Lecture 6

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Application of the continuum model with microstructure (continua with inner rotational degrees of freedom) to description on the macro-level of electro-magnetic processes: theory connecting electro-magnetic and heat processes.

1 Continuum of multi-rotor gyrostats

We consider the material medium (see Fig. 1) consisting of multi-rotor gyrostats. The multi-rotor gyrostat has $N+6$ degrees of freedom which are determined by the following functions:

$$\mathbf{v}(\mathbf{r},t), \quad \mathbf{P}_0(\mathbf{r},t), \quad \beta_i(\mathbf{r},t), \quad i=1,2,...,N. \tag{1}$$

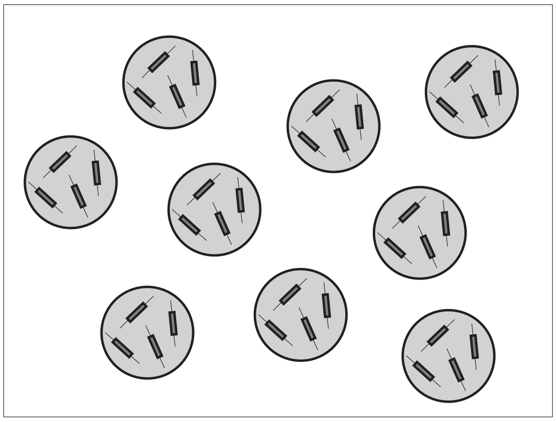

Figure 1: Elementary volume of continuum consisting of multi-rotor gyrostats

1.1 The equations of motion

The momentum balance equation for the gyrostats has the form

$$\nabla\cdot\boldsymbol{\tau}+\rho\mathbf{f}=\rho\frac{\delta\mathbf{v}}{\delta t}. \tag{2}$$

The angular momentum balance equation for the gyrostats is

$$\nabla \cdot \boldsymbol{\mu} + \boldsymbol{\tau}_{\times} + \rho \mathbf{L} = \rho \frac{\delta}{\delta t} \boldsymbol{\mathfrak{L}}(\mathbf{r}, t), \tag{3}$$

where $\boldsymbol{\mathfrak{L}}$ is the mass density of the proper angular momentum of the gyrostat

$$\boldsymbol{\mathfrak{L}} = \mathbf{P}_0(\mathbf{r},t) \cdot \mathbf{C} \cdot \mathbf{P}_0^T(\mathbf{r},t) \cdot \boldsymbol{\omega}_0(\mathbf{r},t) + \sum_{i=1}^{N} \lambda_i \frac{\delta \beta_i(\mathbf{r},t)}{\delta t} \mathbf{P}_0(\mathbf{r},t) \cdot \mathbf{n}(\mathbf{r},t). \tag{4}$$

The projections of the angular momentum balance equations on the axes of the rotors take the form

$$\lambda_i \frac{\delta}{\delta t} \left(\frac{\delta \beta_i(\mathbf{r},t)}{\delta t} + \boldsymbol{\omega}_0(\mathbf{r},t) \cdot \mathbf{P}_0(\mathbf{r},t) \cdot \mathbf{n}(\mathbf{r},t) \right) = L_i, \qquad i = 1, 2, ..., N. \tag{5}$$

Here L_i is the mass density of external moments acting on the rotor number i. Let L_i take the form

$$L_i = -\nu_i \left(\frac{\delta \beta_i}{\delta t} - \Omega_i \right), \qquad \nu_i > 0, \tag{6}$$

where $\Omega_i = \text{const}$ and $\nu_i = \text{const}$ are the parameters of the particle.

1.2 The equation of energy balance

In order to obtain the constitutive equations we should consider the equation of energy balance.

$$\rho \frac{\delta U}{\delta t} = \boldsymbol{\tau}^T \cdot\cdot (\nabla \mathbf{v} + \mathbf{E} \times \boldsymbol{\omega}_0) + \boldsymbol{\mu}^T \cdot\cdot \nabla \boldsymbol{\omega}_0 + \nabla \cdot \mathbf{H} + \rho Q - \rho \sum_{i=1}^{N} L_i \frac{\delta \beta_i}{\delta t}. \tag{7}$$

The right-hand side of Eq. (7) contains the terms characterizing the supply of energy in the form of heat and the supply of energy energy supply due to dynamics of the internal rotors. In addition, The right-hand side of Eq. (7) contains the power of force and moment stresses. Some part of this power is expended on the internal energy change, some other part stays in the body in the form of heat, and part dissipates into the surroundings. In order to separate these parts we use the method by Zhilin. According to this method the tensors of force and moment stresses should be represented as sums

$$\boldsymbol{\tau} = \boldsymbol{\tau}_e + \boldsymbol{\tau}_f, \qquad \boldsymbol{\mu} = \boldsymbol{\mu}_e + \boldsymbol{\mu}_f, \tag{8}$$

where the elastic part of the stresses (the part independent of velocities) is denoted by index "e" and remaining part of the stresses (the dissipative part) is denoted by index "f". By using Eqs. (6), (8) the energy balance equation (7) is rewritten in the form

$$\begin{aligned} \eta \frac{\delta \mathfrak{U}}{\delta t} &= \boldsymbol{\tau}_e^T \cdot\cdot (\nabla \mathbf{v} + \mathbf{E} \times \boldsymbol{\omega}_0) + \boldsymbol{\mu}_e^T \cdot\cdot \nabla \boldsymbol{\omega}_0 + \nabla \cdot \mathbf{H} + \rho Q + \\ &+ \rho \sum_{i=1}^{N} \nu_i \frac{\delta \beta_i}{\delta t} \left(\frac{\delta \beta_i}{\delta t} - \Omega_i \right) + \boldsymbol{\tau}_f^T \cdot\cdot (\nabla \mathbf{v} + \mathbf{E} \times \boldsymbol{\omega}_0) + \boldsymbol{\mu}_f^T \cdot\cdot \nabla \boldsymbol{\omega}_0. \end{aligned} \tag{9}$$

1.3 The heat conduction equation

Now we reduce the energy balance equation (9) to the special form which is called *the reduced equation of energy balance.* With that end in view we introduce new variables ϑ and $\mathcal{H}$ such that

$$\rho\vartheta\,\frac{\delta\mathcal{H}}{\delta t} = \nabla\cdot\mathbf{H} + \rho Q + \rho\sum_{i=1}^{N}\nu_i\frac{\delta\beta_i}{\delta t}\left(\frac{\delta\beta_i}{\delta t} - \Omega_i\right) + \\ + \boldsymbol{\tau}_f^T\cdot\cdot\,(\nabla\mathbf{v} + \mathbf{E}\times\boldsymbol{\omega}_0) + \boldsymbol{\mu}_f^T\cdot\cdot\,\nabla\boldsymbol{\omega}_0. \tag{10}$$

Here ϑ is the temperature; $\mathcal{H}$ is the density of entropy (the entropy per unit mass). Thus, the temperature and entropy are introduced as conjugate quantities, the temperature is understood as the quantity measured with a thermometer. Eq. (10) is *the heat conduction equation.* In fact, this equation represents the definition of temperature and entropy. In view of Eq. (10) the energy balance equation (9) takes the form

$$\rho\frac{\delta\mathcal{U}}{\delta t} = \boldsymbol{\tau}_e^T\cdot\cdot\,(\nabla\mathbf{v} + \mathbf{E}\times\boldsymbol{\omega}_0) + \boldsymbol{\mu}_e^T\cdot\cdot\,\nabla\boldsymbol{\omega}_0 + \rho\vartheta\,\frac{\delta\mathcal{H}}{\delta t}. \tag{11}$$

This form of the energy balance equation is called *the reduced equation of energy balance.*

2 Non-linear theory of electromagnetic field

Now we consider the model of electromagnetic field proposed by P. A. Zhilin. This model of electromagnetic field is based on the continuum of two-spin particles, i. e. particles consisting of a carrier body and a rotor (see Fig. 2). In this particular case

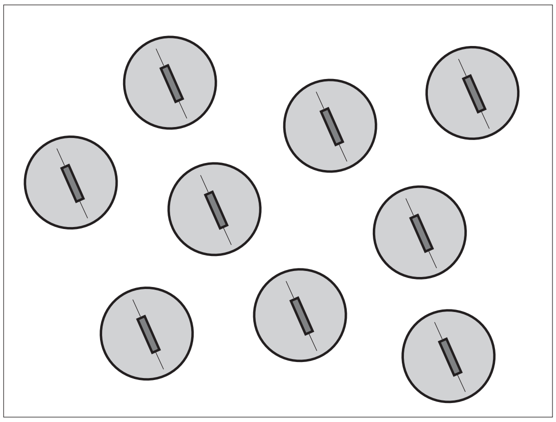

Figure 2: Elementary volume of continuum consisting of one-rotor gyrostats

the expression for the angular momentum (12) is reduced to the form

$$\boldsymbol{\mathcal{L}} = \mathbf{P}_0(\mathbf{r},t)\cdot\mathbf{C}\cdot\mathbf{P}_0^T\,(\mathbf{r},t)\cdot\boldsymbol{\omega}_0\,(\mathbf{r},t) + \lambda_r\frac{\delta\beta\,(\mathbf{r},t)}{\delta t}\,\mathbf{P}_0(\mathbf{r},t)\cdot\mathbf{n}\,(\mathbf{r},t), \tag{12}$$

where λ_r is the axial moment of inertia of the rotor; β is the angle of rotation of the rotor relative to the carrier body. The carrier body is assumed to possess the transversely isotropic inertia tensor with the axis of isotropy $\mathbf{n}$ which coincides with the axis of rotation of the rotor. Furthermore, we consider the mass centers of the carrier body and rotor to be on the line $\mathbf{n}$. In this case the inertia tensor $\mathbf{C}$ is

$$\mathbf{C} = \lambda \mathbf{n} \otimes \mathbf{n} + \mu \left(\mathbf{E} - \mathbf{n} \otimes \mathbf{n} \right). \tag{13}$$

By using the accepted notations the angular momentum (12) can be written as

$$\boldsymbol{\mathfrak{L}} = \left[\mu \boldsymbol{\omega}_0 + \left((\lambda - \mu)(\boldsymbol{\omega}_0 \cdot \mathbf{n}') + \lambda_r \frac{\delta \beta}{\delta t} \right) \mathbf{n}' \right], \qquad \mathbf{n}' = \mathbf{P}(\mathbf{r}, t) \cdot \mathbf{n}. \tag{14}$$

Now we introduce the electric field vector $\boldsymbol{\mathfrak{E}}$ in the same way as was done previously:

$$\boldsymbol{\mathfrak{E}} = \varpi c^2 \rho \boldsymbol{\mathfrak{L}}. \tag{15}$$

We assume the stress tensor $\boldsymbol{\tau}$ and the moment stress tensor $\boldsymbol{\mu}$ to be a sum of the symmetric and antisymmetric parts and the antisymmetric tensor correspondingly

$$\boldsymbol{\tau} = \boldsymbol{\tau}_s + \varpi^{-1} \boldsymbol{\mathfrak{D}} \times \mathbf{E}, \qquad \boldsymbol{\tau}_s = \boldsymbol{\tau}_s^T, \qquad \boldsymbol{\mu} = \varpi^{-1} \boldsymbol{\mathfrak{B}} \times \mathbf{E}. \tag{16}$$

Here vector $\boldsymbol{\mathfrak{B}}$ has the sense of magnetic induction vector (as was accepted previously). Moreover, we assume that

$$\mathbf{v} = \text{const} \qquad \Rightarrow \qquad \nabla \cdot \mathbf{v} = 0 \qquad \Rightarrow \qquad \frac{\delta \rho}{\delta t} = 0. \tag{17}$$

Under condition $\rho \mathbf{f} = \mathbf{0}$ the momentum balance equation (2) takes the form

$$\nabla \cdot \boldsymbol{\tau}_s + \varpi^{-1} \nabla \times \boldsymbol{\mathfrak{D}} = \mathbf{0}. \tag{18}$$

The angular momentum balance equation (3) takes the form

$$\nabla \times \boldsymbol{\mathfrak{B}} - 2 \boldsymbol{\mathfrak{D}} + \varpi \rho \mathbf{L} = \frac{1}{c^2} \frac{\delta \boldsymbol{\mathfrak{E}}}{\delta t}. \tag{19}$$

Eq. (19) should be added by the equation of motion of the rotor

$$\lambda_r \frac{\delta}{\delta t} \left(\frac{\delta \beta}{\delta t} + \boldsymbol{\omega}_0 \cdot \mathbf{n}' \right) + \nu \left(\frac{\delta \beta}{\delta t} - \Omega_0 \right) = 0, \tag{20}$$

where ν and Ω_0 are the parameters of particle.

Now we consider the energy balance equation and obtain the heat conduction equation and the reduced equation of energy balance. In view of Eq. (8) we rewrite the relations (17) in the form

$$\begin{aligned} &\boldsymbol{\tau}_e = \boldsymbol{\tau}_{se} + \varpi^{-1} \boldsymbol{\mathfrak{D}}_e \times \mathbf{E}, \qquad \boldsymbol{\tau}_{se} = \boldsymbol{\tau}_{se}^T, \qquad \boldsymbol{\mu}_e = \varpi^{-1} \boldsymbol{\mathfrak{B}}_e \times \mathbf{E}, \\ &\boldsymbol{\tau}_f = \boldsymbol{\tau}_{sf} + \varpi^{-1} \boldsymbol{\mathfrak{D}}_f \times \mathbf{E}, \qquad \boldsymbol{\tau}_{sf} = \boldsymbol{\tau}_{sf}^T, \qquad \boldsymbol{\mu}_f = \varpi^{-1} \boldsymbol{\mathfrak{B}}_f \times \mathbf{E}. \end{aligned} \tag{21}$$

Substituting Eq. (21) into Eq. (10) we obtain the heat conduction equation

$$\begin{aligned} \rho \vartheta \frac{\delta \mathcal{H}}{\delta t} = \nabla \cdot \mathbf{H} + \rho Q + \rho \nu \frac{\delta \beta}{\delta t} \left(\frac{\delta \beta}{\delta t} - \Omega_0 \right) + \\ + 2 \varpi^{-1} \boldsymbol{\mathfrak{D}}_f \cdot \boldsymbol{\omega}_0 - \varpi^{-1} \boldsymbol{\mathfrak{B}}_f \cdot (\nabla \times \boldsymbol{\omega}_0). \end{aligned} \tag{22}$$

Substituting Eq. (21) into Eq. (11) we obtain the reduced equation of energy balance

$$\rho \frac{\delta \mathcal{U}}{\delta t} = 2\varpi^{-1} \boldsymbol{\mathcal{D}}_e \cdot \boldsymbol{\omega}_0 - \varpi^{-1} \boldsymbol{\mathcal{B}}_e \cdot (\nabla \times \boldsymbol{\omega}_0) + \rho \vartheta \frac{\delta \mathcal{H}}{\delta t}. \tag{23}$$

By some mathematical transformations Eq. (23) can be reduced to the form

$$\begin{gathered} \rho \frac{\delta \mathcal{U}}{\delta t} = \left[\varpi^{-1} \left(\boldsymbol{\mathcal{D}}_e - \frac{1}{2} \mathbf{F} \cdot \boldsymbol{\mathcal{B}}_e + \frac{1}{2} (\operatorname{tr} \mathbf{F}) \, \boldsymbol{\mathcal{B}}_e \right) \times \mathbf{P}_0 \right]^T \cdot\cdot \frac{\delta \mathbf{P}_0}{\delta t} - \\ - \varpi^{-1} \boldsymbol{\mathcal{B}}_e \cdot \frac{\delta \mathbf{F}_\times}{\delta t} + \rho \vartheta \frac{\delta \mathcal{H}}{\delta t}, \end{gathered} \tag{24}$$

where $\mathbf{F}_\times$ is the vector invariant of tensor $\mathbf{F}$; and $\mathbf{F}$ is the strain tensor defined by the equation

$$\nabla \mathbf{P}_0 = \mathbf{F} \times \mathbf{P}_0. \tag{25}$$

Now we can determine the arguments on which functions $\mathcal{U}$, $\boldsymbol{\mathcal{D}}_e$, $\boldsymbol{\mathcal{B}}_e$, ϑ depend

$$\begin{gathered} \mathcal{U} = \mathcal{U}(\mathbf{P}_0, \, \mathbf{F}_\times, \, \mathcal{H}), \qquad \boldsymbol{\mathcal{D}}_e = \boldsymbol{\mathcal{D}}_e(\mathbf{P}_0, \, \mathbf{F}, \, \mathcal{H}), \\ \boldsymbol{\mathcal{B}}_e = \boldsymbol{\mathcal{B}}_e(\mathbf{P}_0, \, \mathbf{F}_\times, \, \mathcal{H}), \qquad \vartheta = \vartheta(\mathbf{P}_0, \, \mathbf{F}_\times, \, \mathcal{H}). \end{gathered} \tag{26}$$

According to Eq. (26) the material derivative of the internal energy density is calculated by the formula

$$\frac{\delta \mathcal{U}}{\delta t} = \frac{\partial \mathcal{U}}{\partial \mathbf{F}_\times} \cdot \frac{\delta \mathbf{F}_\times}{\delta t} + \left(\frac{\partial \mathcal{U}}{\partial \mathbf{P}_0} \right)^T \cdot\cdot \frac{\delta \mathbf{P}_0}{\delta t} + \frac{\partial \mathcal{U}}{\partial \mathcal{H}} \frac{\delta \mathcal{H}}{\delta t}. \tag{27}$$

Substituting Eq. (27) into Eq. (24) we obtain

$$\begin{gathered} \left[\varpi^{-1} \left(\boldsymbol{\mathcal{D}}_e - \frac{1}{2} \mathbf{F} \cdot \boldsymbol{\mathcal{B}}_e + \frac{1}{2} (\operatorname{tr} \mathbf{F}) \, \boldsymbol{\mathcal{B}}_e \right) \times \mathbf{P}_0 - \rho \frac{\partial \mathcal{U}}{\partial \mathbf{P}_0} \right]^T \cdot\cdot \frac{\delta \mathbf{P}_0}{\delta t} - \\ - \left(\varpi^{-1} \boldsymbol{\mathcal{B}}_e + \rho \frac{\partial \mathcal{U}}{\partial \mathbf{F}_\times} \right) \cdot \frac{\delta \mathbf{F}_\times}{\delta t} + \rho \left(\vartheta - \frac{\partial \mathcal{U}}{\partial \mathcal{H}} \right) \frac{\delta \mathcal{H}}{\delta t} = 0. \end{gathered} \tag{28}$$

From Eq. (28) taking into account the restriction on the material derivative of the rotation tensor we obtain the Cauchy–Green relations

$$\begin{gathered} \boldsymbol{\mathcal{B}}_e = - \varpi \rho \frac{\partial \mathcal{U}}{\partial \mathbf{F}_\times}, \qquad \vartheta = \frac{\partial \mathcal{U}}{\partial \mathcal{H}}, \\ 2 \boldsymbol{\mathcal{D}}_e = - \varpi \rho \left(\frac{\partial \mathcal{U}}{\partial \mathbf{P}_0} \cdot \mathbf{P}_0^T \right)_\times + \varpi \rho \mathbf{F} \cdot \frac{\partial \mathcal{U}}{\partial \mathbf{F}_\times} - \varpi \rho \, (\operatorname{tr} \mathbf{F}) \, \frac{\partial \mathcal{U}}{\partial \mathbf{F}_\times}. \end{gathered} \tag{29}$$

As a result, the problem of determination of vectors $\boldsymbol{\mathcal{B}}_e$, $\boldsymbol{\mathcal{D}}_e$ and the temperature is reduced to the assignment of the internal energy density as a function of the rotation tensor of the carrier body, the strain vector $\mathbf{F}_\times$ and the density of entropy.

3 Summary of the basic equations

Now we write down the summary of the basic equations of the continuum modelling electromagnetic field. The second law of dynamics by Euler is formulated in the form of Eqs. (19), (20) where vector $\boldsymbol{\mathcal{E}}$ is determined by Eqs. (14), (15):

$$\nabla \times \boldsymbol{\mathcal{B}} - 2\boldsymbol{\mathfrak{D}} + \varpi\rho\mathbf{L} = \frac{1}{c^2}\frac{\delta\boldsymbol{\mathcal{E}}}{\delta t},$$

$$\lambda_r \frac{\delta}{\delta t}\left(\frac{\delta\beta}{\delta t} + \boldsymbol{\omega}_0 \cdot \mathbf{n}'\right) + \nu\left(\frac{\delta\beta}{\delta t} - \Omega_0\right) = 0, \tag{30}$$

$$\boldsymbol{\mathcal{E}} = \varpi c^2 \rho\left[\mu\boldsymbol{\omega}_0 + \left((\lambda-\mu)(\boldsymbol{\omega}_0 \cdot \mathbf{n}') + \lambda_r\frac{\delta\beta}{\delta t}\right)\mathbf{n}'\right], \qquad \mathbf{n}' = \mathbf{P}_0(\mathbf{r}, t) \cdot \mathbf{n}.$$

The first equation in Eqs. (30) represents one of the Maxwell equations. In the opinion of P. A. Zilin the term $(-2\boldsymbol{\mathfrak{D}})$ in this equation corresponds to Joule heat. The Joule–Lenz law gives the quantitative assessment of the thermal effect of electric current. The law is stated as follows. The power of heat generated per unit volume of the medium during the flow of electric current is proportional to the product of the electric current density on the magnitude of the electric field.

Now we suppose the dissipative part of the stress tensors to be absent. Then the Cauchy–Green relations and the heat conduction equation take the form

$$\boldsymbol{\mathcal{B}} = -\varpi\rho\frac{\partial\mathcal{U}}{\partial\mathbf{F}_\times}, \qquad \vartheta = \frac{\partial\mathcal{U}}{\partial\mathcal{H}},$$

$$2\boldsymbol{\mathfrak{D}} = -\varpi\rho\left(\frac{\partial\mathcal{U}}{\partial\mathbf{P}_0}\cdot\mathbf{P}_0^T\right)_\times + \varpi\rho\mathbf{F}\cdot\frac{\partial\mathcal{U}}{\partial\mathbf{F}_\times} - \varpi\rho\,(\mathrm{tr}\,\mathbf{F})\,\frac{\partial\mathcal{U}}{\partial\mathbf{F}_\times}, \tag{31}$$

$$\rho\vartheta\,\frac{\delta\mathcal{H}}{\delta t} = \nabla\cdot\mathbf{H} + \rho Q + \rho\nu\frac{\delta\beta}{\delta t}\left(\frac{\delta\beta}{\delta t} - \Omega_0\right).$$

These equations should be added by the kinematic and geometric equations

$$\frac{\delta\mathbf{P}_0}{\delta t} = \boldsymbol{\omega}_0 \times \mathbf{P}_0, \qquad \nabla\mathbf{P}_0 = \mathbf{F}\times\mathbf{P}_0. \tag{32}$$

The set of equations (30)–(32) is closed if the concrete dependence of internal energy on the state parameters $\mathbf{P}_0$, $\mathbf{F}$, $\mathcal{H}$ and the constitutive equation for the heat flow $\mathbf{H}$ are assigned. Eqs. (30)–(32) describes the non-linear model of liquid crystal continuum. In order to this continuum would be a model of electromagnetic field it is necessary to take into account some additional data including the results of experimental studies and intuitive considerations. At this stage, experimental studies are difficult because the experiments have always considered the interaction of electromagnetic fields with a substance.

The terminology used in the papers by P. A. Zilin is different from the that accepted in modern physics. In the papers by P. A. Zilin the ether is called electromagnetic field. The equations for disturbance propagating in the ether correspond to the equations which are usually called the equations of electromagnetic field.

4 Historical remarks: from models proposed in XIX century to the model by P. A. Zhilin

The representation of ether properties by means of the properties of solid is important part of works of the scientists of XIX century. Thomas Young (1773–1829), Augustin Jean Fresnel (1788–1827), Augustin Louis Cauchy (1789–1857) and George Green (1793–1841) proposed the models analogous to the classical elastic momentless continuum.

James MacCullagh (1809–1847) proposed the original theory, irreproachable in respect of correspondence with the optics experiments. Introduction of a new type of elastic solids is the distinguishing feature of MacCullagh's theory. From the results obtained by Green, MacCullagh concluded that comparing the ether with the customary elastic solid, which resists to compression and form change, it is impossible to explain the optic phenomena satisfactory. As a result, MacCullagh constructed the model of continuum whose internal energy depends on the rotation of volume elements, i.e. on strain measure $\nabla \times \mathbf{u}$ (rotor of the displacement vector). MacCullagh in fact contrived the medium whose oscillations possesses the same properties that the oscillations of light. However, MacCullagh's theory gave rise to doubts of both contemporaries and the scientists of next generation. This theory was appreciated only in 40 years later when FitzGerald attracted attention to it.

The Maxwell's model proposed in 1862 is based on the concept of magnetism as the phenomenon of rotational character. In accordance with Faraday's ideas, Maxwell supposed that the ether is a medium rotating about the lines of magnetic force, and each unit tube of force can be presented as an isolated vortex. There is an evident problem in this model. Since two neighbouring vortices rotate in the same direction, the particles in the circumference of one vortex must be moving in the opposite direction to the particles contiguous to them in the circumference of the contiguous vortex. Therefore the motion is discontinuous. In order to escape from this difficulty Maxwell used a simple technique. When two wheels should revolve in the same direction, an "idle" wheel is inserted between them (see Fig. 3). In

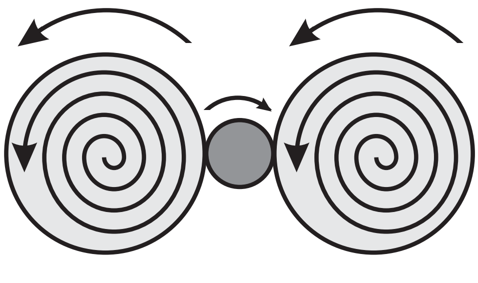

Figure 3: Maxwell's model

fact, this model is a two-component medium. In this model "magnetic medium" is divided into cells by the walls consisting of separated layer of spherical particles which are "electricity". The cell substance is elastic both with respect to compression and with respect to form change. The connection between the cells and the particles forming walls is that rolling without sliding and the tangential action on each other take place. When sells rotating the stress state equivalent to combination of the hydrostatic pressure and the longitudinal stress along the rotation axes arises. On the basis of his model Maxwell proposed the mathematical description of electrodynamics in the form of system of equation which is called by his name. A characteristic feature of Maxwell's theory is the fact that magnetic energy is the kinetic energy and that electric energy is the internal energy. This conception, for which Maxwell was indebted to Faraday and Thomson, brought together the electromagnetic theory and the theories of ether as the elastic solid. Creation of an electromagnetic theory of light was the logical result of that. By that time it had been determined by experiment that the value of constant in Maxwell's equations is identical to the velocity of light. This result was very important since it allowed Maxwell to maintain that the light consists in the transverse wave motion of the same medium which causes electric and magnetic phenomena.

In 1885 FitzGerald proposed a model resembling the Maxwell's model. This model is based on the mechanism constituted of a number of wheels, free to rotate on axes fixed perpendicularly in a plane board (see Fig. 4). The axes are fixed at

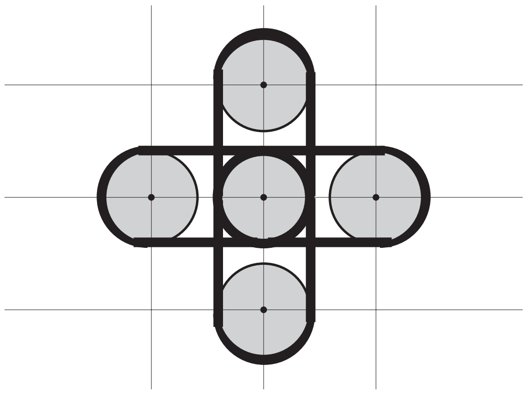

Figure 4: FitzGerald's model

the intersections of two systems of perpendicular lines, and each wheel is connected to each of its four neighbours by an rubber band. If all wheels are rotating with the same velocity then there is no stresses in the system. If some of the wheels are rotating faster than others, the rubber bands are strained. It is evident that the wheels in this model play the same role as the vortices in Maxwell's model. A strain on the bands represents dielectric polarization. Conduction is represented by a slipping of the bands on the wheels.

The models of ether based on the rotational degrees of freedom, unfortunately, have not been developed. The reason is the fact that in the second half of XIX century the level of development of continuum mechanics made it impossible to describe 3D-continua with rotational degrees of freedom.

For a long time there was no followers of the Cosserat's approach to construction of mathematical models of continua with the rotational degrees of freedom. However, starting with the works by C. Truesdell and J. Ericksen, written in the second half of XX century, this approach began to intensively develop. Now this approach is quite well developed and the derivation of basic equations is not difficult. Thus, at the close of XX century the possibility of mathematical realization of the ideas analogous to ideas by Maxwell, FitzGerald and Kelvin appears.

5 Concluding remarks

- According to the ideas of modern theoretical physics the Maxwell equations describe the electromagnetic field which is some kind of abstraction giving a good description of electromagnetic interactions but not having a material carrier.

- The viewpoint accepted in modern physics essentially differs from the viewpoint which was launched by M. Faraday and realized (as far as possible at that time) by J. Maxwell. According to the scientists of the XIX century the electromagnetic field is a special kind of material medium.

- The viewpoint of P. A. Zhilin coincides with viewpoint of M. Faraday and J. Maxwell. P. A. Zhilin proposed a new mechanical model of electromagnetic field that is based only on the rotational degrees of freedom.