THE MOTION OF A RIGID BODY ON THE INERTIAL ELASTIC PLATE. A NONLINEAR FORMULATION OF THE PROBLEM AND SOME EFFECTS

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The motion of a rigid body on the elastic foundation is considered. It is supposed that the inertia tensor of the body is nearly symmetrical one and the mass center of the body is near its axis of symmetry. The elastic foundation is simulated by the ring plate, the external contour of which is fixed and the internal contour of which is connected with the rigid body by the bearing, so that the rigid body can freely rotate around its axis of symmetry. A moment of the electromotor and a dissipative moment act on the body. The system under consideration is one of the simple models of centrifuge. The inertial properties of the elastic foundation is taken into account, that allows to adequately describe the motion of the centrifuge at large angular velocities of the rigid body rotation. Nonlinear formulation of the problem is presented. Proposed formulation of the problem is essentially based on using of the turn-tensor both for description of the rotation of the plate particles and for description of the rigid body rotation. Linear formulation of the problem is studied in detail. The motion of the system at different parameters of the plate is analyzed. In particular, comparative analysis of the motion of the system in the case of thin plate and in the case of thick plate with small shear stiffness is carried out. Influence of the different types of the moment of motor and the dissipative moment is studied.

1. Introduction.

The motion of a rigid body on the nonlinear elastic foundation is investigated. The inertia tensor of the rigid body slightly differs from symmetrical one. The mass center of the body is near its axis of symmetry. The elastic foundation is simulated by a ring plate. The external contour of the plate is fixed. The internal contour of the plate is joined with the rigid body by the bearing (see Fig. 1). When the plate is not deformed, the axis of symmetry of the body is orthogonal to the plate. The rigid body is rotated around its axis of symmetry by the restricted power motor. The nutational vibrations of the rigid body take place. They are caused by the unbalanced mass and errors in the initial conditions. A nonlinear formulation of the problem is considered. The angle of nutation of the rigid body is assumed to be not small. Hence, the deformations of the plate are also assumed to be not small and the nonlinear plate theory should be used. Two formulations of the problem are considered. The first one is: the rigid body has a fixed point. The second one is: the rigid body has no fixed points. To reduce the problem to the system of differential equations, which can be solved by numerical methods, is objective of the investigation. Formulation of the conditions of conjunction of the rigid body and the plate is the main difficulty of the investigation. The problem of formulation of conditions of conjunction is sucessfully solved due to using the identical mathematical technique (the direct tensor

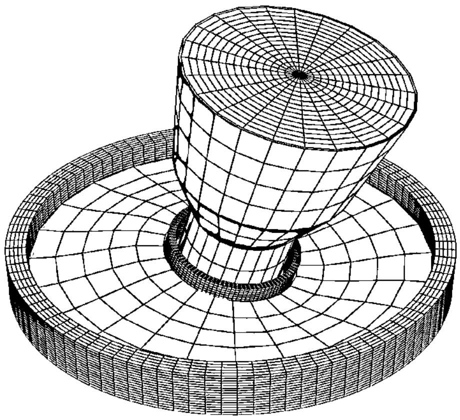

Fig. 1. A rigid body on an elastic plate.

technique) for description of the plate motion and the rigid body motion. Let us note, that it is very important to obtain the system of differential equations in the form, convenient for using in numerical procedures. This problem is solved by the special choice of the basic variables, describing the rotations of the rigid body and the plate particles. (For example, the angles by Euler are not convenient for numerical solution of the problem, because the solution can contain singular points.) In this paper the rotations are described by the turn-tensor. The special representation of the turn-tensor by the turn-vector is used. This representation was proposed by P. A. Zhilin [3], [4]. Let us introduce into consideration the turn-vector. Let us note, that there exist different objects, which are called "turn-vector". We will use the turn-vector associated with the turn-tensor most simple and naturally [4]. Let us consider representation of the turn-tensor in the form by Euler

$$\underline{\underline{P}}(\theta \underline{m}) = (1-\cos\theta)\,\underline{m}\,\underline{m} + \cos\theta\,\underline{\underline{E}} + \sin\theta\,\underline{m}\times\underline{\underline{E}} \tag{1.1}$$

Here θ is the angle of turn and $\underline{m}$ is the unit vector, directed along the axis of turn. The vector

$$\underline{\theta} = \theta\,\underline{m} \tag{1.2}$$

is called the turn-vector. Using the definition (1.2) the turn-tensor (1.1) can be expressed in terms of the turn-vector $\underline{\theta}$

$$\underline{\underline{P}}(\underline{\theta}) = \frac{1-\cos\theta}{\theta^2}\,\underline{\theta}\,\underline{\theta} + \cos\theta\,\underline{\underline{E}} + \frac{\sin\theta}{\theta}\,\underline{\theta}\times\underline{\underline{E}} \tag{1.3}$$

The left angular velocity vector $\underline{\omega}$ and the right angular velocity vector $\underline{\Omega}$ are defined as follows

$$\underline{\omega} = -\frac{1}{2}\Big[\dot{\underline{\underline{P}}}\cdot\underline{\underline{P}}^T\Big]_\times, \qquad \underline{\Omega} = -\frac{1}{2}\Big[\underline{\underline{P}}^T\cdot\dot{\underline{\underline{P}}}\Big]_\times \tag{1.4}$$

The angular velocity vectors $\underline{\omega}$ and $\underline{\Omega}$ are expressed in terms of the turn-vector $\underline{\theta}$ by the formulae

$$\underline{\omega} = \underline{\underline{Z}}(\underline{\theta})\cdot\dot{\underline{\theta}}, \qquad \underline{\Omega} = \underline{\underline{Z}}^T(\underline{\theta})\cdot\dot{\underline{\theta}} \tag{1.5}$$

where tensor $\underline{\underline{Z}}(\underline{\theta})$ has the form

$$\underline{\underline{Z}}(\underline{\theta}) = \underline{\underline{E}} + \frac{1-\cos\theta}{\theta^2}\,\underline{\underline{R}} + \frac{\theta-\sin\theta}{\theta^3}\,\underline{\underline{R}}^2, \qquad \underline{\underline{R}} = \underline{\theta}\times\underline{\underline{E}} \tag{1.6}$$

2. A nonlinear plate theory.

The Reissner's type nonlinear plate theory is used [3]. This theory is described by the 10 th order system of differential equations. Five boundary conditions should be formulated in each point of the plate contour.

The positions and the orientations of the plate particles are determined by the position vector $\underline{R}$ and the turn-tensor $\underline{\underline{P}}$. Using cylindrical coordinate system, vector $\underline{R}$ and tensor $\underline{\underline{P}}$ are represented as follows

$$\underline{R}(r,\theta,t) = (r+u_r)\,\underline{e}_r + u_\theta\,\underline{e}_\theta + u_z\,\underline{n}$$

$$\underline{\underline{P}}(r,\theta,t) = \underline{\underline{E}} + \frac{\sin\varphi}{\varphi}\,\underline{\varphi}\times\underline{\underline{E}} + \frac{1-\cos\varphi}{\varphi^2}\,\underline{\varphi}\times\underline{\underline{E}}\times\underline{\varphi} \tag{2.1}$$

$$\underline{\varphi}(r,\theta,t) = \varphi_r\,\underline{e}_r + \varphi_\theta\,\underline{e}_\theta\,, \qquad \varphi = \sqrt{\varphi_r^2+\varphi_\theta^2}$$

Projection of the turn-vector $\underline{\varphi}$ on the unit vector $\underline{n}$, orthogonal to the nondeformed plate, is considered to be equal to zero, because there is no reaction of the plate to rotation around the axis, directed along the vector $\underline{n}$.

The equations of the motion of the plate are

$$\tilde{\nabla}\cdot\underline{\underline{T}} = \rho(\underline{v} + \underline{\underline{\theta}}_1^T\cdot\underline{\omega})^{\cdot}$$

$$\tilde{\nabla}\cdot\underline{\underline{M}} + \underline{\underline{T}}_\times = \rho(\underline{\underline{\theta}}_1\cdot\underline{v} + \underline{\underline{\theta}}_2\cdot\underline{\omega})^{\cdot} + \rho\,\underline{v}\times\underline{\underline{\theta}}_1^T\cdot\underline{\omega} \tag{2.2}$$

Here $\underline{\underline{T}}$ and $\underline{\underline{M}}$ are the force tensor and the moment tensor, $\tilde{\nabla}$ is the twodimensional Hamilton's operator. The linear and angular velocities of the plate particles are calculated by the formulae

$$\underline{v} = \dot{\underline{R}}\,, \qquad \underline{\omega} = -\frac{1}{2}(\dot{\underline{\underline{P}}}\cdot\underline{\underline{P}}^T)_\times \tag{2.3}$$

The inertia tensors of the plate particles have the form

$$\rho\,\underline{\underline{\theta}}_1^T = \rho_0\,\theta_1\,\underline{\underline{E}}\times(\underline{\underline{P}}\cdot\underline{n})\,, \qquad \rho\,\underline{\underline{\theta}}_2 = \rho_0\,\theta_2\,\underline{\underline{P}}\cdot(\underline{\underline{E}} - \underline{n}\underline{n})\cdot\underline{\underline{P}}^T \tag{2.4}$$

The relations by Cauchy – Green are

$$\underline{\underline{T}}_e = \sqrt{\frac{A}{a}}\,\frac{\partial(\rho_0 U)}{\partial\underline{\underline{A}}^x}\,, \qquad \underline{\underline{M}}_e = \sqrt{\frac{A}{a}}\,\frac{\partial(\rho_0 U)}{\partial\underline{\underline{K}}^x} \tag{2.5}$$

Here $\underline{\underline{T}}_e$ and $\underline{\underline{M}}_e$ are the energetic tensors. They are expressed in terms of the tensors $\underline{\underline{T}}$ and $\underline{\underline{M}}$ as follows

$$\underline{\underline{T}}_e = (\tilde{\nabla}\underline{r})^T\cdot\underline{\underline{T}}\cdot\underline{\underline{P}}\,, \qquad \underline{\underline{M}}_e = (\tilde{\nabla}\underline{r})^T\cdot\underline{\underline{M}}\cdot\underline{\underline{P}} \tag{2.6}$$

The tensors $\underline{\underline{A}}^x$ and $\underline{\underline{K}}^x$ are called the first and the second strain measures. They are calculated by the formulae

$$\underline{\underline{A}}^x = \nabla\underline{R}\cdot\underline{\underline{P}}\,, \qquad \underline{\underline{K}}^x = (\underline{e}_r\underline{\Phi}_r + \underline{e}_\theta\underline{\Phi}_\theta)\cdot\underline{\underline{P}} + \underline{e}_\theta\underline{n}/r$$

$$\underline{\Phi}_r = -\frac{1}{2}\left[\frac{\partial\underline{\underline{P}}}{\partial r}\cdot\underline{\underline{P}}^T\right]_\times, \qquad \underline{\Phi}_\theta = -\frac{1}{2}\left[\frac{1}{r}\,\frac{\partial\underline{\underline{P}}}{\partial\theta}\cdot\underline{\underline{P}}^T\right]_\times \tag{2.7}$$

The density of the strain energy has the form

$$\rho_0 U = \tfrac{1}{2}\,\underline{\underline{E}}\cdot\cdot\,{}^{(4)}\underline{\underline{C}}_1\cdot\cdot\,\underline{\underline{E}} + \underline{\underline{E}}\cdot\cdot\,{}^{(4)}\underline{\underline{C}}_2\cdot\cdot\,\underline{\underline{\Phi}} + \tfrac{1}{2}\,\underline{\underline{\Phi}}\cdot\cdot\,{}^{(4)}\underline{\underline{C}}_3\cdot\cdot\,\underline{\underline{\Phi}} + \tfrac{1}{2}\,\underline{\gamma}\cdot\underline{\underline{\Gamma}}\cdot\underline{\gamma}$$

$$\underline{\underline{E}} = \tfrac{1}{2}\left(\underline{\underline{A}}^x\cdot\underline{\underline{A}}^{xT} - \underline{\underline{a}}\right), \qquad \underline{\underline{\Phi}} = \underline{\underline{K}}^x\cdot\underline{\underline{a}}\cdot\underline{\underline{A}}^{xT}\,, \qquad \underline{\gamma} = \underline{\underline{A}}^x\cdot\underline{n} \tag{2.8}$$

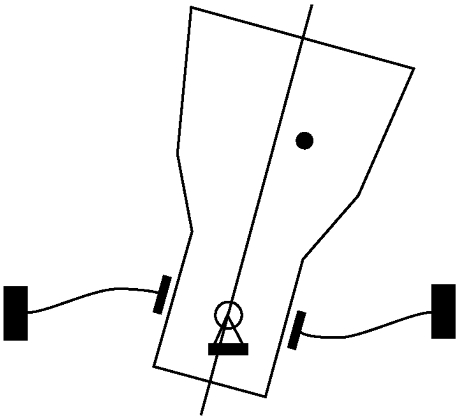

Fig. 2. A rigid body on an elastic plate. The body has a fixed point.

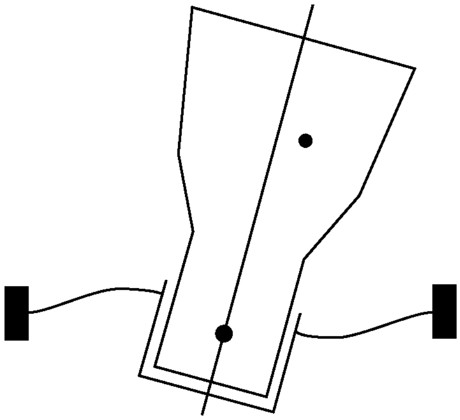

Fig. 3. A rigid body on an elastic plate. The body has no fixed points.

where $\underline{\underline{a}} = \underline{e}_r \underline{e}_r + \underline{e}_\theta \underline{e}_\theta$. The tensors $\underline{\underline{E}}$, $\underline{\underline{\Phi}}$, $\underline{\gamma}$ are called the strain tensors. The tensors ${}^{(4)}\underline{\underline{C}}_1$, ${}^{(4)}\underline{\underline{C}}_2$, ${}^{(4)}\underline{\underline{C}}_3$, $\underline{\underline{\Gamma}}$ are called the elastic modulus tensors; the expressions for them can be found in [3].

The boundary conditions on the external plate contour are formulated as follows

$$\underline{u}|_{r=r_2} = 0, \qquad \underline{\varphi}\Big|_{r=r_2} = 0. \tag{2.9}$$

3. The motion of the rigid body. Case 1: the rigid body has a fixed point.

The rigid body is joined with the internal plate contour by the bearing (see Fig. 2). Hence, the turn-tensors of particles of the internal plate contour differ from the turn-tensor of the external bearing ring by the turn around the bearing axis only. The rigid body has a fixed point, fastened by the spherical hinge. The fixed point is situated on the axis of symmetry of the rigid body and coincides with the plate center.

The turn-tensor of the external bearing ring has the form

$$\underline{\underline{\tilde{P}}}(t) = \underline{\underline{P}}[\underline{\Psi}(t)] \cdot \underline{\underline{P}}[\alpha(t)\underline{n}]\,, \qquad \underline{n} \cdot \underline{\Psi}(t) = 0 \tag{3.1}$$

The turn-vectors and the position vectors of particles of the internal plate contour are related with the quantities, defining the motion of the external bearing ring, as follows

$$\underline{\varphi}(r,\theta,t)\Big|_{r=r_1} = \underline{\Psi}(t), \qquad \underline{R}(r,\theta,t)|_{r=r_1} = \underline{\underline{\tilde{P}}}(t) \cdot r_1 \underline{e}_r \tag{3.2}$$

The turn-tensor of the rigid body $\underline{\underline{P}}_*$ differs from the turn-tensor of the external bearing ring $\underline{\underline{\tilde{P}}}$ by the turn around the bearing axis

$$\underline{\underline{P}}_*(t) = \underline{\underline{P}}[\underline{\Psi}(t)] \cdot \underline{\underline{P}}[\beta(t)\underline{n}]\,, \qquad \underline{\omega}_* = \dot{\underline{\Psi}} + \underline{\underline{P}}(\underline{\Psi}) \cdot \dot{\beta}\underline{n} \tag{3.3}$$

The elastic force and the elastic moment, acting from the plate on the external bearing ring, have the form

$$\underline{F}_{pl} = -\int\limits_0^{2\pi} \left(\underline{\nu} \cdot \underline{\underline{T}}\right)\Big|_{r=r_1} r_1\, d\theta\,, \qquad \underline{\nu} = -r_1^{-1}\, \underline{R}|_{r=r_1}$$

$$\underline{M}_{pl} = -\int\limits_0^{2\pi} \left[\underline{\nu} \cdot \underline{\underline{M}} + \underline{R} \times \left(\underline{\nu} \cdot \underline{\underline{T}}\right)\right]\Big|_{r=r_1} r_1\, d\theta \tag{3.4}$$

The first law of dynamics by Euler as applied to the motion of the rigid body has the following form

$$m\ddot{\underline{R}}_C = \underline{F}_{pl} + \underline{F}_R + m\underline{g}\,, \qquad \underline{R}_C(t) = \underline{\underline{P}}_*(t) \cdot [\,a\underline{n} + \varepsilon \underline{j}\,] \tag{3.5}$$

where $\underline{R}_C$ is the position vector of the mass center, $m\underline{g}$ is the gravity force, $\underline{F}_R$ is the reaction in the hinge. The equations (3.5) allow to find the vector $\underline{F}_R$, if the motion of the rigid body is known.

Let us consider the rotation of the external bearing ring. The bearing mass is assumed to be equal to zero. Therefore, the second law of dynamics by Euler as applied to the motion of the external bearing ring takes the form of the equation of balance of moments. Three moments act on the external bearing ring: the elastic moment from the plate $\underline{M}_{pl}$, the moment of friction between the rings of the bearing $\underline{M}_{fr} = -b(\dot{\alpha} - \dot{\beta})\,\underline{\underline{P}}_* \cdot \underline{n}$ and the moment of the reaction of the rigid body $\underline{M}_{rb}$, which is orthogonal to the bearing axis: $(\underline{\underline{P}}_* \cdot \underline{n}) \cdot \underline{M}_{rb} = 0$. Hence, the projection of the second law of dynamics by Euler on the axis, directed along the vector $\underline{\underline{P}}_* \cdot \underline{n}$, takes the form

$$(\underline{\underline{P}}_* \cdot \underline{n}) \cdot \underline{M}_{pl} - b(\dot{\alpha} - \dot{\beta}) = 0 \tag{3.6}$$

The equation (3.8) relates the angles α and β.

The second law of dynamics by Euler as applied to the motion of the rigid body is formulated as follows

$$\left(\underline{\underline{P}}_* \cdot \underline{\underline{J}_o} \cdot \underline{\underline{P}}_*^T \cdot \underline{\omega}_*\right)^{\cdot} = \left[\underline{\underline{E}} - (\underline{\underline{P}}_* \cdot \underline{n})(\underline{\underline{P}}_* \cdot \underline{n})\right] \cdot \underline{M}_{pl} + \underline{R}_C \times m\underline{g} -$$

$$-b(\dot{\beta} - \dot{\alpha})\,\underline{\underline{P}}_* \cdot \underline{n} - \underline{\underline{P}}_* \cdot \underline{\underline{B}}_{vf} \cdot \underline{\underline{P}}_*^T \cdot \underline{\omega}_* + \underline{L}_{mt} \tag{3.7}$$

$$\underline{\underline{B}}_{vf} = b_3\underline{n}\underline{n} + b_{12}(\underline{\underline{E}} - \underline{n}\underline{n})\,, \qquad \underline{L}_{mt} = L\,(\omega_0 - \dot{\beta})\left[\,\eta\,\underline{n} + (1-\eta)\underline{\underline{P}}_* \cdot \underline{n}\,\right]$$

The equation (3.10) is written with respect to the fixed point of the rigid body. Here $\underline{\underline{J}_o}$ is the ineria tensor of the rigid body, $\underline{\underline{B}}_{vf}$ is the tensor coefficient of viscous friction, $\underline{L}_{mt}$ is the moment of the restricted power motor. Taking into account the equation (3.8), the equation (3.10) can be rewritten in the form

$$\left(\underline{\underline{P}}_* \cdot \underline{\underline{J}_o} \cdot \underline{\underline{P}}_*^T \cdot \underline{\omega}_*\right)^{\cdot} = \underline{M}_{pl} + \underline{R}_C \times m\underline{g} - \underline{\underline{P}}_* \cdot \underline{\underline{B}}_{vf} \cdot \underline{\underline{P}}_*^T \cdot \underline{\omega}_* + \underline{L}_{mt} \tag{3.8}$$

Thus, the boundary conditions on the internal plate contour plate are given by the equations (2.20), (3.8), (3.11).

4. The motion of the rigid body. Case 2: the rigid body has no fixed points.

The rigid body is joined with the internal contour of the plate by the bearing (see Fig. 3). The turn-tensor of the external bearing ring has the form the same as in the case when the rigid body has a fixed point. The turn-tensors of particles of the internal plate contour differ from the turn-tensor of the external bearing ring by the turn around the bearing axis only.

$$\tilde{\underline{\underline{P}}}(t) = \underline{\underline{P}}[\underline{\Psi}(t)] \cdot \underline{\underline{P}}[\alpha(t)\underline{n}]\,, \qquad \underline{\varphi}(r,\theta,t)\big|_{r=r_1} = \underline{\Psi}(t) \tag{4.1}$$

The position vector of particles of the internal plate contour is expressed in terms of the quantities, defining the motion of the external bearing ring by the formulae

$$\underline{R}(r,\theta,t)|_{r=r_1} = \underline{R_o}(t) + \tilde{\underline{\underline{P}}}(t) \cdot r_1 \underline{e}_r, \qquad \underline{R_o}(t) = x(t)\underline{i} + y(t)\underline{j} + z(t)\underline{n} \tag{4.2}$$

where $\underline{R_o}(t)$ is the position vector of the point of the rigid body, which coincides with the plate center, if the plate is not deformed. The turn-tensor of the rigid body and the position vector of its mass center have the form

$$\underline{\underline{P}}_*(t) = \underline{\underline{P}}[\underline{\Psi}(t)] \cdot \underline{\underline{P}}[\beta(t)\underline{n}]\,, \qquad \underline{R}_C(t) = \underline{R_o}(t) + \underline{\underline{P}}_*(t) \cdot [\,a\underline{n} + \varepsilon\underline{j}\,] \tag{4.3}$$

The elastic force and the elastic moment, acting from the plate on the external bearing ring, are calculated by the formulae

$$\begin{aligned} \underline{F}_{pl} &= -\int\limits_0^{2\pi} \left(\underline{\nu} \cdot \underline{\underline{T}}\right)\Big|_{r=r_1} r_1\, d\theta\,, \qquad \underline{\nu} = -r_1^{-1}\Big(\underline{R}|_{r=r_1} - \underline{R_o}\Big) \\ \underline{M}_{pl} &= -\int\limits_0^{2\pi} \left[\underline{\nu} \cdot \underline{\underline{M}} + (-\underline{R}_C + \underline{R_o} - r\underline{\nu}) \times \left(\underline{\nu} \cdot \underline{\underline{T}}\right)\right]\Big|_{r=r_1} r_1\, d\theta \end{aligned} \tag{4.4}$$

The first law of dynamics by Euler as applied to the motion of the rigid body has the following form

$$m\ddot{\underline{R}}_C = \underline{F}_{pl} + m\underline{g} \tag{4.5}$$

The projection of the second law of dynamics by Euler as applied to the motion of the external bearing ring on the axis, directed along the vector $\underline{\underline{P}}_* \cdot \underline{n}$, relates the angles α and β. It has the form (3.8).

Taking into account the relation (3.8), the second law of dynamics by Euler as applied to the motion of the rigid body is formulated as follows

$$\left(\underline{\underline{P}}_* \cdot \underline{\underline{J}}_C \cdot \underline{\underline{P}}_*^T \cdot \underline{\omega}_*\right)^{\cdot} = \underline{M}_{pl} - \underline{\underline{P}}_* \cdot \underline{\underline{B}}_{vf} \cdot \underline{\underline{P}}_*^T \cdot \underline{\omega}_* + \underline{L}_{mt} \tag{4.6}$$

where the tensor coefficient of viscous friction $\underline{\underline{B}}_{vf}$ and the moment of the restricted power motor $\underline{L}_{mt}$ have the form the same as in the case when the rigid body has a fixed point (see formulae (3.10)).

Thus, the boundary conditions on the internal plate contour are given by the equations (3.8), (3.3), (3.8), (4.1).

5. Some results of analysis of the linear formulation of the problem.

The stationary regime of the motion of the rigid body on the elastic plate is investigated ($\dot{\beta} = \omega_0$). The turn-vectors, defining the rotations of the plate particles, and the turn-vector, defining the nutation vibrations of the rigid body, are assumed to be small. The linear formulation of the problem is considered. This problem has an exact analytical solution. Numerical analysis of the solution was carried out for two types of the plate: the thin plate, the shearing thickness of which $Gh\Gamma$ is much more than its bending stiffness D, and the thick plate, the shearing thickness of which $Gh\Gamma$ is less than its bending stiffness D. Influence of the different types of the moment of motor and the dissipative moment was analyzed. The following results were obtained.

Free vibrations of the system

The plate is thin, $Gh\Gamma \gg D$. In this case the eigenfrequencies and the eigenfunctions are essentially depend on the parameters of the rigid body and its angular velocity ω_0. It is known that the following property of the eigenvalues takes place: if $\omega_0 = 0$, then $p_{2n} = -p_{2n-1}$, if $\omega_0 \neq 0$, then $p_{2n} \neq -p_{2n-1}$. As ω_0 increases the difference between $|p_{2n}|$ and $|p_{2n-1}|$ increases. The difference between $|p_{2n}|$ and $|p_{2n-1}|$ depends on the number n. When the number n becomes large so that $|p_{2n-1}| \gg \gg \omega_0$, then the difference between $|p_{2n}|$ and $|p_{2n-1}|$ tends to zero. The following property of several first eigenfunctions takes place: the displacements and the stresses vary along the coordinate r so that they take the largest values on the internal plate contour, joined with the rigid body. Let us note, that all these results are the same as the results of the investigation of analogous problem of the motion of the rigid body on the elastic rod [4].

The plate is thick, $Gh\Gamma < D$. As the plate thickness h increases and the cross shear coefficient Γ decreases the difference between $|p_{2n}|$ and $|p_{2n-1}|$ decreases and when $Gh\Gamma < D$ the difference becomes small. If $Gh\Gamma < D$ eigenfrequencies little depend on the angular velocity of the rigid body ω_0, if any. The form of the eigenfunctions is essentially differs from it in the case of the thin plate. This fact is caused by the difference of the asymptotic properties of solution in the cases of the thin plate and the thick plate with small Γ. If the plate is thin and $Gh\Gamma \gg D$, then $|\underline{\varphi}| \sim u_z$, $|\underline{\gamma}| \ll u_z$. If the plate is thick and $Gh\Gamma < D$, then $|\underline{\gamma}| \sim u_z$, $|\underline{\varphi}| \ll u_z$. In the case of the thick plate with small Γ, the displacements vary along the coordinate r so that the cross displacement u_z takes the largest values in the middle points of the plate and smallest value on the internal plate contour, joined with the rigid body. Projections of the turn-vector $\underline{\varphi}$ take the largest values on the internal plate contour, however they are much less than the largest values of the cross displacement u_z. As the plate thickness increases and the cross shear coefficient Γ decreases the displacements on the internal plate contour decrease and the stresses increase so that the increase of the stresses is proportional to the decrease of the displacements.

Forced vibrations of the system, caused by the unbalanced mass of the rigid body

As in the case of free vibrations, in the case of forced vibrations when the plate thickness increases and the cross shear coefficient decreases the displacements on the internal plate contour decrease and the stresses on the internal plate contour increase. However, unlike the case of free vibrations, the decrease of the displacements is not directly proportional to the increase of the stresses. In the case of forced vibrations the decrease of the displacements is much more than the increase of the stresses. Let us suppose that free vibrations of the system are damped by the friction and the forced vibrations of the system are caused by the unbalanced mass only. In this case using of the thick plate, having small shearing stiffness, allows to essentially decrease the amplitude of the nutational vibrations of the rigid body, and the increase of the stresses will not be large.

The influence of the friction on the free and forced vibrations of the system

It is determined that the influence of the friction depends on the type of the moment of motor. If the moment of motor is the following one ($\eta = 0$), then the rotation of the rigid body around its axis of symmetry is stable at any values of the coefficients of viscous friction b_{12} and b_3 . In the case of the moment of motor, having the constant direction ($\eta = 1$), stability of the rotation of the rigid body around its axis of symmetry essentially depends on the ratio of the coefficients of viscous friction b_{12} and b_3 . The increase of the coefficient b_{12} is the stabilizing factor, the increase of the coefficient b_3 is the destabilizing factor. At $b_{12} = 0$ the motion is unstable. When b_{12} is large and b_3 is small, the motion becomes stable. In the case of the moment of motor being the superposition of the constantly directed moment and the following moment ($0 < \eta < 1$), the influence of the destabilizing coefficient of friction b_3 is less than in the case of the constantly directed moment of motor.

6. The motion of a multi-rotor gyrostst on the elastic plate.

It is known, that the angular velocities of high-speed centrifuges are more than the first eigenfriquencies of them. Hence, as accelerating and decelerating the centrifuge has to pass over the critical angular velocities $\dot\beta_j = p_j(\dot\beta_j)$. Let us compare the processes of deceleration of two models of the centrifuge: the rigid body on the elastic plate and the multi-rotor gyrostst on the elastic plate (see Fig. 4).

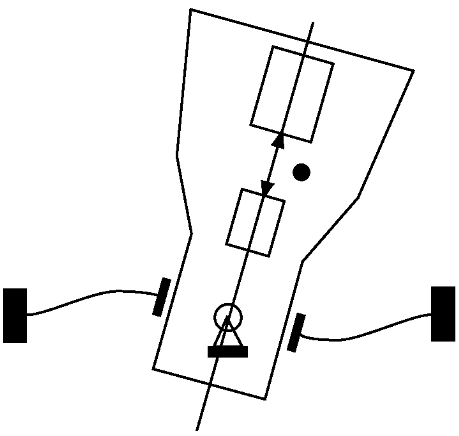

Fig. 4. A multi-rotor gyrostat on an elastic plate.

The rigid body without additional rotors. If the angular velocity of the rigid body ω_0 is large, then the difference between the eigenfrequencies $|p_{2n}|$ and $|p_{2n-1}|$ is large. Because of that several first eigenfrequencies are small. As the angular velocity of the rigid body $\dot\beta$ decreases the difference between $|p_{2n}|$ and $|p_{2n-1}|$ decreases and the first eigenfrequencies increase. As a result the values of the first critical angular velocities $\dot\beta_j = p_j(\dot\beta_j)$ are much more than the values of the first eigenfrequencies $p_j(\omega_0)$. When the rigid body passes over the large critical angular velocities, the large dynamical loads act on the plate and the amplitude of the nutational vibrations of the rigid body quickly increases.

The multi-rotor gyrostat. Let us suppose, that several rotors, the direction of rotation of which coincides with the direction of the carrier body rotation, are added in the system. Then the moment of momentum of the system increases. Hence, the difference between the eigenfrequencies $|p_{2n}|$ and $|p_{2n-1}|$ increases so that the first eigenfrequency becomes small and the second eigenfrequency becomes large. If the moment of momentum of the system is rather large, then the angular velocity of the carrier body at the working regime will be less than the second eigenfrequency. Let us suppose that the angular velocity of the carrier body decreases and the angular velocities of the additional rotors increase so that the moment of momentum of the system is constant. In this case the critical angular velocities are equal to the eigenfrequencies of the system. Hence, during decelerating, the carrier body will pass over the first low-frequency resonance only. When the carrier body passes over the low-frequency resonance, the dynamical loads, acting on the plate are not very large and the amplitude of the nunational vibration of the carrier body does not highly increase.

The one-rotor gyrostat. Let us suppose that one rotor, the direction of the rotation of which is opposite to the direction of the carrier body rotation, is added in the system. Then the moment of momentum of the system increases. Hence, the difference between the eigenfrequencies $|p_{2n}|$ and $|p_{2n-1}|$ decreases and the first eigenfrequency increases. Thus, using of the additional rotor allows to increase the working angular velocity of the carrier body, so that it will be less than the first eigenfrequency.

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