Topologically nontrivial loop monopoles

I. A. Ovid'ko
M. I. Kalinin Polytechnic Institute, Leningrad

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The loop monopoles—new type of topological excitations in gauge theories—are studied by the algebraic-topology methods. Each loop monopole with a continuous kernel is shown to be a combination of an ordinary (spherical) monopole and a string-shaped configuration of the gauge field.

The analysis of topologically nontrivial gauge-field configurations—topological excitations—attracts considerable interest in the Yang-Mills-Higgs gauge theories. Only the excitations with a relatively simple geometry—monopoles (spherical configurations),[1] string-like configurations,[1] strings that terminate at the monopoles,[2] strings that have monopoles in their kernels[3]—have so far been studied. The fact that there is no *a priori* restriction on the existence in nature of other types of topological excitations provides an incentive to analyze them. In the present letter we study the stability and classification of loop-shaped excitations which we call loop monopoles.

We consider the stationary Yang-Mills-Higgs theory, in which the gauge group G is disrupted by the Higgs mechanism to the group H (G and H are Lie groups, $H \subset G$). In such a theory the kernel of a loop monopole is a bagel-shaped region L (Fig. 1), which contains a local (gauge) G symmetry, whereas outside the bagel there is only a local H symmetry. Let us consider the classification of the loop monopoles and their (topological) stability.

In terms of the stratified-space formalism—the geometrical analog of the gauge theories[1]—the loop monopoles are in a one-to-one correspondence with such principal G stratifications, ξ, above the baseline R^3 (R^3 is the three-dimensional Euclidean space) which reduce to the H stratifications above the region $R^3 \backslash L$. To find the loop-monopole classification, it is therefore sufficient to calculate the set of stratification-equivalence classes ξ. This problem reduces the problem of a homotopic classification of the mappings $L \rightarrow B_G$ and $R^3 \backslash L \rightarrow B_H$ (here B_G is the baseline of a universal G stratification, and B_H is the baseline of a universal H stratification) which are continuously extended on each other.[4] Solving the above problem by means of the obstacle theory,[5,6] we find that the loop monopoles can be classified by the (α, β) sets—the elements of the group

$$\Gamma^0(G, H) = \pi_1(G, H) \times \pi_2(G, H) \tag{1}$$

[here π_r (G,H) is the relative homotopic group], where $\alpha \in \pi_1(G,H)$ and $\beta \in \pi_2(G,H)$. For example, $G = SO(3)$ and $H = U(1)$,

$$\Gamma^0(SO(3), U(1)) = Z_2 \times Z\ . \tag{2}$$

Here $\alpha \in Z_2$ and $\beta \in Z$.

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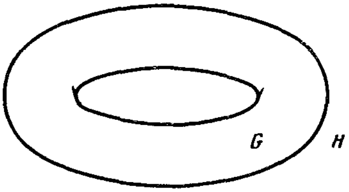

FIG. 1. A loop monopole. A local G symmetry prevails in the monopole kernel and a local H symmetry prevails outside of its kernel.

According to (1), each loop monopole is a combination of a spherical monopole and a string-like configuration. The properties of a string like those of a loop monopole are characterized by the subcharge α, while the properties of a spherical monopole are characterized by the subcharge β. A similar result holds for loop defects in condensed media.[6] The stability of a loop monopole depends in different ways on the topological subcharges α and β: (a) If α is a trivial subcharge ($\alpha = 0$) and β is a nontrivial subcharge, the loop monopole can be "torn" by means of a continuous transformation of the gauge field strength, after which it becomes a spherical monopole (characterized by only β). (b) α is nontrivial and $\beta = 0$. The loop monopole is simply a string coiled into a ring. This configuration cannot be "broken," but the monopole can be eliminated completely by reducing the radius of its loop (by tightening it). (c) α and β are nontrivial. A loop monopole cannot be "torn," but it can be transformed into a spherical monopole by reducing the radius of its loop.

Let us now consider the Yang-Mills-Higgs theory with a double symmetry breaking: $P \to G \to H$ (P, G, and H are the Lie groups, $H \subset G \subset P$). Some of the possible topologically nontrivial configurations in such a theory are strings, which contain spherical monopoles in their kernels[3] (Fig. 2), and loop monopoles, which also contain spherical monopoles in their kernels (Fig. 2). A topological analysis of such loop monopoles (which is similar to the analysis of loop defects in condensed media[6]) shows that these stable monopoles are classified by the sets $(\gamma,\delta,\epsilon_1,...,\epsilon_m)$—the elements of the group

$$\Gamma^m(P, G, H) = \pi_1(H) \times \operatorname{Im}(\pi_1(G, H) \xrightarrow{\varphi} \pi_0(H)) \times [A(P, G, H)]^m. \tag{3}$$

Here m is the number of spherical monopoles ($m \geqslant 2$), $\gamma \in \pi_1(H)$, δ is an element of the group $\operatorname{Im}\varphi$—a form of homomorphism φ, ϵ_i is an element of the group

$$(A(P, G, H))_i = ([\pi_1(G)/\operatorname{Ker}(\pi_1(G) \xrightarrow{\psi} \pi_1(G, H))] \cap \operatorname{Ker}(\pi_1(G) \xrightarrow{\theta} \pi_1(P)))_i \tag{4}$$

($i = 1,...,m$), and the homomorphisms φ, ψ, and θ belong to exact sequences of the

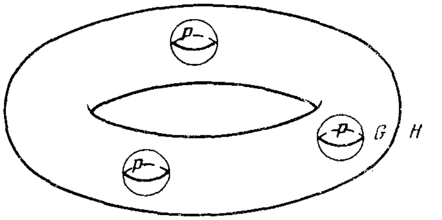

FIG. 2. A loop monopole which contains m spherical monopoles ($m = 3$) in its kernel. A local (gauge) P symmetry prevails in the kernel of each spherical monopole. A local G symmetry prevails in the remaining part of the kernel (bagel) of the loop monopole and a local H symmetry prevails outside of its kernel.

homotopic groups and of their homomorphisms (Kerψ and Kerθ are the homomorphism kernels). The subcharge γ describes the spherical monopole and the subcharge δ describes the string characteristic of a loop monopole. Each subcharge ϵ_i characterizes a gauge-field configuration at the kernel (bagel) of the i-th to $(i+1)$ "inner" spherical monopole. The subcharges ϵ_{i-1} and ϵ_i, together describe the i-th spherical monopole. If $\epsilon_{i-1} = \epsilon_i$, the spherical monopole is unstable.

Accordingly, the loop monopoles with continuous kernels (which are combinations of spherical monopoles and strings) and the loop monopoles with kernels that contain spherical monopoles are (topologically) stable gauge-field configurations.

[1]A. Jaffe and C. Taubes, Vortices and Monopoles, 1980, Boston, Birkhäuser, p. 288; Geometric Techniques in Gauge Theories, Lec. Not. Math., 1982, Vol. 926, Berlin, Springer, p. 320.

[2]A. Vilenkin, Nucl. Phys. **B196**, 240 (1982).

[3]M. Hindmarsh and T. W. B. Kibble, Phys. Rev. Lett. **55**, 2398 (1986).

[4]D. Husemoller, Rassloennye prostranstva (Stratified Spaces), Russ. transl., Mir, Moscow, 1970, Chaps. 4 and 6.

[5]I. A. Ovid'ko and A. E. Romanov, Comm. Math. Phys. **105**, 443 (1986).

[6]I. A. Ovid'ko, Zh. Eksp. Teor. Fiz. **91**, 1427 (1986) [Sov. Phys. JETP **64**, No. 4 (1986)].

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