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NONLINEAR DYNAMICS OF PERCUSSIVE DRILLING OF HARD MATERIALS

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ABSTRACT

It is postulated that the main mechanism of the enhancement of material removal rate (MRR) in percussive drilling is associated with generating impact forces, which act on the workpiece and help to develop micro-cracking in the cutting zone. The inherent non-linearity of the discontinuous impact process is modelled as a frictional pair, to generate the pattern of the impact forces. A novel formula for calculating the MRR is proposed, which explains the experimentally observed fall in MRR at higher static forces.

INTRODUCTION

Recently, nonlinear dynamics approaches have increasingly been used to explain complexities occurring in manufacturing systems. Theoretical studies have been carried out in the area of ductile metal cutting (e.g. (Grabec, 1986; Wiercigroch, 1997)), where periodic (chatter) and aperiodic (chaos) behaviour of simple models has been demonstrated. Despite the fact that strong nonlinear dependencies have been observed in cutting brittle materials, this area has been given little attention so far. For example, one of the best known anomalies in ultrasonic percussive drilling is the decrease in material removal rate for higher values of static forces, contradicting a classical perception of the efficiency of the process mechanism. A study of this phenomenon was the stimulus for the work described in this paper.

Percussive drilling offers a solution to the expanding need for an efficiency drilling of rocks or other brittle materials, which is very similar to ultrasonic machining of brittle materials. The actual cutting is performed either by abrasive particles suspended in a fluid, or by a rotating diamond-plated tool. These variants are known respectively as traditional ultrasonic machining, and rotary ultrasonic machining (RUM). The RUM technology was developed in the early 1960s by U.K.A.E.A.-Harwell in England. Some years later quite similar methods were studied by (Markov et al, 1972; Markov, 1977; Markov, 1980) and (Petrukha, 1980), but details of the methods were not revealed. Other workers carried out experimental studies on the basic characteristics of the process, for example (Kubota et al, 1977). Their tests established the influences of working conditions such as grain size, amplitude of vibration, rotational speed and feed pressure on the material removal rate (MRR), which is defined as volume of material removed in a unit of time. A particular feature of these experiments is that plots of MRR versus static load presented in (Markov, 1980) and (Kubota et al, 1977) show a maximum for a certain value of static load. (Komaraiah et al, 1988) also conducted experimental studies on the ultrasonic machining of different workpiece materials. Their work confirmed the superiority of the rotary technique over traditional slurry-type machining. The first theoretical approach to modelling USM was put forward by (Saha et al, 1988). They attempted to develop a comprehensive analytical model for the estimation of the MRR in order to make an in-depth study of the material removal process and its

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dependence on major influencing parameters. Satisfactory agreement was reported between theory and experiment, apparently explaining the fall in MRR for higher static loads. However, their model uses only Hertzian theory to explain the mechanism of material removal; it seems that this approach should be more suitable for workpieces comprising ductile rather than brittle materials. Moreover, using Hertzian theory alone to explain the relationship between MRR and static force, it appears impossible to obtain a function of the form obtained from experimental test (Kubota et al, 1977). Accordingly, this paper adopts the nonlinear dynamics approach to modelling MRR for brittle materials, which is phenomenologically different to any others previously undertaken, and hopefully would be of interest to both nonlinear dynamics and applied physics communities. It is based on applying impacting oscillator theory (Thompson et al, 1982; Shaw, 1985; Wiercigroch et al, 1998) to explain the main mechanism occurring in ultrasonic drilling. In particular, we shall address the formulation of a simple model of the non-linear dynamic interactions encountered in the machine tool – ultrasonic cutting process system, which could explain the fall in the MRR for higher static loads.

The proposed model will investigate the dependences of the static force and the amplitude of the dynamic force on the material removal rate, assuming that the drilling resistance is modeled by a dry friction element.

DRY FRICTION MODEL

The presented model is shown in Figure 1, where m is mass of the tool, $F(t)$ is overall drilling force, $P(\dot{y})$ is the resistive (dry friction) force, x is coordinate of the tool's tip, and y is coordinate

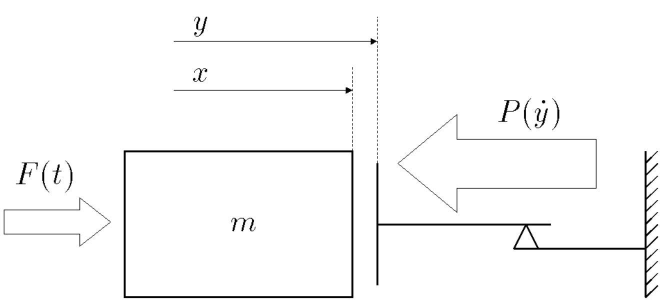

Figure 1. The dry friction model.

of the dry friction element, which represents the progression of the drilling surface. The equation of motion of the mass takes the following form

$$
\begin{aligned}
x < y \quad &\Rightarrow \quad m\ddot{x} = F(t)\,, \\
x = y \quad &\Rightarrow \quad m\ddot{x} = F(t) - P(\dot{y})\,,
\end{aligned}
\tag{1}
$$

which depends on the relative position between x and y coordinates. In turn, the equation of motion for the slider can be expressed as

$$
\begin{aligned}
\dot{x} \geq 0 \quad &\Rightarrow \quad y = x\,, \\
\dot{x} < 0 \quad &\Rightarrow \quad \dot{y} = 0\,.
\end{aligned}
\tag{2}
$$

For the simplicity of the further analysis it was assumed the overall drilling force $F(t)$ has the form

$$
F(t) = A \sin \omega(t - t_0) + B\,, \tag{3}
$$

where A and ω are the amplitude and frequency of harmonic force, B is the static force, t is time, and t_0 is a time constant. The resistive force $P(\dot{y})$ is modelled by the Coulomb dry friction fulfilling the following conditions

$$
\begin{aligned}
\dot{y} > 0 \quad &\Rightarrow \quad P(\dot{y}) = Q\,, \\
\dot{y} = 0 \quad &\Rightarrow \quad P(\dot{y}) \leq Q\,,
\end{aligned}
\tag{4}
$$

where Q stands for the modulus of the dry friction force. From the practical viewpoint one may consider that the dry slider progression (material removal rate) is due to generation of the dynamic impacts, while decelerating the mass. That is why we can limit our consideration to the case when $F(t) < Q$. From the equations (1)–(4) it can be easily deduced that the considered system is in one of three unique states (see Tab. 1).

State	x	y	Condition
Free motion	$m\ddot{x} = F(t)$	$\dot{y} = 0$	$x < y$
Drilling	$m\ddot{x} = F(t) - Q$	$y = x$	$\dot{x} > 0$
Stop	$\dot{x} = 0$	$y = x$	$F(t) \geq 0$

Table 1. Three unique states of the system.

In order to gain some extra flexibility of the analysis new dimensionless variables and parameters are introduced. Let the dimensionless time to be denoted as $\tau = \omega t$, and prime stands for derivation with the dimensionless time. It is worth to note that the resistive coefficient, Q has a distinct value for a drilled material. Contrary the amplitude of the harmonic force and the static force can vary, and can be used as control parameters for the drilling process. Hence, let us divide all the drilling forces by the resistive coefficient, Q

$$
f(\tau) \stackrel{def}{=} \frac{F(t)}{Q}; \qquad a \stackrel{def}{=} \frac{A}{Q}, \qquad b \stackrel{def}{=} \frac{B}{Q}, \tag{5}
$$

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which leads to the dimensionless drilling force

$$f(\tau) = a\sin(\tau - \omega t_0) + b \ < \ 1\,. \qquad (6)$$

To complete dimentionlising process, coordinates ξ and η are defined as follows

$$\xi \stackrel{def}{=} \frac{\omega^2 m}{Q}x\,, \qquad \eta \stackrel{def}{=} \frac{\omega^2 m}{Q}y\,. \qquad (7)$$

PROGRESSIVE STATIONARY MOTION

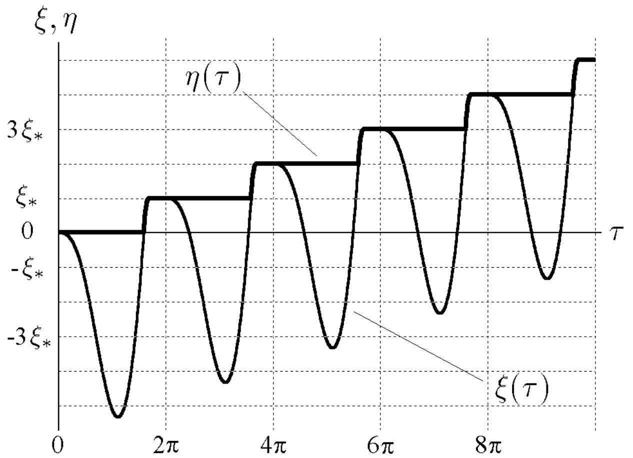

Figure 2. The progressive stationary motion.

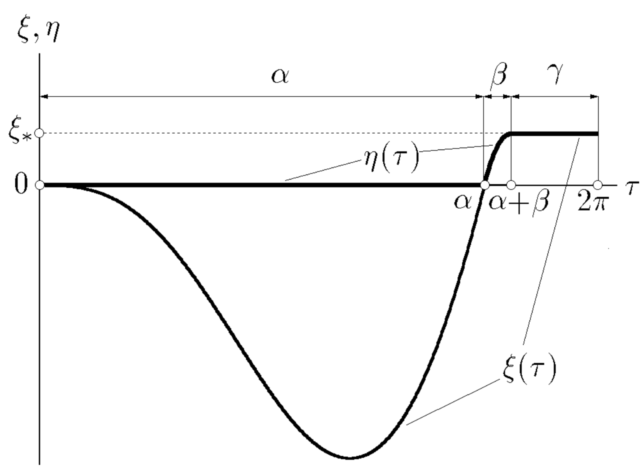

Figure 3. One period of the motion.

Let us consider a progressive stationary motion of the system depicted in Figure 2, that satisfies the following equations

$$\xi(\tau + 2\pi) = \xi_* + \xi(\tau)\,, \qquad \eta(\tau + 2\pi) = \xi_* + \eta(\tau)\,, \qquad (8)$$

where ξ_* is the dimensionless penetration of the drillbit during one excitation period, which is equal to 2π. The dimensionless material removal rate, r calculated per one period is

$$r = \frac{\eta(2\pi) - \eta(0)}{2\pi} = \frac{\xi_*}{2\pi}\,. \qquad (9)$$

Stage	Time limits	ξ	η
Free motion	$0 < \tau < \alpha$	$\xi'' = f(\tau)$	$\eta = 0$
Drilling	$\alpha < \tau < \alpha + \beta$	$\xi'' = f(\tau) - 1$	$\eta = \xi$
Stop	$\alpha + \beta < \tau < 2\pi$	$\xi = \xi_*$	$\eta = \xi_*$

Table 2. Three distinct sequential stages of the stationary motion.

If one breaks up one period of the progressive stationary motion into distinct intervals, three sequential time intervals can be specified. They are schematically shown in Figure 3 and are specified in Table 2. The time intervals α, β and γ satisfy the identity $\alpha + \beta + \gamma = 2\pi$.

The dimensionless excitation force expressed by (6) can be represented in the following form

$$f(\tau) = -a\cos(\tau - \varphi) + b\,, \qquad (10)$$

where $\varphi \stackrel{def}{=} \omega t_0 - \pi/2$. The sum $\varphi + \pi$ is a phase shift between the overall drilling force and the progressive stationary motion.

The four parameters α, β, γ and φ, specified above, are used to determine the progressive stationary motion. They are unknown and to be found by integrating the equations of motion. The initial conditions to be chosen, are given in Table 3, where $\xi'_-|_{\tau=\alpha}$ and $\xi'_+|_{\tau=\alpha}$ are derivatives calculated in the left and right vicinity of the time when $\tau = \alpha$. The above mentioned initial conditions are also in this case the boundary conditions where the solution switches between the different stages of stationary motion.

Time	Coordinate	Velocity
$\tau = 0$	$\xi = 0$	$\xi' = 0$
$\tau = \alpha$	$\xi = 0$	$\xi'_- = \xi'_+$
$\tau = \alpha + \beta$	—	$\xi' = 0$

Table 3. Boundary conditions.

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During the stop period which is between $2\pi-\gamma$ and 2π (or $-\gamma<\tau<0$), the overall drilling force should be positive ($f(\tau)>0$). If this can not be satisfied then there is no stop in the system, and the free motion of the tool (stage 1) starts immediately after the drilling (stage 2). Mathematically, the stop absence condition can be reduced to the inequality $f(2\pi)<0 \Leftrightarrow f(0)<0$. The necessary condition for the stop existence is $f(0)=0$. In addition, to have the stop it is required to fulfill $f'(0)<0$, which in practical terms means that the overall drilling force changes its sign from positive to negative at $\tau=0$. Generally, a third type of motion exists, when $f(\tau)>0$ for all values of τ, this is, the variant with the total stop. The condition for the total stop can be reduced to $f(0)>0$. Thus, there are three different types of the stationary motion, which are summarized in Table 4. Only two first types are interesting from the practical point of view. Conditions (2) specified in Table 4 were obtained from the Conditions (1) using the introduced earlier representation of the drilling force, (10). From Table 4, it can be easily deduced that drilling is possible for $b<a$, and for $b>a$ total stop occurs.

Type	Motion	γ	Conditions (1)	Conditions (2)
I	no stop	$\gamma=0$	$f(0)<0$	$\cos\varphi>b/a$
II	with stop	$\gamma>0$	$f(0)=0$, $f'(0)<0$	$\cos\varphi=b/a$, $\sin\varphi>0$
III	total stop	$\gamma=2\pi$	$f(0)>0$	$b/a>1$

Table 4. Three types of the stationary motion.

By solving the equations of motion two relations between the parameters α, β and φ of the stationary motion can be obtained

$$a\cos(\alpha-\varphi)-a\cos\varphi-a\alpha\sin\varphi+\tfrac{1}{2}b\alpha^2=0\,, \qquad (11)$$

$$a\sin(\alpha+\beta-\varphi)+a\sin\varphi-b\alpha+(1-b)\beta=0\,. \qquad (12)$$

However, to determine all unknowns, one more equation is required, and this is substantiated from the stop conditions (see Table 4)

$$\text{I.}\ \ \gamma=0; \qquad \text{II.}\ \ \cos\varphi=b/a\,. \qquad (13)$$

Identity I is used for the motion without stop (type I of the stationary motion), where identity II is appropriate for the motion with stop (variant II). Moreover, the stop conditions (see Table 4) provide two inequalities

$$\text{I.}\ \ \cos\varphi>b/a; \qquad \text{II.}\ \ \sin\varphi>0\,, \qquad (14)$$

that are needed to choose a correct solution for the phase shift, φ. Thus there are three equations, (11)–(13), with three unknown parameters α, β and φ. If the quantities α and β are known explicitly, γ can be calculated from the identity $\gamma=2\pi-(\alpha+\beta)$.

If the motion parameters α, β, γ and φ are found from (11)–(13) then the dimensionless MRR can be obtained as

$$r=\xi_*/(2\pi),\ \ \xi_*=a\cos(\gamma+\varphi)-a\cos(\alpha-\varphi)- \\ -a\beta\sin(\gamma+\varphi)+\tfrac{1}{2}(1-b)\beta^2\,. \qquad (15)$$

SMALL PARAMETERS ANALYSIS

To simplify the process of finding solution of the equations for parameters some additional assumptions are needed. From the practical point of view the most interesting case is when the drilling force is small with respect to resistive force ($a,\ b\ll 1$).

For the motion without stop, the small parameters approach provides an elegant solution for equations (11)–(13)

$$\varphi=\mathrm{Arcsin}\,(\pi b/a)\,, \qquad (16)$$

$$\alpha=2\pi\,, \qquad \beta=2\pi b\,, \qquad \gamma=0\,. \qquad (17)$$

From (14), using condition I, one can estimate a ratio range between the static force and the amplitude of the harmonic force

$$\frac{b}{a}<\frac{1}{\sqrt{1+\pi^2}}\stackrel{def}{=}s_*\approx 0.30\,. \qquad (18)$$

Thus, if there is no stop in the motion the parameters a and b satisfy the inequality (18).

For the motion with stop the small parameters approach reduces equations (11)–(13) to the following form

$$\varphi=\mathrm{Arccos}\,(b/a)\,, \qquad (19)$$

$$(\cos\alpha-1+\tfrac{1}{2}\alpha^2)/(a-\sin\alpha)=\mathrm{tg}\,\varphi\,, \qquad (20)$$

$$\beta=b\alpha-a\sin(\alpha-\varphi)-a\sin\varphi\,, \qquad (21)$$

$$\gamma=2\pi-\alpha\,. \qquad (22)$$

Equation (20) can not be solved analytically in the terms of α, but it can be easily evaluated numerically. If α is calculated from (20) then β and γ can be determined from (21) and (22).

Expanding (15) in power series in terms of the small parameters a, b and β, one can obtain the first non zero approximation

$$r=\frac{1}{4\pi}\beta^2\,, \qquad \xi_*=\tfrac{1}{2}\beta^2\,, \qquad (23)$$

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where r is the dimensionless material removal rate, ξ_* is dimensionless penetration of the drill during one period of the excitation force. For experimental verification purposes it is useful to operate with dimensional MRR, which can be calculated as

$$R = \frac{SQ}{\omega m} r. \quad (24)$$

RESULTS AND DISCUSSION

Now we examine the influences of the main drilling parameters, A and B on the dynamic system behaviour, keeping in mind, that A is amplitude of the harmonic force, B is the static force. All the results presented here are calculated for the small parameters approximation.

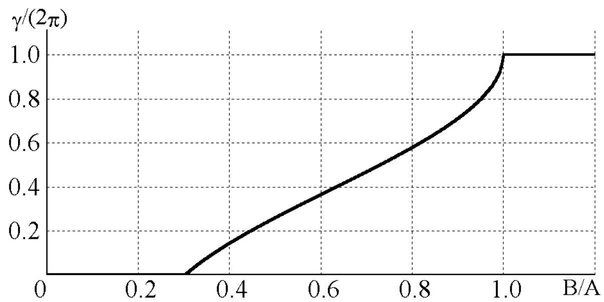

Figure 4. Dependence of the stop period $\gamma/(2\pi)$ on the drilling forces ratio B/A.

Consider dependence of the tool stop period on the parameters of the drilling force. The dimensionless period of the overall drilling force is equal to 2π, hence $\gamma/(2\pi)$ is the part of the period when the tool is in the stop mode. Parameter γ is function of the drilling forces ratio $b/a = B/A$. The functional dependence of the drilling forces ratio B/A on the relative stop period is depicted in Figure 4. From the graph it can be seen that for $B/A < s_* \approx 0.3$ there is no stop, which practically means that the mass is in motion regardless to the interactions with the slider. Then the stop period monotonically increases until $B/A \geq 1$, where total stop occurs and there is no drilling.

The phase shift φ is also a function of the drilling forces ratio B/A (see Figure 5) and monotonically increases from zero (at $B/A = 0$) up to $\varphi_{max} \approx 0.4\pi$ (at $B/A = s_*$) and then decreases down to zero (at $B/A = 1$).

The MRR is a nontrivial function of two force parameters A and B. The most interesting from the practical point of view is the dependence of the static force, B, on the MRR, while the amplitude of the harmonic force, A is kept constant (Wiercigroch et al, 1993). The relationship between the MRR and the static force B is demonstrated in Figure 6 and has clearly pronounced

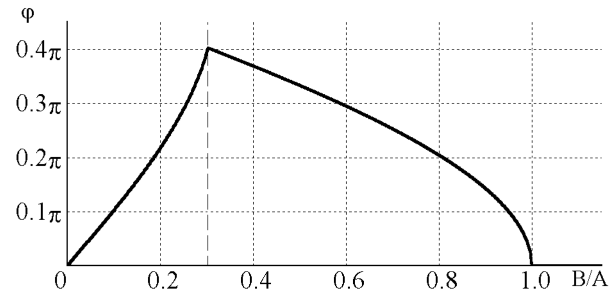

Figure 5. Dependence of the phase shift parameter φ on the drilling forces ratio B/A.

maximum as indicated earlier (Neilson et al, 1993). It can be seen that the MRR increases in the area of the motion without stop, and it reaches its maximum value in the area of the motion with stop at $B \approx 0.4A$. But note that the maximum is close to the border between these two areas. For the further increase of B/A the MRR decreases down to zero at $B = A$. The maximum value of the MRR is

$$R_{max} = \frac{SA^2}{\omega m Q} p_{max}. \quad (25)$$

where $p_{max} \approx 0.360$. The maximum value of MRR is obtained at $B/A \approx 0.387$. The shape of the curve corresponds well to the experimental results reported previously in (Markov, 1977; Wiercigroch et al, 1993).

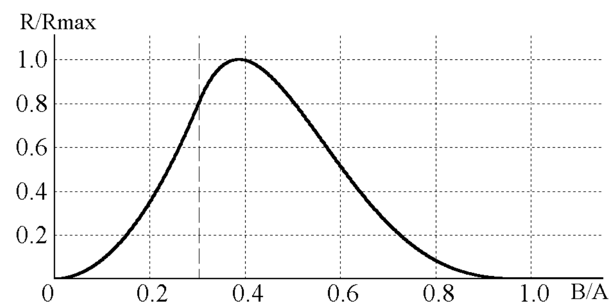

Figure 6. Dependence of the MRR on the relative static force B/A ($A = \text{const}$).

Consider now dependence of the amplitude of the harmonic force A on the MRR while the static force B is constant, see Figure 7. The MRR is equal to zero for $A \leq B$, then it increases monotonically in the area of the motion with stop ($1 < A/B < \sqrt{1+\pi^2} \approx 3.3$), and finally reaches a constant value for $A/B > \sqrt{1+\pi^2}$. Hence, practically there is no need to increase

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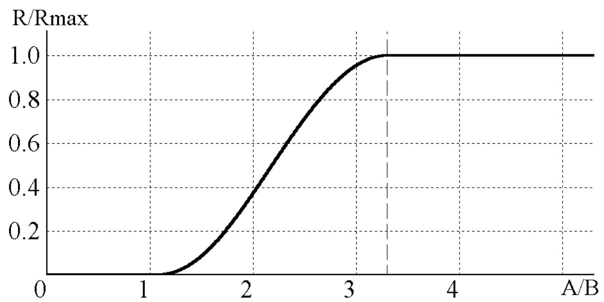

Figure 7. Dependence of the MRR on the relative amplitude of the harmonic force A/B ($B = \text{const}$).

the amplitude of harmonic force more than $3.3B$. Thus the maximum value of the MRR is

$$R_{max} = \pi \frac{SB^2}{\omega m Q}. \qquad (26)$$

CONCLUDING REMARKS

The developed model allows to calculate the material removal rate R in the form

$$R = \frac{SQ}{\omega m} r\left(\frac{A}{Q}, \frac{B}{Q}\right) = \frac{S}{\omega m Q} r(A,B), \qquad (27)$$

where S is the cross section of the drillbit, Q is the resistive force of the material, ω is the frequency of harmonic force, m is the tool mass, and A and B are the controllable parameters of the drilling force; $r = r(A,B)$ is a known function of two variables. Mathematically the MRR is a nontrivial function of two variables, amplitude of the harmonic force, A and the static force, B. Note that this formulation is valid only if the drilling force is substantially smaller than the resistive force, i. e., A and $B \ll Q$.

An investigation of the MRR function given by equation (27) provides the following conclusions.

1. Drilling action is only possible if the static force is smaller than the amplitude of the harmonic force, $B < A$. If B is greater than A the drill is stopped motionless by the static force.
2. The MRR function on the static force, B (while A is kept constant) has a well pronounced maximum, which is taken at $B \approx 0.39A$.
3. The MRR function on the amplitude of the harmonic force, A (while B is kept constant) is monotonically increasing up to some maximum, which is taken at the optimum value of the amplitude, $A \approx 3.3B$. For the amplitude A greater than this optimum value the MRR becomes constant, so any further increase of the amplitude has no effect.
4. The theoretically predicted MRR as a function of B/A corresponds well to the experimental results (Markov, 1977; Wiercigroch et al, 1993).

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