Astron. Astrophys. 335, 1077–1084 (1998)

ASTRONOMY AND ASTROPHYSICS

Determination of the full velocity vector based on vector magnetograph measurements in an asymmetric sunspot

A.M. Krivtsov[1], A. Hofmann[2], J. Staude[2], M. Klvaňa[3], and V. Bumba[3]

[1] St.-Petersburg State Technical University, Department of Theoretical Mechanics, Politechnicheskaya Street 29, 195251 St.-Petersburg, Russia (krivtsov@mail.wplus.net)
[2] Astrophysikalisches Institut Potsdam, Sonnenobservatorium Einsteinturm, D-14473 Potsdam, Germany (ahofmann@aip.de, jstaude@aip.de)
[3] Astronomical Institute, Czech Academy of Sciences, Ondřejov, Czech Republic (mklvana@asu.cas.cz, bumba@asu.cas.cz)

Received 2 March 1998 / Accepted 27 April 1998

Abstract. A new method to determine the distribution of the full velocity vector in an asymmetric sunspot is presented. Measurements of the Doppler velocity and of the vector of the magnetic field are used as initial data for these calculations. The determination is subdivided into two stages: in a first step we obtain the distribution of the velocity projection onto the solar surface, and in the second step the orthogonal component of the velocity field is calculated.

The method has been applied to a sunspot observed from two observatories over 7 days, that is at different positions on the solar disk and in different phases of its development. The resulting vector velocity field is in good agreement with the basic features of the siphon flow model of penumbral flux tubes. The velocities are directed outwards from the sunspot, and they have small values in the umbra and maximum values in the central part of the penumbra, while in the outer penumbra the velocities decrease rapidly. The vertical velocities are mainly localized at the umbral boundary (upflows) and at the outer parts of the penumbra (downflows).

Key words: MHD – Sun: photosphere – Sun: magnetic fields – sunspots

1. Introduction

In order to understand the underlying physics of most phenomena observed in the solar atmosphere it is necessary to know the spatial distribution and development of the vectors of both the magnetic and velocity fields. For the magnetic field it is possible in principle to determine the full vector from photoelectric spectro-polarimetric measurements in Zeeman-sensitive lines – see e. g. Staude et al. (1991) and Hofmann (1991). But for velocities only the line-of-sight component can be obtained directly from Doppler shift measurements.

The motions in the plane perpendicular to the line-of-sight can be estimated from the shifts of photospheric details (local correlation tracking), but these motions seem to be connected rather with the subphotospheric flows, so it is hard to use them together with the Doppler measurements for deriving the full velocity vector. For example, Shine et al. (1994) have shown that the proper motion velocity is generally larger than the Doppler velocity if both are interpreted as projections of horizontal motions. The horizontal velocities derived from local correlation tracking sometimes have even the opposite direction (inward) to that of the photospheric Evershed flow (Wang & Zirin 1992, Kitai et al. 1997). A velocity field comparable to the relative motions of sunspots can also be determined from a time sequence of magnetograms (Levine & Nakagawa 1974). For a symmetric sunspot the full velocity vector can be estimated from the Doppler measurements alone (Klvaňa et al. 1995, 1996).

Stanchfield et al. (1997) have computed the absolute values of the velocity vector from the measured Doppler velocity using the assumption that the flow is everywhere aligned with the vector of the magnetic field. These results have been confirmed by the latest high-resolution observations of Westendorp Plaza et al. (1997) which have been interpreted by means of an inversion of the radiative transfer equations for polarized light. According to these data the Evershed flow follows along flux tubes which dive below the surface near the outer edge of the penumbra or just outside, in excellent agreement with the predictions of the siphon-flow model. This model has been proposed three decades ago by Meyer & Schmidt (1968); it has been elaborated in several papers by Montesinos & Thomas (1997, and references given there).

In the present paper the full velocity vector in an asymmetric sunspot is determined by a new method based on the measurement of the Doppler velocity and of the vector of the magnetic field. We will show that our results are fully consistent with the latest observations of Stanchfield et al. (1997) and Westendorp Plaza et al. (1997) as well as with the siphon flow model of Montesinos & Thomas (1997).

2. The observed region and the initial data

Observations of the active region NOAA 6716 during the period July 06–12, 1991 will be considered. In Fig. 1 maps of the distribution of the continuum brightness for each of five days

Send offprint requests to: A.M. Krivtsov

Table 1. The heliographic coordinates of the maps for different dates of measurement

Date	July 6	July 7	July 8	July 11	July 12
Latitude	12^o N	13^o N	12^o N	11^o N	11^o N
Longitude	40^o E	28^o E	15^o E	27^o W	39^o W

are presented. Note that the sunspot is not totally symmetric. Beside the main umbra, it has two small additional nuclei: the first one, in the south-east, is more visible on the first three maps, while the second, in the north-east, can be seen on the last two maps. The first three maps (July 6, 7, 8) correspond to disk positions before the central meridian passage (CMP), the two last maps (July 11, 12) after the CMP. The heliographic coordinates of the maps are shown in Table 1.

Fig. 2 shows maps of the distribution of the Doppler velocity (line-of-sight component) measured in the Fe I 524.76 nm spectral line. The black (white) color corresponds to velocities from (toward) the observer. The change of the signature of the Evershed effect from black-white before to the white-black after CMP is clearly evident. All the maps presented are cuttings from larger fields of view. The zero point of the velocities was defined to be the mean velocity in the quiet sun outside the sunspot.

In Fig. 3 we present maps of the measured full vector of the magnetic field viewed from the line-of-sight direction.The background shows the radial component of the magnetic field, and the arrows depict the projection of the magnetic field onto the solar surface plane. The projection effect is clearly visible.

Continuum brightness (shown in Fig. 1), Doppler velocities (Fig. 2), and magnetic field vectors (Fig. 3) were measured at the Einsteinturm Solar Observatory in Potsdam. For the magnetic field vectors the measurements were taken in the wings of the Fe I 525.02 nm line, which is formed in photospheric layers a bit higher than the Fe I 524.76 nm line. A total polarizer and a quarterwave plate in front of the first coelostat mirror were used to calibrate the instrument, including an elimination of parasitic instrumental polarization, before and after the completion of each magnetogram. During the measurements an inclined glass plate was used to compensate for the main part of the linear polarization caused by the telescope. The Stokes vectors measured during the scan were corrected by taking into account the time variation of the calibrational Müller matrix of the instrument (instrumental polarization is eliminated in this way) and the influence of the parasitic stray light. The corrected Stokes vectors were transformed into magnetic field vectors by means of the theoretical calibration functions of Staude (1970a, b; Staude et al. 1991), which take into account the true photometer slit dimensions, the Faraday effect, and different atmospheric models for different solar features such as the umbra or the quiet photosphere.

3. The method of determining the horizontal velocity

Let the horizontal plane be the plane of the solar surface and the vertical direction be the direction orthogonal to the solar surface (radial direction). To find out the velocity vector field the following assumptions will be used

1. The line-of-sight component of the velocity vector is given by the measured Doppler velocity.
2. The vertical component of the velocity vector is small compared to the horizontal component.
3. The horizontal component of the velocity vector is parallel to horizontal component of the magnetic field vector.

Assumption 2 is a widely accepted observational fact (see Maltby 1964, Schröter 1965, Title et al. 1993, Shine et al. 1994). In particular, according to Shine et al. (1994) the inclination of Evershed velocities is $\lesssim 6^o - 11^o$. Results obtained for a symmetric sunspot (Klvaňa et al. 1995, 1996) give the same estimate for the average velocity inclination. Physical arguments for horizontal motions in sunspots have been given by Unno et al. (1981) and Berton (1985). Recent observations by Rimmele (1995a, b) have shown that the Evershed flow is extended along arched magnetic flux tubes. In Fe spectral lines such as those used in the present paper the flow along the nearly horizontal, upper portions of these flux tubes is seen (Stanchfield et al. 1997).

Assumption 3 is suggested by observations showing that the Evershed flows are concentrated in the horizontal dark filaments with a nearly horizontal magnetic field; the inclined magnetic field in the white filaments is not correlated with the Evershed velocities (Title et al. 1993, Rimmele 1995a). A mathematical argument in favour of Assumption 3 in a special case was given by Berton (1985). Note that for symmetric sunspots this assumption is fulfilled with high accuracy (Klvaňa & Bumba 1996).

Let us calculate the horizontal component of the velocity vector from the values of the Doppler velocity and the vector of magnetic field using the Assumptions 1–3. We introduce the notations

$\boldsymbol{v}$, $\boldsymbol{B}$	—	vectors of velocity and magnetic field,
$\boldsymbol{v}_{\rm h}$, $\boldsymbol{B}_{\rm h}$	—	horizontal components of velocity and magnetic field vectors, and
$\boldsymbol{k}$, $\boldsymbol{e}$	—	unit vectors of the vertical direction and of the line-of-sight, respectively.

Assumption 3 results in

$$\boldsymbol{v}_{\rm h} = \lambda \boldsymbol{B}_{\rm h}\,, \tag{1}$$

where λ is an unknown scalar multiplier. To find out the value of λ let us project Eq. (1) to the line-of-sight direction (i. e. take the scalar product with the unit vector $\boldsymbol{e}$):

$$\boldsymbol{e}\cdot\boldsymbol{v}_{\rm h} = \lambda \boldsymbol{e}\cdot\boldsymbol{B}_{\rm h} \quad \Rightarrow \quad \lambda = \frac{\boldsymbol{e}\cdot\boldsymbol{v}_{\rm h}}{\boldsymbol{e}\cdot\boldsymbol{B}_{\rm h}} \tag{2}$$

Neglecting the vertical component of the velocity (Assumption 2) the quantity $\boldsymbol{e}\cdot\boldsymbol{v}_{\rm h} = \boldsymbol{e}\cdot\boldsymbol{v} \stackrel{def}{=} v_e$ is the known value of

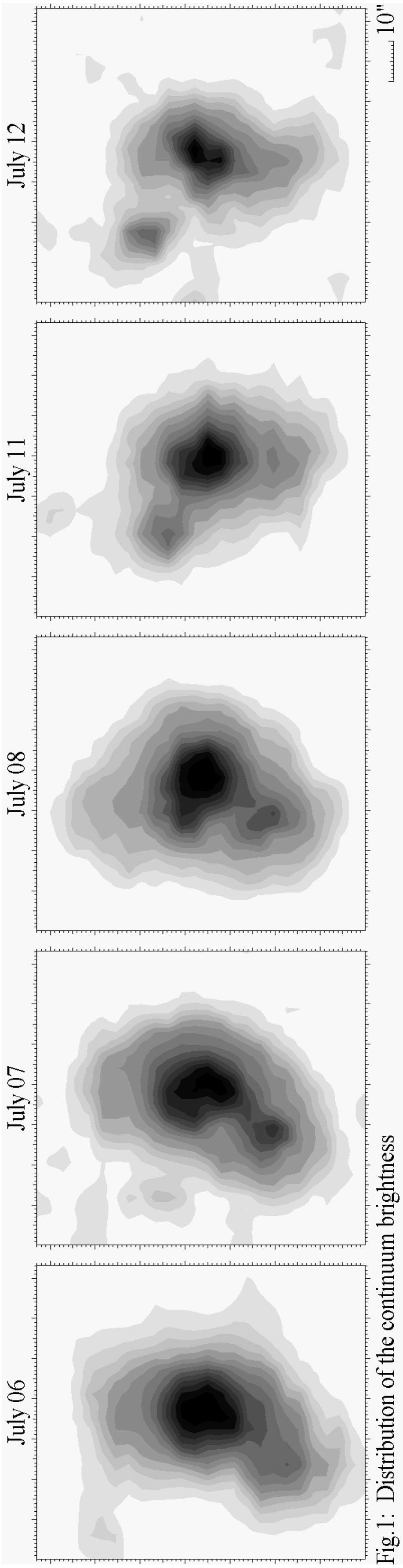

Fig.1: Distribution of the continuum brightness

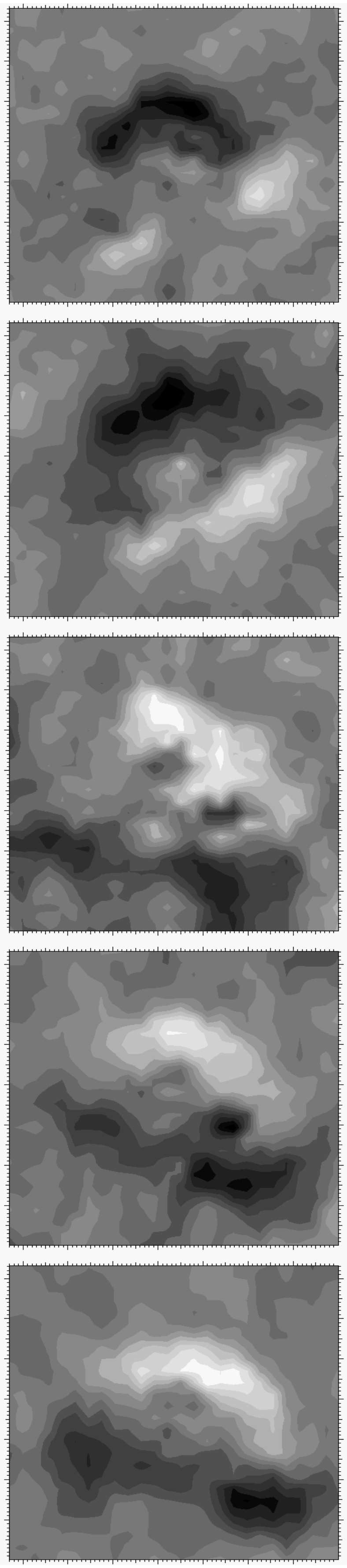

Fig.2: Distribution of the line-of-sight component of the velocity, Fe I 5247 spectral line

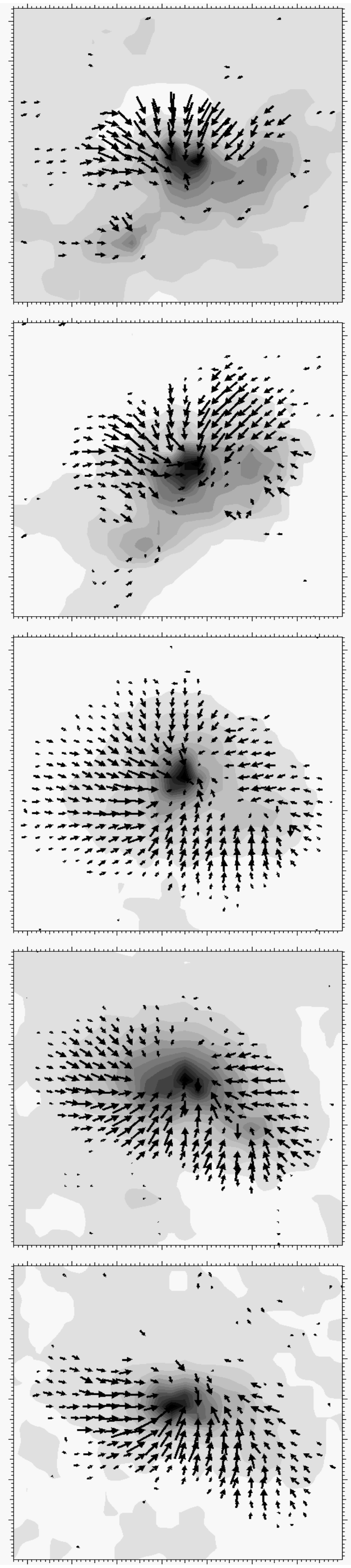

Fig.3: Distribution of the whole vector of the magnetic field

the Doppler velocity. The quantity $\boldsymbol{e}\cdot\boldsymbol{B}_\mathrm{h}$ can be calculated as follows:

$$\boldsymbol{B} = \boldsymbol{B}_\mathrm{h} + B_z\boldsymbol{k} \quad \Rightarrow \quad \boldsymbol{e}\cdot\boldsymbol{B}_\mathrm{h} = \boldsymbol{e}\cdot\boldsymbol{B} - B_z\boldsymbol{k}\cdot\boldsymbol{e}\,. \tag{3}$$

Here $\boldsymbol{e}\cdot\boldsymbol{B} \overset{def}{=} B_e$ is the measured line-of-sight projection of the magnetic field vector; B_z is the vertical projection of the magnetic field vector; and $\boldsymbol{k}\cdot\boldsymbol{e} = \cos\vartheta$, where ϑ is the angle between the line-of-sight and the radial direction. Thus from (1), (2), and (3) we have

$$\boldsymbol{v}_\mathrm{h} = \lambda\boldsymbol{B}_\mathrm{h}\,, \qquad \lambda = \frac{\boldsymbol{e}\cdot\boldsymbol{v}_\mathrm{h}}{\boldsymbol{e}\cdot\boldsymbol{B}_\mathrm{h}} = \frac{v_e}{B_e - B_z\cos\vartheta}\,. \tag{4}$$

In Eq. (4) the desired horizontal velocity vector is expressed in terms of the measured quantities. (B_z is not measured directly but it can be obtained from the measured magnetic field vector $\boldsymbol{B}$.)

4. Computation of the horizontal velocity distribution in the sunspot

Let us consider the measurement from July 8 (the third maps in Figs. 1–3) as an example to demonstrate the calculation techniques. Figs. 4a, b show maps of the Doppler velocity and the magnetic field vector, respectively. Note that in spite of the proximity of the area to the solar center (distance $\approx 19^o$) the signature of the Evershed effect is clearly visible in the Doppler velocities (Fig. 4a). For the magnetic field, however, projection effects are unimportant (Fig. 4b). This is the well known effect of the anticorrelation of velocity and magnetic fields (Berton 1985).

Consider Eq. (4). To find out the velocity it is necessary to calculate the scalar coefficient λ. The denominator of the coefficient is the longitudinal projection of the horizontal magnetic field, $\boldsymbol{e}\cdot\boldsymbol{B}_\mathrm{h} = B_e - B_z\cos\vartheta$. The map of this quantity obtained from the magnetic field vector distribution (Fig. 4b) is shown in Fig. 4c. Compare this map with the distribution of the Doppler velocity in Fig. 4a: the velocity field (Fig. 4a) and the magnetic field (Fig. 4c) are in good correlation. This fact confirms the assumption of parallelism of the horizontal components of the velocity and magnetic field vectors (Assumption 3).

In order to determine the velocity it is necessary to calculate the ratio of the velocity field (Fig. 4a) and the magnetic field (Fig. 4c). Of course, there will be problems in the vicinity of the line where both quantities are close to zero — we will obtain a ratio of two small parameters. Such a line will be called a central line; it is close to the line with a direction from bottom to top with a little inclination to the right in Figs. 4a, 4b. The result of the simple computation by Eq. (4) is shown in Fig. 4d. The background shows the original Doppler velocities, and the arrows show the resulting horizontal velocities. Far from the central line the velocity field looks quite reasonable, but there are large errors in the vicinity of the central line. To avoid such errors a median filter with 3×3 pixels has been applied to the field of the coefficients λ. This filter removes such values which sharply differ from their neighbors. The result of the computation with the corrected λ coefficients is shown in Fig. 4e. The final result – after a weak additional filtration of the vector field – is shown in Fig. 4f.

The same technique was used for the measurements of the other four days. The results are presented in Fig. 5. In order to eliminate the projection effects due to different positions on the disk, in Fig. 5 the areas are rescaled to appear as if they were observed from the radial direction. All the maps show that the velocities are directed outwards from the sunspot, the velocities have small values in the umbra, and they achieve maximal values near the center of the penumbra and rapidly decrease outwards in the sunspot (see Fig. 1 for brightness). A comparison of the brightness (Fig. 1) and the velocity maps (Fig. 5) for the last measurement (July 12) shows that in the area between the two nuclei (north-east from the main umbra) the velocities have very small values.

5. The method of determining the vertical velocity

The method described in Sect. 3 only allows us to obtain the horizontal projection of velocity vector. To find out the vertical velocity let us use the equation of mass conservation assuming a steady state ($\partial\rho/\partial t = 0$):

$$\nabla\cdot(\rho\boldsymbol{v}) = 0 \quad \Longleftrightarrow \quad v_z\frac{\partial\rho}{\partial z} + \rho\,\frac{\partial v_z}{\partial z} = -\nabla_\mathrm{h}\cdot(\rho\boldsymbol{v}_\mathrm{h})\,, \tag{5}$$

where ρ is the density, ∇_h is the horizontal component of the ∇ operator, and z is the coordinate along the vertical axis. Denote the inverse vertical velocity scale height as

$$\gamma \overset{def}{=} \frac{1}{v_z}\,\frac{\partial v_z}{\partial z}\,;$$

then the derivative $\partial v_z/\partial z$ can be represented in the form

$$\frac{\partial v_z}{\partial z} = \gamma v_z\,. \tag{6}$$

In general ρ and γ are functions of the coordinates $(x,\ y,\ z)$, where (x, y) are the coordinates in the plane of the solar surface. It is quite evident that the dependence of ρ on z is much stronger than that on (x, y). The same can be expected for γ. Neglecting the dependence of γ and ρ on the coordinates (x, y), we have

$$\nabla_\mathrm{h}\cdot(\rho\boldsymbol{v}_\mathrm{h}) = \rho\nabla_\mathrm{h}\cdot\boldsymbol{v}_\mathrm{h}\,,$$

and from Eqs. (5), (6) we obtain

$$v_z = -\frac{1}{C(z)}\,\nabla_\mathrm{h}\cdot\boldsymbol{v}_\mathrm{h}\,, \qquad C(z) \overset{def}{=} \gamma(z) + \frac{d}{dz}\ln\rho(z)\,, \tag{7}$$

where C is a scalar coefficient that has a constant value for the whole map. Thus, using Eq. (7) it is possible to find the distribution of the vertical velocity from the known distribution of the horizontal velocity.

6. Computation of the vertical velocity distribution in the sunspot

For the computation of the vertical velocity, let us use the maps (Fig. 5) of the horizontal velocity distribution obtained from the

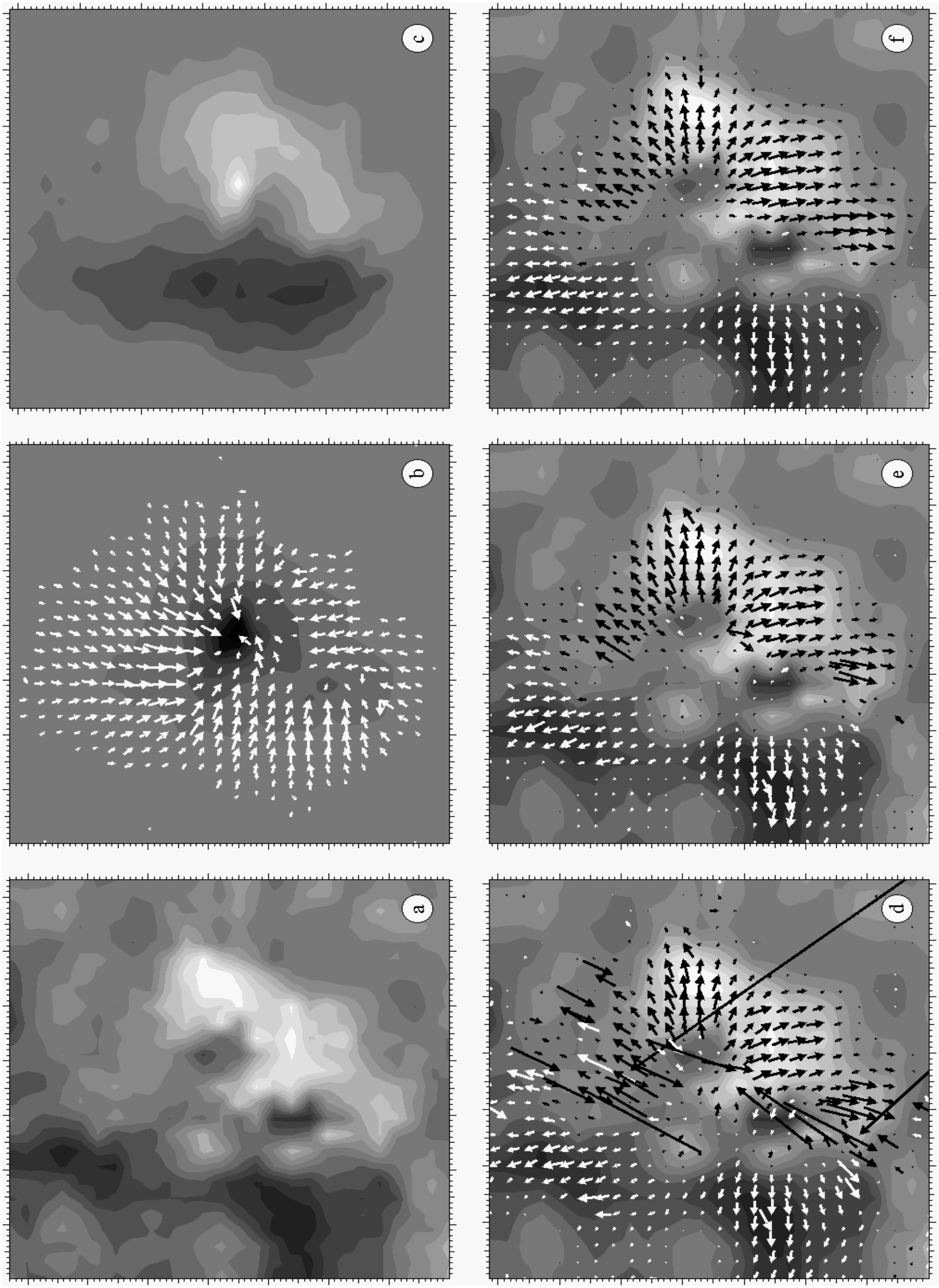

Fig.4: Computation of the horizontal velocity distribution, the measurement in Fe I 5247 spectral line, July 8

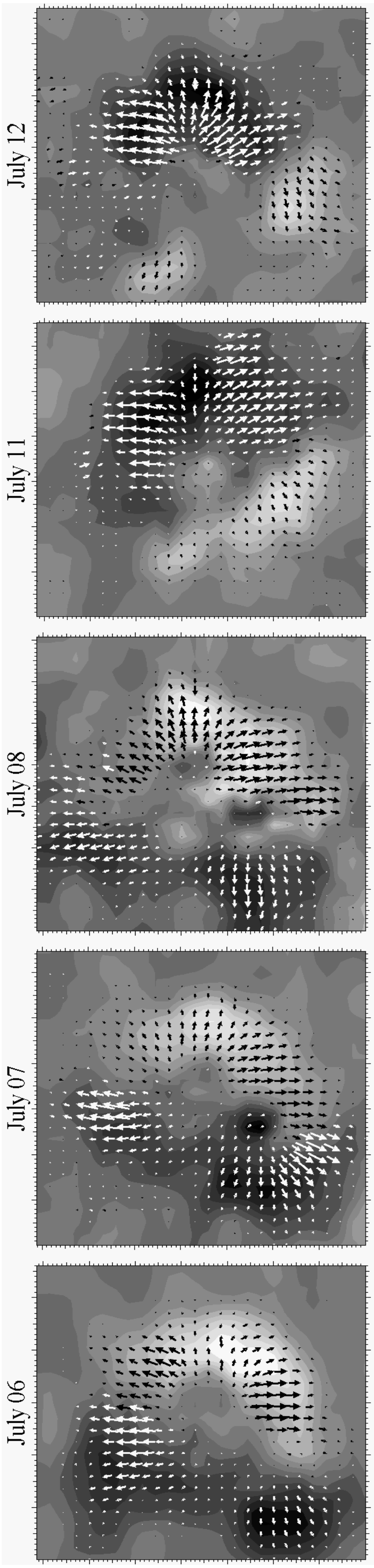

Fig.5: Distribution of the horizontal velocity on the map of the Doppler velocity, Fe I 5247 spectral line

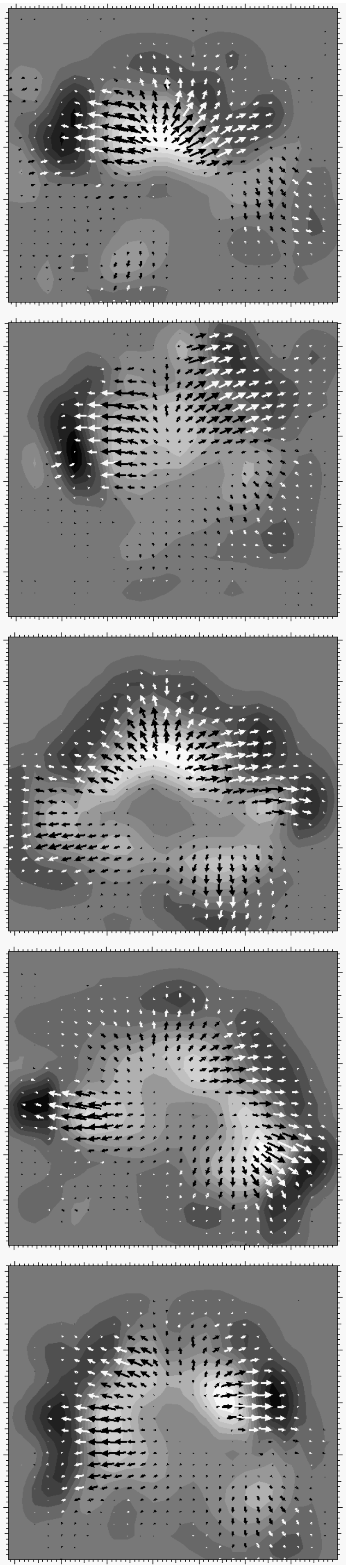

Fig.6: Distribution of the horizontal velocity on the map of the vertical velocity, Fe I 5247 spectral line

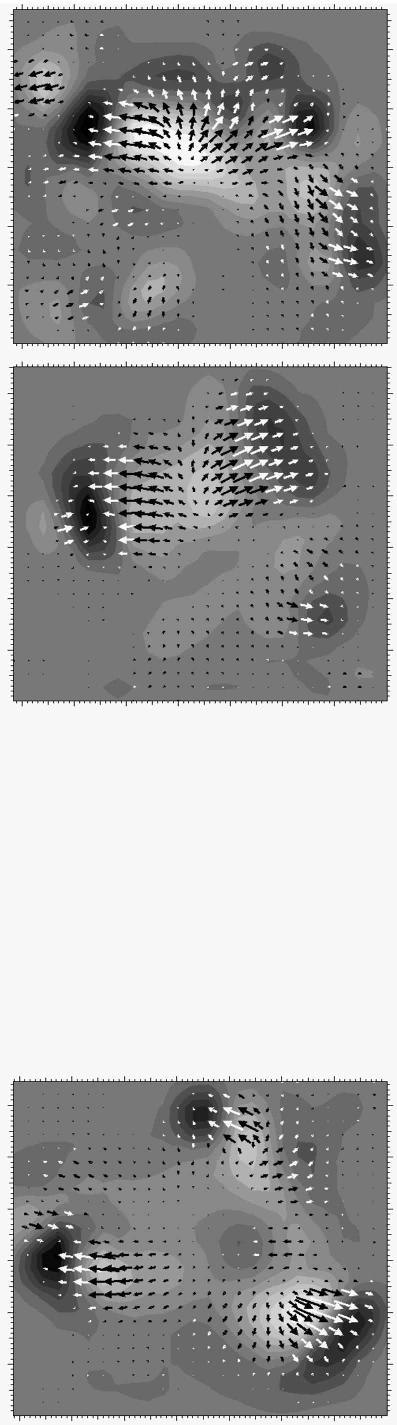

Fig.7: Distribution of the horizontal velocity on the map of the vertical velocity, Fe I 5253.47 spectral line

Doppler measurements in the spectral line Fe I 524.76 nm. The result is shown in Fig. 6. The background shows the distribution of the vertical velocity, and the arrows show the distribution the horizontal velocity. From the maps it follows that the vertical velocity is practically zero in the umbra, upflows are mainly localized at the umbra boundary, and downflows appear at the outer parts of the penumbra.

7. Comparison with the measurements from the Ondřejov Observatory

At the Ondřejov Observatory (Czech Republic) Doppler velocities were measured in the Fe I 525.347 nm spectral line for the same active region; our aim was to look for possibilities to use Doppler measurements from the Ondřejov Observatory together with magnetic field measurements from the Einsteinturm Observatory to compute the whole velocity vector distribution in the active region. A basic problem with such a comparison of measurements from different places is the choice of a common system of coordinates. For this purpose the brightness maps were used. After some averaging of the maps, the point of minimum brightness was used as the origin of the coordinate system for both maps. Then this common coordinate system was used for the Doppler and magnetic field maps. The computations show that this method gives the maximum cross-correlation for the measurements. In Fig. 7 the results of these computations of the full velocity vector are shown for July 7, 11, and 12. The basic structures of the velocity field are very similar to those in Fig. 6, where the computations of the vertical velocities were made using the Doppler measurements from the Einsteinturm Observatory (in the Fe I 524.76 nm spectral line). Thus, the use of Doppler velocity and magnetic field measurements from different observatories can give good results; the flows in the Fe I 525.347 nm and Fe I 524.76 nm spectral lines looks very similar to each other.

8. Discussion and conclusions

We have presented a method of determining the distribution of the full velocity vector in an asymmetric sunspot. One of the assumptions in our computation of the horizontal velocities is the condition that the horizontal component of the velocity vector is parallel to the horizontal component of the magnetic field vector. Recent observations (Title et al. 1993, Rimmele 1995a) have confirmed earlier results showing that the Evershed flows are concentrated in the horizontal dark filaments where velocity vectors and magnetic field vectors are parallel. In the bright filaments the magnetic field is more highly inclined, but the velocities are negligible there. In our observations the spatial resolution was not very high ($\approx 3"$), and values inherent to dark and bright filaments are averaged. Therefore the inclination of the observed magnetic field (averaged over dark and bright filaments) differs from the inclination of the velocity vector (measured mainly in the dark filaments which are almost horizontal). That is why in the method presented we have separated the horizontal component $\boldsymbol{B}_{\rm h}$ of the magnetic field that is connected with the dark filaments and the Doppler velocities. The map of the line-of-sight projection of $\boldsymbol{B}_{\rm h}$ (Fig. 4c) shows a very good correlation with the map of the Doppler velocity (Fig. 4a). This fact confirms that Doppler velocities in the middle photosphere are directed along horizontal channels which are the upper portions of the magnetic arches carrying the Evershed flows.

For the computation of the horizontal velocities the vertical velocities were neglected in the first step. However, after determination of the horizontal velocity field the method allows us to compute the small vertical components of the velocity. Indeed, due to mass conservation the radial flow can't exist without upflow in the inner sunspot area and downflow in the outer area. The computed maps of the vertical velocities for the Fe I 525.347 nm and Fe I 524.76 nm spectral lines (Fig. 6) show that the vertical velocity is practically zero in umbra, upflows are mainly localized at the umbra boundary, and downflows appear at the outer parts of the penumbra. The regions where the obtained vertical flows are localized correspond to those areas were the footpoints of the magnetic arches have been observed (Rimmele 1995b, Stanchfield et al. 1997). The level where we have measured the velocities is too high in the atmosphere to observe the footpoints of the magnetic arches, and our spatial resolution is not high enough. Nevertheless, the method presented allows us to find weak vertical flows that can't be measured directly. Our results are in excellent agreement with the interpretation of the latest observations of Stanchfield et al. (1997) and Westendorp Plaza et al. (1997) as well as with the predictions of the siphon flow model of Montesinos & Thomas (1997).

The sunspot we have observed is not symmetric; it has two small additional nuclei. Our computation shows that the velocity vectors almost vanish in the penumbra between the main umbra and the additional nuclei.

Our investigation has demonstrated that Doppler velocity data and vector magnetic field data needed for the vector velocity computations can be taken at different observatories, as has been done here with magnetic field measurements from the Potsdam Einsteinturm Observatory and Doppler measurements from the Ondřejov Observatory. The comparison of these results with the velocity vector fields computed from the Einsteinturm Observatory data alone shows a good agreement (see Figs. 6 and 7). It would be highly desirable to repeat this analysis with data of much better spatial resolution, obtained, for example, at the vacuum solar telescopes at Tenerife.

Acknowledgements. We are very thankful to the referee, John H. Thomas, for drawing our attention to related recent papers and for suggesting improvements of the language of the paper. M. K. and V. B. gratefully acknowledge support of the present work by the grant GACR 205/97/0500.

References

Berton R., 1985, A&A, 149, 291

Hofmann A., 1991, Astron. Nachr., 312, 49

Kitai R., Funakoshi Y., Ueno S., Sano S., Ichimoto K., 1997, PASJ, 49, 513

Klvaňa M., Bumba V., Krivtsov A. M., 1995, Zborníc ref. z 12 celošt. slunečn. seminára, Teplý Vrch, B. Lukáč (ed.), 12 (in Czech)

Klvaňa M., Bumba V., 1996, Joso Annual Reports 1995, M. Saniga (ed.), 144
Levine R. H., Nakagawa Y., 1974, ApJ, 190, 703
Maltby P., 1964, Astrophys. Norvegica, 8, 205
Meyer F., Schmidt H. U, 1968, Z. Angew. Math. Mech., 48, 218
Montesinos B., Thomas J. H., 1997, Nature, 390, 485
Rimmele T. R., 1995a, A&A, 298, 260
Rimmele T. R., 1995b, ApJ, 445, 511
Shine R. A., Title A. M., Tarbell T. D., Smith K., Frank Z. A., 1994, ApJ., 430, 413
Schröter E. H., 1965, Zs. f. Ap., 62, 228.
Stanchfield II, D. C. H., Thomas J. H., Lites B. W., 1997, ApJ, 477, 485
Staude J., 1970a, Solar Phys., 12, 84
Staude J., 1970b, Solar Phys., 15, 102
Staude J., Hofmann A., Bachmann G., 1991, in L. November (ed.), Solar Polarimetry, Sunspot, New Mexico, p. 49
Title A. M., Frank Z. A., Shine R. A. et al., 1993, ApJ, 403, 780
Unno W., Tanaka K., Semel M., 1981, PASP, 33, 495
Wang H., Zirin H., 1992, Solar Phys., 140, 41
Westendorp Plaza C., del Toro Iniesta J. C., Ruiz Cobo B. et al., 1997, Nature, 389, 47