u\omega(x_1, u), \tag{10}$$

where $\ = \ \ ^{\mathrm{T}} > 0$,

$$\omega(x, u) = (\nabla_x Q)^{\mathrm{T}} \overline{f}(x_1, u),$$

$$\overline{f}(x_1, u) = f_1(x_1, \eta(x_1, u), u),$$

and $\overline{x}_2 = \eta(x_1, u)$ is a root of the equation $f_2(x_1, x_2, u) = 0$.

The main result of the paper is as follows.

Theorem 1. Given a system (7), (8) and (10). Let the functions $f_1(\cdot)$, $Q(\cdot)$ and $f_2(\cdot)$ be twice continuously differentiable, and satisfy the following conditions:

A0) *The functions $\overline{f}(x_1, u)$, $Q(x_1)$, and their first and second derivatives are bounded in the set $\Omega_R = \{x_1 : Q(x_1) \leq R\}$ for all u satisfying (10).*

A1) *for any x_1 and u there exists a unique root $\overline{x}_2 = \eta(x_1, u)$ of the equation $f_2(x_1, x_2, u) = 0$ and the function $\eta(x_1, u)$ is twice continuously differentiable;*

A2) *the function ω is convex in u, and there exists a constant vector u_* such that for all $x_1 \in \Omega_R$*

$$\omega(x_1, u_*) < \mu(x \ \ _1)$$

for some $\mu(x_1) \geq 0$ and there exist numbers $\alpha_i > 0$ $(i = 0, 1, 2)$ such that

$$0 \leq \alpha_1 \mu(x_1) \leq |\nabla_{x_1} Q(x_1)| \leq \alpha_2 Q(x_1)^{1/2}$$

A3) *there exist a continuously differentiable function $V_2(\widetilde{x}_2)$, where $\widetilde{x}_2 = x_2 \ \ \ \eta(x_1, u)$, and numbers β_i $(i = 0, 1, 2)$ such that*

$$\partial_2 V_2 \leq \ \ \ \beta_0 |\widetilde{x}_2|^2, \quad \beta_1 |\widetilde{x}_2| \leq |\nabla V_2| \leq \beta_2 |\widetilde{x}_2|,$$

where

$$\partial_2 V_2 = (\nabla V_2(\widetilde{x}_2))^{\mathrm{T}} f_2(x_1, \widetilde{x}_2 + \eta(x_1, u), u).$$

Then:

i) *for any bounded set D of the initial states x_2 there exists a number $\varepsilon_0 > 0$ such that for $0 < \varepsilon < \varepsilon_0$ the* *solutions of the system (7), (8) and (10) with $x_1 \in \Omega$ satisfy the following relations*

$$\lim_{t\to\infty} \mu \;\; x_1(t))=0,\;\; \lim_{t\to\infty}\Big(x_2(t) \quad \eta(x_1(t),u(t))\Big)=0;$$

ii) *let*

$$B_R=\{(x_1,x_2):\ Q(x_1)\le R, x_2\in D\}\subset\mathbb{R}^{n+m}, \tag{11}$$

and let the values

$$L_1(R)=\sup_{B_R}\Big|\frac{\partial f_1(x_1,x_2,u)}{\partial x_2}\Big|$$
$$L_2=\sup_{B_R}\Big|\frac{\partial}{\partial x_1}(\partial_1\eta(x_1,u))\Big|,$$
$$L_3(R)=\sup_{B_R}\Big|\frac{\partial}{\partial x_1}\eta(x_1,u)\Big|.$$

be bounded for some $R>0$ and bounded $D\subset\mathbb{R}^m$. Additionally, if

$$x(0)\in D_R=\Big\{(x_1,x_2):\ Q(x_1)+\frac{\alpha_1 L_1(R)}{\beta_1 L_2(R)}V_2(\widetilde{x}_2)\ \le\ R\Big\}$$

then ε_0 can be chosen in the form

$$\varepsilon_0(R)=\frac{\alpha_0\alpha_1\beta_0\beta_1}{\beta_2 L_1(R)(\alpha_2 L_2(R)+\alpha_0\alpha_1 L_3(R))}. \tag{12}$$

Remark 2. *If $\alpha_0\alpha_1 L_3(R) \ll \alpha_2 L_2(R)$, then the right-hand side of (12) can be approximated as*

$$\varepsilon_0(R)\approx k_1k_2(k_{12}k_{21})^{1} \quad , \tag{13}$$

where the coefficients $k_1=\alpha_0\alpha_1/\alpha_2$, $k_2=\beta_0\beta_1/\beta_2$ are proportional to the stability degrees of the fast and slow subsystems, and the numbers $k_{12}=L_2(R)$ and $k_{21}=L_1(R)$ can be interpreted as degrees of interconnection between the subsystems.

In brief, the above theorem means the following. If the fast subsystem (8) is exponentially stable on x_1 for $u=0$ and the reduced-order system (9), (10) is partially asymptotically stable with respect to the function $\mu(x_1)$, then the algorithm (10) ensures partial stabilization of the initial system (7), (8) with respect to the function $\mu(x_1)$ for sufficiently small parameter $\varepsilon>0$. The particular value of ε depends on initial conditions $x(0)$, since the right-hand sides of (7), (8) may be locally, but not globally, Lipschitz. Thus, we can conclude that the considered control algorithm (10) is robust with respect to unmodeled fast dynamics (singular perturbations).

The above result applies to the controlled system is the Hamiltonian form

$$\dot p_i=\ \ \frac{\partial H(p,q,u)}{\partial q_i},\ \dot q_i=\frac{\partial H(p,q,u)}{\partial p_i},\ i=1,\ldots,n, \tag{14}$$

where $p=\mathrm{col}(p_1,\ldots,p_n)$, $q=\mathrm{col}(q_1,\ldots,q_n)$ are the vectors of generalized coordinates and momenta, $H=H(p,q,u)$ is the Hamiltonian function, and $u(t)\in\mathbb{R}^m$ is the input (generalized force).

Consider the problem of approaching the prespecified level of energy of the free (unforced) system

$$H_0(p(t),q(t))\to H_*\quad\text{as } t\to\infty, \tag{15}$$

where $H_0(p,q)=H(p,q,0)$ is the "internal" Hamiltonian describing the unforced system

The speed-gradient algorithm for the posed problem is as follows:

$$u=\psi\quad (H_0\quad H_*)[H_0,H_1]^{\mathrm{T}}), \tag{16}$$

where ψ is a smooth vector function with values in $\mathbb{R}^m$ which satisfies the strict pseudogradient condition $\psi(z)^{\mathrm{T}}z>0$ for $z\neq 0$, where

$$[f,g]=\sum_{i=1}^{n}\left(\frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}\quad \frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i}\right).$$

is the Poisson bracket of smooth functions $f(p,q)$ and $g(p,q)$ (if the functions f,g are the vector functions then the Poisson bracket is defined componentwise).

Theorem 1 applies to the energy control problem for singularly perturbed Hamiltonian system (14). In that case $Q(x_1)=\frac{1}{2}\ H(x_1)\quad H_*)^2$, $x_1=\mathrm{col}(p,q)$, x_2 is a state vector of the perturbed system (1), (2). The condition A2 of the theorem holds for $\mu(x_1)=[H_0(x_1),H_1(x_1)]^2Q(x_1)$. It follows from Theorem 1 and Corollary 1 that the goal (3) is achieved with the control algorithm (16) for initial conditions from B_R and sufficiently small $\varepsilon>0$, if $[H_0(x),H_1(x)]\neq 0$ for $x\in B_R$ for some $R>0$.

3. EXAMPLE. FLEXIBLY COUPLED PENDULUMS

3.1 Model of the controlled system

Consider the two pendulums coupled by the spring (see Fig. 1). Such a system is the special case of the diffusively coupled oscillators model, which is often used for modeling various physical and mechanical systems (Jackson, 1990). To take into account the dynamics of the coupling unit, it is assumed that the coupling torque between the pendulums depends dynamically on the difference between the pendulum angles. The coupling unit is conceived here of a small flywheel, mounted on the torsion spring. The both ends of the spring are connected with the pendulum rods. The system dynamics can be described as

$$\begin{cases} J_p\ddot\varphi_1+R_p\dot\varphi_1+mgl\sin\varphi_1\\ \qquad =K(\mu\quad \varphi_1)+M(t),\\ J_p\ddot\varphi_2+R_p\dot\varphi_2+mgl\sin\varphi_2\\ \qquad =K(\mu\quad \varphi_2),\\ J_s\ddot\mu+R_s\dot\mu+2K\mu=K(\varphi_1+\varphi_2),\end{cases} \tag{17}$$

where $\varphi_i(t)$ are the pendulum rotation angles ($i=1,2$); $M(t)$ is the external torque (the control action), applied to the first pendulum; m, l are the mass and the length of the rod for each pendulum; J_p is the pendulum moment of inertia ($J_p = ml^2$ for the point mass pendulum); g is the acceleration of gravity; K is the coupling parameter (the stiffness of the spring); R_p and R_s are the viscous friction coefficients for the pendulums and the coupling unit, respectively; J_s is the moment of inertia of the coupling unit (of the "flywheel"); $\mu(t)$ is the twist angle (the flywheel rotation angle). The mass and the damping of the coupling unit are assumed to be small.

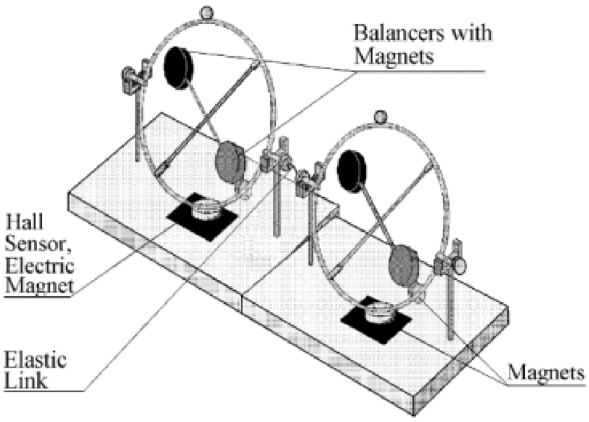

Fig. 1. Two pendulums, coupled by the spring.

Consider the problem of excitation oscillations with the desired amplitude, understood as achieving the given energy level. An additional goal may be posed as the requirement that pendulums have either coinciding or opposite phases of oscillation (in-phase or anti-phase synchronization). To design the control law we use speed-gradient method for the reduced plant model, neglecting the coupling dynamics.

3.2 Plant model reduction

Assuming that the coupling dynamics are "fast", introduce the "small parameter" ε into the third of Eqs. (17). To this end, the following notations are introduced: $J_s = \varepsilon^2 \bar{J}_s$, $R_s = \varepsilon \bar{R}_s$. Then the plant model (17) takes the form

$$\begin{cases} \ddot{\varphi}_1 + \rho\dot{\varphi}_1 + \omega^2 \sin\varphi_1 = k(\mu - \varphi_1) + u(t), \\ \ddot{\varphi}_2 + \rho\dot{\varphi}_2 + \omega^2 \sin\varphi_2 = k(\mu - \varphi_2), \\ \varepsilon^2\ddot{\mu} + \varepsilon\rho_s\dot{\mu} + 2k_s\mu = k_s(\varphi_1 + \varphi_2), \end{cases} \tag{18}$$

where $u(t) = M(t)/J_p$ is the rescaled external torque (the control action); ω is the natural frequency of small oscillations for the uncoupled pendulums, $\omega^2 = mgl/J_p$; $k = K/J_p$ is the coupling coefficient; $\rho = R/J_p$ is the friction parameter; $\rho_s = \bar{R}_s/\bar{J}_s$, $k_s = K/\bar{J}_s$. Evidently, Eq. (18) is a particular case of Eqs. (1), (2), where $x_1 = \mathrm{col}\,(\varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2) \in \mathbb{R}^4$ is the vector of slow variables and $x_2 = \mathrm{col}\,(\mu, \dot{\mu}) \in \mathbb{R}^2$ is the vector of fast variables.

To design a control law, the full-order initial system (18) is replaced by a reduced-order one, obtained by substitution $\varepsilon = 0$ and described by the following equations

$$\begin{cases} \ddot{\varphi}_1 + \rho\dot{\varphi}_1 + \omega^2 \sin\varphi_1 + 0.5\,k(\varphi_1 - \varphi_2) \\ \qquad = u(t), \\ \ddot{\varphi}_2 + \rho\dot{\varphi}_2 + \omega^2 \sin\varphi_2 + 0.5\,k(\varphi_2 - \varphi_1) = 0, \end{cases} \tag{19}$$

(the notations are given above, see Eqs. (17), (18).)

The simplified model (19) is used further on for energy control law design.

3.3 Control law design for the reduced plant model

The total energy $H(x_1)$ of the system (19) can be written as follows

$$\begin{aligned} H(x_1) = {} & \frac{1}{2}\dot{\varphi}_1 + \omega^2(1 - \cos\varphi_1) + \frac{1}{2}\dot{\varphi}_2^2 \\ & + \omega^2(1 - \cos\varphi_2) + \frac{k}{4}\left(\varphi_1 - \varphi_2\right)^2, \end{aligned} \tag{20}$$

$x_1(t) = \mathrm{col}\,(\varphi_1, \dot{\varphi}_1, \varphi_2, \dot{\varphi}_2)$.

In order to apply the speed-gradient procedure of Sec. 2, introduce the two objective functions as follows:

$$\begin{aligned} Q_\varphi(\dot{\varphi}_1, \dot{\varphi}_2) &= \frac{1}{2}\left(\delta_\varphi\right)^2 \\ Q_H(x_1) &= \frac{1}{2}(H(x_1) - H_*)^2. \end{aligned} \tag{21}$$

where $\delta_H = H(x_1) - H_*$ is referred to as *energy error*; $\delta_\varphi = \dot{\varphi}_1 + \sigma\dot{\varphi}_2$ is referred to as *synchronization error*; $\sigma \in \{-1, 1\}$ is a reference phase-shift parameter; H_* is the prescribed value of the total energy.

Apparently, minimization of Q_H means achievement of the desired oscillations magnitude. The minimum value of the function Q_φ allows, additionally, to meet the "inphase/antiphase" requirement (at least for small initial phases $\varphi_1(0), \varphi_2(0)) : Q_\varphi(\dot{\varphi}_1, \dot{\varphi}_2) \equiv 0$ if and only if $\dot{\varphi}_1 \equiv -\sigma\dot{\varphi}_2$. Hence option $\sigma = 1$ sets the *antiphase* desired pendulums oscillations, while $\sigma = -1$ sets the *inphase* ones.

To design the control algorithm, the objective function $Q(x_1)$ as the weighted sum of Q_φ and Q_H is introduced:

$$Q(x_1) = \alpha Q_\varphi(\dot{\varphi}_1, \dot{\varphi}_2) + (1 - \alpha)Q_H(x_1), \tag{22}$$

where $\alpha \in [0, 1]$ is a *weighting coefficient*.

The speed-gradient procedure of Sec. 2 leads to the following control laws:

–the *proportional form*

$$u = -\gamma\left(\alpha\delta_\varphi + (1 - \alpha)\delta_H\dot{\varphi}_1\right), \tag{23}$$

–the *relay form*

$$u = -\gamma\,\mathrm{sign}\left(\alpha\delta_\varphi + (1 - \alpha)\delta_H\dot{\varphi}_1\right), \tag{24}$$

where $\gamma > 0$ is a gain factor. The case $\alpha = 0$ corresponds to the energy control problem Application of the results of Sec. 2 (Corollary 1) yields that for

$\alpha = 0$ the sufficient conditions for the achievement of the control goal $Q(x_1(t)) \to 0$ are valid if the desired level of energy does not exceed the value $H_* = 2\omega^2$, corresponding to the upper equilibrium of one pendulum and the lower equilibrium of the other one.

It is worth noting that the control law (23) for the system (19) with $\sigma = 1$ has been proposed and numerically examined in (Andrievsky and Fradkov, 1999). In the paper (Kumon *et al.*, 2002) the case of $\sigma =$ 1 and nonlinear coupling function in (19) is considered and the results of analytical, numerical and experimental study of the closed-loop system are presented. Numerical analysis of the system with the control law (24) for the both $\sigma = \{0, 1\}$ and for the cases of conservative ($\rho = 0$) and dissipative ($\rho > 0$) oscillators is performed in (Fradkov and Andrievsky, 2003).

3.4 Comparative examination of the closed-loop full-order and reduced systems

In this subsection some numerical results of examination the closed-loop system with the proportional control law (23) are presented. The parameter values are as follows:

– the control law parameters: $\gamma = 1$, $\alpha = 0.7$, $\sigma = 1$ (the anti-phase steady-state oscillations are required), $H_* = 10\ \mathrm{s}^2$;

– the plant model parameters: $\omega^2 = 10\ \mathrm{s}^2$, $k = 1.75$ s^2 , $k_s = 8.57 \cdot 10^2$ s^2 , $\rho_s = 0.01\ \mathrm{s}^1$, $\varepsilon = 0.05$.

The phase variables have zero initial values. Two cases of the damping parameter ρ are studied: $\rho = 0$ (the conservative system), and $\rho = 0.1\ \mathrm{s}^1$ (the dissipative system).

The simulation results are depicted in Figs. 2–7. First consider the energy control problem ($\alpha = 0$) for conservative case ($\rho = 0$). It is seen from Fig. 2 and Fig. 3 that the energy approaches the desired value both for reduced and for the full order systems. This result confirms the theoretical statements.

Now consider more complex problem of energy control with synchronization ($\alpha > 0$). Since the existing theoretical results do not apply, simulation is the only way of its analysis. It is seen that for the lossless case ($\rho = 0$) and the "ideal" plant model ($\varepsilon = 0$) the control goal is achieved: both pendulums fall in anti-phase oscillatory mode, the total energy $H(x_1(t))$ tends to the desired value H_*, and the control torque $u(t)$ tends to zero, see Fig. 4. Note, that the relation between transient times for H and for Q_φ can be changed by means of changing the weight coefficient α. In the lossless case the control amplitude can be arbitrarily decreased by means of decreasing the gain γ. For the the initial plant model (18), the small amplitude oscillations of the energy and the control action around the steady-state values occur, see Fig. 5. In the case of the damped pendulums ($\rho > 0$), for both initial and reduced plant models some steady-state error in the energy $H(x_1)$ appears and the control action $u(t)$ does not vanish, see Figs. 6, 7. Additionally, oscillations with a small amplitude occur in the full-order system (18), see Fig. 7. The simulations show that the examined speed-gradient energy control law (23) possesses the robustness with respect to unmodeled dynamics of the coupling unit. Moreover, using more sophisticated goal function allows to achieve, additionally, in-phase or anti-phase synchronization and this property is also robust with respect to dynamical disturbances.

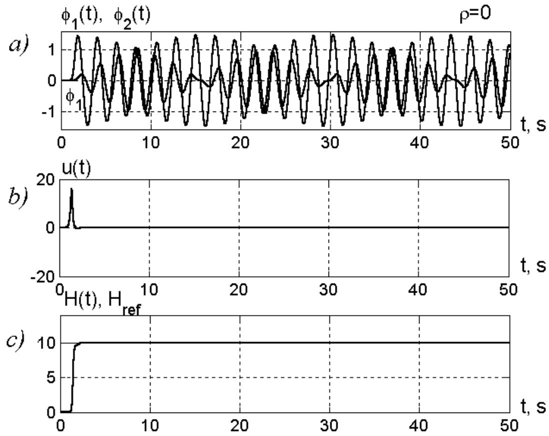

Fig. 2. Energy control; reduced plant model (19), $\rho = 0$.

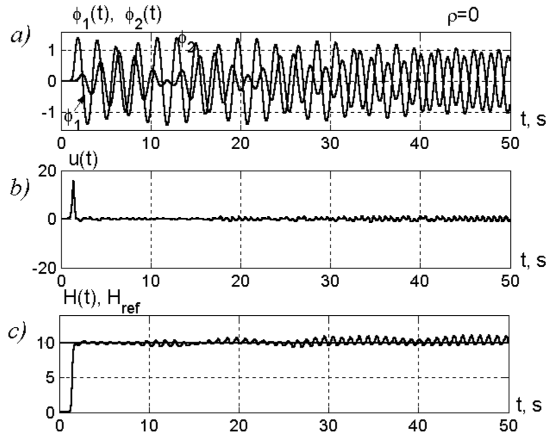

Fig. 3. Energy control; full plant model (18), $\rho = 0$.

4. CONCLUSIONS

In the present work an energy speed-gradient control of singularly perturbed Hamiltonian systems is studied both theoretically and by computer simulation. Previous results on stability of speed-gradient control of singularly perturbed systems are extended to the case of partial stability. Quantitative results are obtained for synchronization of two coupled pendulums, taking into account the coupling link dynamics. The simulations show that the examined speed-gradient energy control law possesses the robustness with respect to unmodeled dynamics of the coupling link. Moreover, using more sophisticated goal function allows to achieve, additionally, in-phase or anti-phase

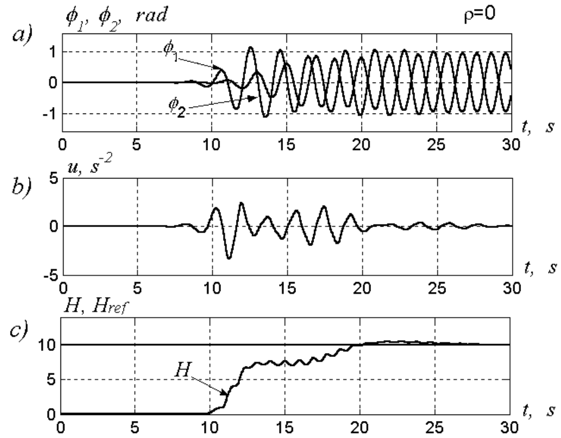

Fig. 4. Excitation of anti-phase oscillations; reduced plant model (19), $\rho = 0$.

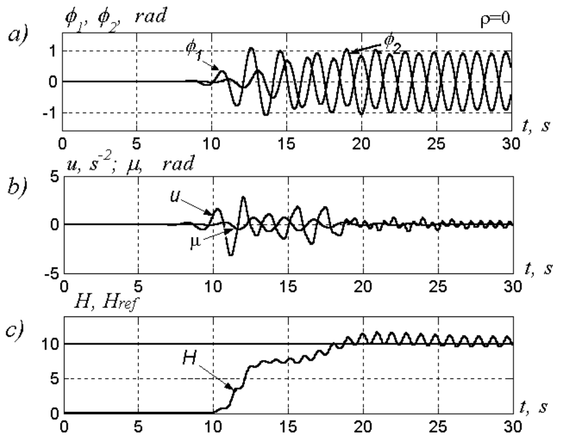

Fig. 5. Excitation of anti-phase oscillations; full plant model (18), $\rho = 0$.

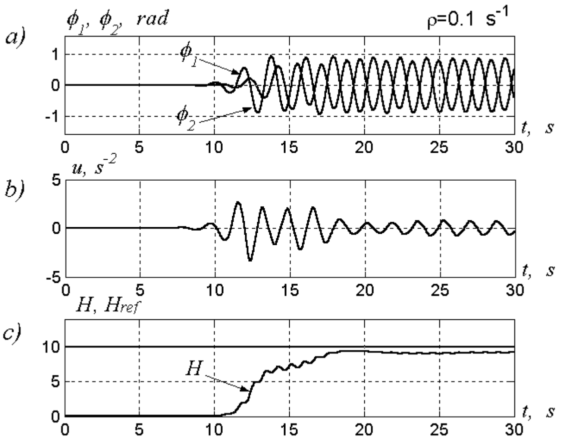

Fig. 6. Excitation of anti-phase oscillations; reduced plant model (19), $\rho = 0.1\,s^1$.

synchronization and this property is also robust with respect to dynamical disturbances.

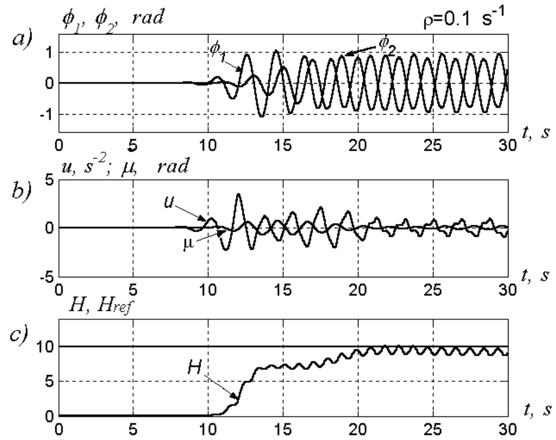

Fig. 7. Excitation of anti-phase oscillations; full plant model (18), $\rho = 0.1\,s^1$.

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