Author's typeset of the paper published in Q. Jl Mech. appl. Math. ***55(1)****: 127–140, 2002.*

On the non-existence of surface waves trapped by submerged obstructions having exterior cusp points

by O.V. MOTYGIN*

(*Department of Mathematical Sciences, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK*)

Trapped modes, i.e. localized unforced oscillations of fluid in presence of floating structures and bottom topography, have been a topic of considerable interest over many years and substantial effort has been put, for a variety of different geometries, into finding the solutions and conditions of their existence or absence. However, the class of geometries having cusp points has been avoided in the known proofs of non-existence of trapped modes, though water-wave problems for such structures have been considered and examples of trapped modes are known. In the work we consider submerged obstructions having exterior cusp points and establish absence of trapped modes for some classes of such geometries. For this purpose we derive a generalization of the so-called Maz'ya's integral identity, which in the case of geometries with cusps also includes algebraic terms containing coefficients of local asymptotics of the trapped mode potential near cusp points of the contour.

1 Introduction

In the paper we consider a three-dimensional ocean with cylindrical boundaries; examples of such geometry are a fluid layer with long canyons or ridges, long circular cylinders submerged parallel to the free surface and a plane beach. We are interested in modes of fluid oscillation which are trapped by the structure, i.e. in unforced time harmonic motions which do not radiate energy to large distances and which are spatially periodic along the generator of the cylindrical geometry.

In the context of the standard linearised surface wave theory it has been found that some cylindrical structures can support trapped modes; the origin of the theory goes back to Stokes edge waves of 1846 and the results of Ursell [1, 2] and Jones [3] in 1951–1953. Since that time a number of papers have appeared concerned with constructing trapped modes and proving their existence or establishing their non-existence in a variety of problems of the surface wave theory. We shall not discuss the results here for reasons

*Supported by Royal Society/NATO/FCO Chevening under a Poctdoctoral Fellowship Award grant.
Permanent address: Institute of Mechanical Engineering Problems, Russian Academy of Sciences, V.O., Bol'shoy pr. 61, 199178 St.Petersburg, Russia.

of space and because an extensive review of the subject can be found in [4]. It is worth noting that in many cases non-existence of trapped modes is equivalent to uniqueness for non-homogeneous problems with wave propagation to infinity.

In the present work we consider the case of submerged obstructions having exterior cusp points. Water-wave problems for this class of obstructions have been under consideration and examples of trapped modes for this class of geometry have been constructed (see [5, 6] and references therein). However, this class of obstructions is avoided in the known proofs of non-existence of trapped modes. As an exception, the paper [7] should be mentioned, where a uniqueness proof is given for the water-wave problem with a vertical shell in a layer of a constant depth.

In this paper we find some classes of obstructions with exterior cusp points which do not support trapped modes. These results are presented in Theorems 4.1, 5.1 and 5.2. First of the assertions deals with geometries having horizontal exterior cusps and guarantees absence of trapped modes if horizontal cross-section of the water domain at any depth h consists of one interval γ_h and if all the intervals γ_h, where h comes through the fluid depth, have a common point. An example of such a geometry is shown in fig. 1. Theorems 5.1 and 5.2 are devoted to the case of normal incidence of waves to obstructions having non-horizontal cusps. Formulations of the theorems involves special notations and, hence, we only mention here one particular consequence of the assertions: one flat barrier adjoint and inclined at any angle to a horizontal bottom cannot support trapped modes. All the results are valid for any frequency of harmonic oscillations of the fluid.

In order to prove the theorems we apply a generalization of the integral identity, which was suggested by Maz'ya in [8, 9] (in a weaker form in [10]). The identity involves an arbitrary vector field and an arbitrary function and states that a quadratic form in the potential of trapped mode is equal to zero. Thus, the potential is equal to zero if the function and the vector field in the domain containing fluid can be chosen such that the quadratic form is non-negative, or, in other words, the vector field nowhere enters the obstruction. Interesting analysis of the scheme can be found in [4, 11, 12, 13], where some interpretations and generalizations of the Maz'ya identity are given. For the geometries considered in the present paper the identity also includes coefficients of local asymptotics of the potential near the cusp points of the contour. The local asymptotics derived in the present work can also be used to generalize other known proofs based on integral identities, e.g. the scheme applied by Simon and Ursell in [14].

Now, we give a brief exposition of the paper. In § 2 we introduce notations and present the mathematical problem for trapped modes. Maz'ya's identity is presented in § 3. The identity is generalized for the case of contours with horizontal cusp points in § 4, where, thus, a class of contours, for which the identity guarantees non-existence of trapped modes, is described. Some classes of geometries having non-horizontal cusp points and not supporting trapped modes are found in § 5. The local asymptotics of trapped mode potential near a corner point of the contour is derived in Appendix A.

Author's typeset of the paper published in Q. Jl Mech. appl. Math. ***55(1)****: 127–140, 2002.*

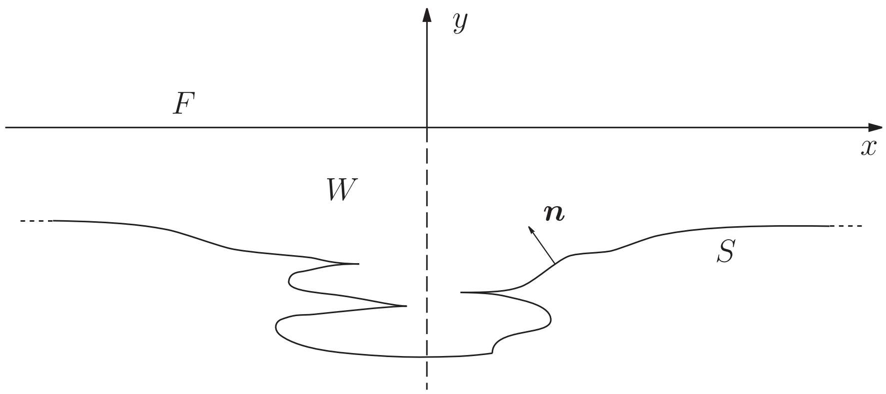

Figure 1: A sketch of geometric notation

2 Statement of the problem

We suppose that the fluid occupies a cylindrical domain of the form $W \times \mathbb{R}$, where W denotes an unbounded open set in $\mathbb{R}^2$. We decompose the boundary of W as $F \cup S$, where F is the mean free surface and S is the wetted part of submerged obstruction including the bottom and some submerged obstacles. The surface S is assumed to be piecewise smooth and is allowed to have corner and cusp points, so that any finite part of the contour contains only a finite number of corner points.

Let t be a time variable and a Cartesian coordinate system (x, y, z) be attached to the free surface of the fluid so that y is the vertical coordinate decreasing with depth and equal to zero in the free surface and x, z are horizontal coordinates, such that the geometry of obstruction is constant in z. The notation is illustrated in fig. 1, where the case of constant-depth ocean with a deepening is shown.

The fluid is assumed to be ideal incompressible. Under these assumptions its irrotational unforced motion can be described in frame of the linearised surface wave theory (see e.g. [15, ch. 3]) by a velocity potential $U(x, y, z, t)$ satisfying the equations

$$\left(\partial_x^2 + \partial_y^2 + \partial_z^2\right) U = 0 \quad \text{in} \quad W \times \mathbb{R}, \tag{2.1}$$

$$\partial_t^2 U + g\partial_y U = 0 \quad \text{on} \quad F \times \mathbb{R}, \tag{2.2}$$

$$\partial_n U = 0 \quad \text{on} \quad S \times \mathbb{R}, \tag{2.3}$$

where we use the notation ∂_a for the partial derivative in variable a, g is the acceleration due to gravity and $\boldsymbol{n}$ is the unit normal vector directed into the domain W.

In the present note we are interested in trapped modes, i.e. solutions of the form

$$U(x, y, z, t) = \mathrm{Re}\left\{u(x, y)\mathrm{e}^{\mathrm{i}(kz-\omega t)}\right\}, \tag{2.4}$$

where k and ω are real numbers and $u(x, y)$ is a real-valued function submitted to the 'localization property' i.e. the condition of finiteness of energy over the cross-section of

Author's typeset of the paper published in Q. Jl Mech. appl. Math. ***55(1)****: 127–140, 2002.*

the fluid domain:

$$\int_W \left(|\nabla u|^2 + \nu^2 u^2\right) \mathrm{d}x\, \mathrm{d}y < \infty. \tag{2.5}$$

A solution to (2.1)–(2.5) corresponds to a harmonic wave propagating in the z-direction without distortion and having finite transverse energy.

It follows by substituting (2.4) into (2.1)–(2.3), that the potential u, which is defined on W, satisfies the following boundary value problem

$$\nabla^2 u - k^2 u = 0 \quad \text{in} \quad W, \tag{2.6}$$

$$\partial_y u - \nu u = 0 \quad \text{on} \quad F, \tag{2.7}$$

$$\partial_n u = 0 \quad \text{on} \quad S, \tag{2.8}$$

where $\nabla = (\partial_x, \partial_y)$ and $\nu = \omega^2/g$. We mean a solution of (2.5)–(2.8) in the classical sense, so that $u \in \mathrm{C}^2(W) \cap \mathrm{C}(\overline{W})$ and the potential u has regular normal derivative at all regular points of contours where the normal is well-defined.

The problem (2.6)–(2.8) also arises when considering surface waves in an infinitely long channel with vertical walls $z = \pm b$ spanned by a horizontal cylindrical obstruction. In this case the velocity potential can be fixed in the form $U(x, y, z, t) = \mathrm{Re}\{u(x, y)\mathrm{e}^{\mathrm{i}\omega t}\} \cos kz$, where $kb = n\pi$, $(n = 1, 2, \ldots)$, so as to satisfy (2.8) on the walls. A similar solution with a sine dependence on z can be taken provided $kb = (2n - 1)\pi/2$, $(n = 1, 2, \ldots)$.

It is of note that non-existence of the trapped modes is closely related to the uniqueness in problems describing radiation and diffraction phenomena. Let, for example, the domain W be a layer of fluid (see fig. 1), having a constant depth $h_\pm$ for $\pm x > \pm a_\pm$. Consider the problem of water waves radiated on the depth profile and having wavenumber ν and whose crests make an angle θ with the plane Oxy, so that $k = \nu \sin\theta$ in (2.4). The problem for the potential u consists of (2.6), (2.7), the non-homogeneous Neumann condition $\partial_n u = f$ on S and the condition at infinity $\partial_x u \mp \mathrm{i}\ell_\pm u \to 0$ as $x \to \pm\infty$, where $\ell_\pm = \left(\lambda_\pm^2 - k^2\right)^{\frac{1}{2}}$ and $\lambda_\pm$ is the unique, real, positive root of the equation $\nu = \lambda_\pm \tanh \lambda_\pm h_\pm$. Since the contours are assumed to have corner and cusp points we also demand $u \in H^1_{\mathrm{loc}}(W)$. Then, following the scheme of [10, § 2] or [11, § 2] and taking into account the asymptotics at the cusp points in the way we use in § 4 of this work, it can be shown that the difference of two solutions to the problem in question satisfies the condition (2.5) and, thus, represents a trapped mode.

3 Maz'ya's integral identity

In this section we present a version of the integral identity known as Maz'ya's one. In order to obtain the variant of the identity we follow the derivation of [4, § 4.2], where the identity was applied for the case $k \neq 0$ in the condition (2.6), and the scheme of [10] to make the identity applicable for obstacles having outlets to infinity. We consider a

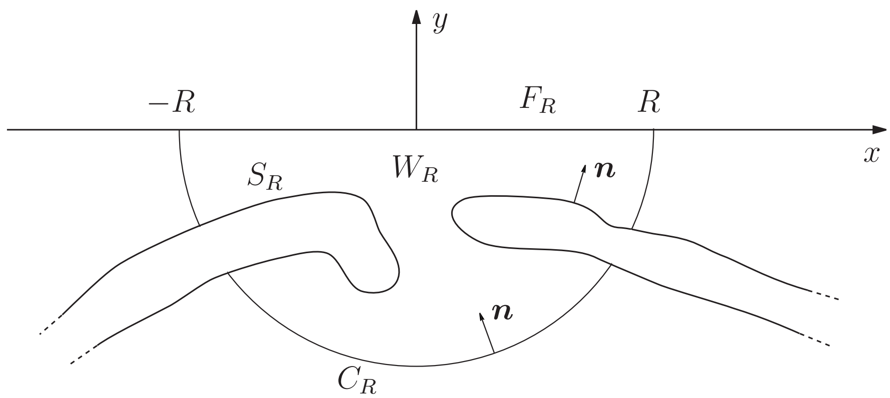

Figure 2: Auxiliary geometrical notation

real vector field $\boldsymbol{V} = (V_1, V_2)$ and a real function H defined on $\overline{W}$. We also assume that $V_i \in \mathrm{C}^1\big(\overline{W}\big)$, $H \in \mathrm{C}^2\big(\overline{W}\big)$ and start with the identity

$$\begin{aligned} 2\big(\boldsymbol{V}\cdot\nabla u + Hu\big)\nabla^2 u =& 2\,\nabla\cdot\big\{\big(\boldsymbol{V}\cdot\nabla u + Hu\big)\nabla u\big\} + (\boldsymbol{Q}\,\nabla u)\cdot\nabla u \\ &- \nabla\cdot\big\{|\nabla u|^2\boldsymbol{V} + u^2\nabla H\big\} + u^2\nabla^2 H, \end{aligned} \tag{3.1}$$

where the elements of the matrix $\boldsymbol{Q}$ are defined as follows:

$$\boldsymbol{Q}_{ij} = \big(\nabla\cdot\boldsymbol{V} - 2H\big)\delta_{ij} - \big(\partial_{x_j}V_i + \partial_{x_i}V_j\big), \quad i,j=1,2, \quad x_1 = x, \quad x_2 = y, \tag{3.2}$$

and δ_{ij} is the Kroneker delta.

Further we shall integrate the identity (3.1) over W_R, where W_R is a domain bounded internally by the obstruction D with the wetted surface $S = \partial D$, externally by a part of semicircle $C_R = \{|x+\mathrm{i}y| = R,\ y \leqslant 0\} \setminus D$ and by a part of the free surface F_R from above (see fig. 2). We also use the notation $S_R = S \cap \{(x,y) : |x+\mathrm{i}y| \leqslant R,\ y \leqslant 0\}$. Then, taking into account the condition (2.6) we have

$$\begin{aligned} 0 = &\int_{W_R} \big\{(\boldsymbol{Q}\,\nabla u)\cdot\nabla u - 2k^2 u\,(\boldsymbol{V}\cdot\nabla u + Hu) + u^2\nabla^2 H\big\}\,\mathrm{d}x\,\mathrm{d}y \\ &+ \int_{C_R\cup S_R} \big\{|\nabla u|^2\,\boldsymbol{V}\cdot\boldsymbol{n} + u^2\partial_n H - 2\partial_n u\,(\boldsymbol{V}\cdot\nabla u + Hu)\big\}\,\mathrm{d}s \\ &+ \int_{F_R} \big\{2\partial_y u\,(\boldsymbol{V}\cdot\nabla u + Hu) - |\nabla u|^2\,V_2 - u^2\partial_y H\big\}\,\mathrm{d}x. \end{aligned} \tag{3.3}$$

It is to note at this point that though the potential u is continuous over the surface of obstruction, its derivatives can be unbounded at corners. From results of Appendix A it follows that the potential is $O(\rho^{\pi/\alpha})$ and its derivatives are $O(\rho^{\pi/\alpha-1})$ as $\rho\to 0$, where ρ denotes the radial distance from the corner of angle α, the angle is measured through the fluid domain. Under the assumption that $\alpha\in(0,2\pi)$ for any corner point of S, the latter estimates are sufficient to ensure the existence of integrals in (3.3) and those appearing subsequently in this section.

Using the condition (2.7) we write

$$\int_{F_R} \partial_y u \left[V_1 \partial_x u + V_2 \partial_y u + H u \right] \mathrm{d}x = \nu \int_{F_R} \left[u^2 \left(\nu V_2 + H \right) + u\, V_1\, \partial_x u \right] \mathrm{d}x, \tag{3.4}$$

where

$$2 \int_{F_R} u\, V_1\, \partial_x u \,\mathrm{d}x = \int_{F_R} V_1\, \partial_x \left(u^2 \right) \mathrm{d}x = \left[u(x,0) \right]^2 V_1(x,0) \Big|_{x=-R}^{x=R} - \int_{F_R} u^2 \partial_x V_1 \,\mathrm{d}x. \tag{3.5}$$

Also, we have

$$\begin{aligned} 2 \int_{W_R} u\, \boldsymbol{V} \cdot \nabla u \,\mathrm{d}x\,\mathrm{d}y &= \int_{W_R} \boldsymbol{V} \cdot \nabla \left(u^2 \right) \mathrm{d}x\,\mathrm{d}y \\ &= - \int_{W_R} u^2\, \nabla \cdot \boldsymbol{V} \,\mathrm{d}x\,\mathrm{d}y + \int_{F_R} u^2\, V_2 \,\mathrm{d}x - \int_{C_R \cup S_R} u^2\, \boldsymbol{V} \cdot \boldsymbol{n} \,\mathrm{d}s \end{aligned} \tag{3.6}$$

Using the formulas (3.4)–(3.6) to transform (3.3) we find

$$\begin{aligned} &\int_{F_R} \Big\{ u^2 \left[2\nu^2 V_2 + 2\nu H - \nu \partial_x V_1 - k^2 V_2 - \partial_y H \right] - |\nabla u|^2 V_2 \Big\} \mathrm{d}x \\ &+ \int_{S_R} \Big\{ \boldsymbol{V} \cdot \boldsymbol{n} \big(|\nabla u|^2 + k^2 u^2 \big) + u^2 \partial_n H - 2 \partial_n u \left(\boldsymbol{V} \cdot \nabla u + H u \right) \Big\} \mathrm{d}s \\ &+ \int_{W_R} \Big\{ (\boldsymbol{Q}\, \nabla u) \cdot \nabla u + u^2 \big[k^2 \big(\nabla \cdot \boldsymbol{V} - 2H \big) + \nabla^2 H \big] \Big\} \mathrm{d}x\,\mathrm{d}y = \alpha(R; u, \boldsymbol{V}, H), \end{aligned} \tag{3.7}$$

where

$$\begin{aligned} \alpha(R; u, \boldsymbol{V}, H) = &\int_{C_R} \Big\{ 2 \partial_n u \left(\boldsymbol{V} \cdot \nabla u + H u \right) - \boldsymbol{V} \cdot \boldsymbol{n} \big(|\nabla u|^2 + k^2 u^2 \big) - u^2 \partial_n H \Big\} \mathrm{d}s \\ &+ \nu \left[u(-R,0) \right]^2 V_1(-R,0) - \nu \left[u(R,0) \right]^2 V_1(R,0). \end{aligned} \tag{3.8}$$

Asymptotics of the term α for large R is established in the following assertion.

Proposition 3.1. *Let u be a solution to the problem (2.5)–(2.8) and let the functions V_i, H and $|\nabla H|$ have estimate $O(R)$ as $R \to \infty$ uniform in θ, where $R\mathrm{e}^{\mathrm{i}\theta} = x + \mathrm{i}y$. Then,*

$$\liminf_{R \to \infty} \alpha(R; u, \boldsymbol{V}, H) = 0. \tag{3.9}$$

Proof. By the assumption on finiteness of energy we have

$$\int_{W \setminus W_R} \big(|\nabla u|^2 + \nu^2 u^2 \big) \mathrm{d}x\,\mathrm{d}y = \int_R^\infty \mathrm{d}\rho \int_{C_\rho} \big(|\nabla u|^2 + \nu^2 u^2 \big) \mathrm{d}s < \infty.$$

From the convergence of the integral over (R, ∞) in the last formula it obviously follows that there exists a sequence $\{R_n\}$, such that $R_n \to \infty$ as $n \to \infty$ and,

$$R_n \int_{C_{R_n}} \left(|\nabla u|^2 + \nu^2 u^2\right) \mathrm{d}s \to 0 \quad \text{as} \quad n \to \infty. \tag{3.10}$$

Further we consider the integral identity

$$\int_{W_{R_n}} u \nabla^2 u \,\mathrm{d}x\,\mathrm{d}y = -\int_{W_{R_n}} |\nabla u|^2 \,\mathrm{d}x\,\mathrm{d}y + \int_{F_{R_n}} u\,\partial_y u \,\mathrm{d}x - \int_{C_{R_n} \cup S_{R_n}} u\,\partial_n u \,\mathrm{d}s,$$

which can be rewritten with the help of (2.6)–(2.8) as follows

$$\int_{W_{R_n}} \left(|\nabla u|^2 + k^2 u^2\right) \mathrm{d}x\,\mathrm{d}y + \int_{C_{R_n}} u\,\partial_n u \,\mathrm{d}s = \nu \int_{F_{R_n}} u^2 \,\mathrm{d}x. \tag{3.11}$$

Using the Cauchy-Buniakowsky inequality and the formula (3.10) it can be shown that the second integral in (3.11) tends to zero as $n \to \infty$. Thus, from the condition (2.5) it follows that $\int_{F_{R_n}} u^2 \,\mathrm{d}x$ has a finite limit as $n \to \infty$. Since $\int_{F_R} u^2 \,\mathrm{d}x$ is monotonic in R, the function $\int_{F_R} u^2 \,\mathrm{d}x$ has the same limit as $R \to \infty$, so that $\int_F u^2 \,\mathrm{d}x < \infty$ and we write

$$\int_{W \setminus W_R} \left(|\nabla u|^2 + \nu^2 u^2\right) \mathrm{d}x\,\mathrm{d}y + \nu \int_{F \setminus F_R} u^2 \,\mathrm{d}x \\ = \int_R^\infty \mathrm{d}\rho \left\{\nu\big[u(\rho, 0)\big]^2 + \nu\big[u(-\rho, 0)\big]^2 + \int_{C_\rho} \left(|\nabla u|^2 + \nu^2 u^2\right) \mathrm{d}s\right\} < \infty,$$

and convergence of the integral implies that

$$\liminf_{\rho \to \infty} \rho \left\{\nu\big[u(\rho, 0)\big]^2 + \nu\big[u(-\rho, 0)\big]^2 + \int_{C_\rho} \left(|\nabla u|^2 + \nu^2 u^2\right) \mathrm{d}s\right\} = 0.$$

Under the assumption on behaviour of V_i, H and $|\nabla H|$ at infinity, the latter formula obviously substantiates the formula (3.9). □

4 Non-existence of modes trapped by obstructions with non-overlapping horizontal exterior cusps

We note that under the particular choice $V = (-x, 0)$, $H = -1/2$, many of the terms in the formula (3.7) disappear and we arrive at the identity (cf. (18.1) in [10]):

$$2 \int_{W_R} |\partial_x u|^2 \,\mathrm{d}x\,\mathrm{d}y - \int_{S_R} x n_x \left(|\nabla u|^2 + k^2 u^2\right) \mathrm{d}s + \int_{S_R} \partial_n u \big(2x \partial_x u + u\big) \,\mathrm{d}s = \alpha(R; u), \tag{4.1}$$

where $\boldsymbol{n} = (n_x, n_y)$ and the function α is given in (3.8).

In the section we shall assume that $xn_x \leqslant 0$ at all regular points of S. Consider first the case when the contours do not contain cusp points. The first two integrals in (4.1) are monotonic in R, the third integral vanishes in view of (2.8), and, thus, from (3.9) it follows that $\lim_{R\to\infty} \alpha(R; u) = 0$ and we arrive at

$$\int_S xn_x\big(|\nabla u|^2 + k^2u^2\big)\,\mathrm{d}s = 2\int_W |\partial_x u|^2\,\mathrm{d}x\,\mathrm{d}y. \tag{4.2}$$

Since $xn_x \leqslant 0$ on S, the identity (4.2) obviously leads to the conclusion that $\partial_x u \equiv 0$ in W and, then, in view of (2.5) we have $u \equiv 0$ in W. The condition $xn_x \leqslant 0$ can be satisfied e.g. for deepening (not raising) of the depth profile and for a system of non-overlapping bodies of semi-infinite extent (docks). The above assertion was proven in [10] and in this section we shall derive its natural generalization for the case of exterior cusps with horizontal one-side tangents.

In the case of contours with cusp points we cannot apply the identity (4.1) directly. First we shall consider the identity for the domain W with small vicinities of cusp points excluded and then shrink their size to zero. We denote by $P_i^+ = (x_i^+, y_i^+)$, $i = 1,\dots,N_+$, and $P_j^- = (x_j^-, y_j^-)$, $j = 1,\dots,N_-$ the cusp points turning to the right and to the left resp. The numeration of the points is e.g. as follows

$$x_1^+ \leqslant \ldots \leqslant x_{N_+}^+, \quad x_1^- \leqslant \ldots \leqslant x_{N_-}^-.$$

We introduce discs of radius ε with centres at the cusp points

$$B_i^\pm(\varepsilon) = \big\{(x,y) : \big|x - x_i^\pm + \mathrm{i}\,\big(y - y_i^\pm\big)\big| \leqslant \varepsilon\big\}, \quad i = 1,\dots,N_\pm,$$

and denote by $W(\varepsilon)$ the fluid domain with ε-vicinities of the cusp points excluded and by $S(\varepsilon)$ the wetted surface of the bodies with ε-vicinities of the cusp points added

$$W(\varepsilon) = W \setminus \bigcup\nolimits_\pm \bigcup\nolimits_{i=1}^{N_\pm} B_i^\pm(\varepsilon), \quad S(\varepsilon) = \partial W(\varepsilon) \setminus F.$$

Following the arguments we used to obtain (4.2) from (4.1), we arrive at the identity

$$2\int_{W(\varepsilon)} |\partial_x u|^2\,\mathrm{d}x\,\mathrm{d}y - \int_{S(\varepsilon)} xn_x\big(|\nabla u|^2 + k^2u^2\big)\,\mathrm{d}s \\ + \sum_\pm \sum_{i=1}^{N_\pm} \int_{S_i^\pm(\varepsilon)} \partial_n u\big(2x\,\partial_x u + u\big)\,\mathrm{d}s = 0, \tag{4.3}$$

where $S_i^\pm(\varepsilon) = \partial B_i^\pm(\varepsilon) \cap W$ (see fig. 3). Further we shall make use of the asymptotic representation (A.5) to compute asymptotics as $\varepsilon \to 0$ of the integrals over contours $S_i^\pm(\varepsilon)$ in the latter formula. Omitting the index i for brevity we define the polar system of coordinates with origin at the point $P_i^\pm$

$$x = x^\pm \mp \rho^\pm \cos\theta^\pm, \quad y = y^\pm - \rho^\pm \sin\theta^\pm,$$

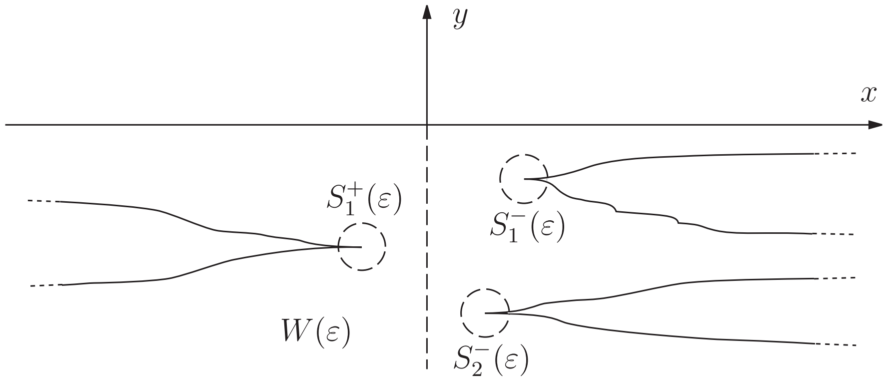

Figure 3: Auxiliary geometrical notation

so that the values $\theta^\pm = 0$ and $\theta^\pm = 2\pi$ correspond to the lower and the upper sides of the cusp as $\rho^\pm \to 0$.

We use (A.1) and (A.5) and write

$$\begin{aligned} u(\rho^\pm, \theta^\pm) &= u(P^\pm) + c^\pm \left[\rho^\pm\right]^{1/2} \cos(\theta^\pm/2) + O\big(\left[\rho^\pm\right]^{3/2}\big), \\ \partial_x u(\rho^\pm, \theta^\pm) &= \mp 2^{-1} c^\pm \left[\rho^\pm\right]^{-1/2} \cos(\theta^\pm/2) + O\big(\left[\rho^\pm\right]^{1/2}\big), \\ \partial_y u(\rho^\pm, \theta^\pm) &= -2^{-1} c^\pm \left[\rho^\pm\right]^{-1/2} \sin(\theta^\pm/2) + O\big(\left[\rho^\pm\right]^{1/2}\big). \end{aligned} \tag{4.4}$$

By (4.4) we have

$$\begin{aligned} &\int_{S^\pm(\varepsilon)} \partial_n u \big(2x\partial_x u + u\big)\, \mathrm{d}s \xrightarrow[\varepsilon\to 0]{} \mp \frac{x^\pm\, (c^\pm)^2}{2} \int_0^{2\pi} \cos^2 \frac{\theta^\pm}{2}\, \mathrm{d}\theta^\pm = \mp \frac{\pi x^\pm\, (c^\pm)^2}{2}, \\ &\int_{S^\pm(\varepsilon)} x n_x \big(|\nabla u|^2 + k^2 u^2\big)\, \mathrm{d}s \xrightarrow[\varepsilon\to 0]{} \frac{x^\pm\, (c^\pm)^2}{4} \int_0^{2\pi} \cos\theta^\pm\, \mathrm{d}\theta^\pm = 0. \end{aligned} \tag{4.5}$$

Finally, combining (4.3), (4.5) and taking the limit $\varepsilon \to 0$ we arrive at the following generalization of Maz'ya's identity involving coefficients of singularities of the velocity field:

$$\frac{\pi}{2} \sum_{\pm} \sum_{i=1}^{N_\pm} \pm x_i^\pm \left(c_i^\pm\right)^2 = \int_W |\partial_x u|^2\, \mathrm{d}x\, \mathrm{d}y - \int_S x n_x \big(|\nabla u|^2 + k^2 u^2\big)\, \mathrm{d}s. \tag{4.6}$$

We apply the equality (4.6) to prove the following assertion.

Theorem 4.1. *Consider the problem (2.5)–(2.8) for a submerged obstruction with a piece-wise smooth wetted surface S having a finite number of horizontal cusp points. Let for any horizontal line $\gamma_h = \{y = h\}$, where $h < 0$ and $\gamma_h \cap W \neq \emptyset$, the set $\gamma_h \cap W$ consist of only one segment and all the intervals γ_h contain a common point. Then the problem (2.5)–(2.8) for the geometry S has only the trivial solution.*

Proof. We fix the origin of coordinate system in such a way that the common point of the intervals γ_h corresponds to $y = 0$. From the assumptions imposed it follows that $xn_x \leqslant 0$ at all regular points of the contour S, the latter was preposed in derivation of the identity (4.6). Besides, under the assumptions of the assertion, we have $\pm x_i^{\pm} \leqslant 0$. Hence, from (4.6) we find

$$2\int_W |\partial_x u|^2 \,\mathrm{d}x\,\mathrm{d}y - \int_S xn_x\big(|\nabla u|^2 + k^2u^2\big)\,\mathrm{d}s \leqslant 0. \tag{4.7}$$

From the last inequality in view of (2.5) it obviously follows that $u \equiv 0$ in W. □

Two important cases of the geometry, for which Theorem 4.1 yields non-existence of trapped modes are the case of deepening of bottom (see e.g. fig. 1), when Theorem 4.1 generalizes results of Vainberg and Maz'ya [10], and the case when the submerged obstruction is a system of non-overlapping horizontal semi-infinite barriers. Consideration of the problem for semi-infinite barriers can be found in [5] (see also references therein).

Let the potential u satisfy the following general form of the Helmholtz equation

$$\nabla^2 u + \lambda u = 0 \quad \text{in} \quad W. \tag{4.8}$$

For the problem with the equation (4.8) we can repeat literally the arguments of the Sections 3 and 4 leading to the inequality (4.7) where the term k^2 should be replaced by $-\lambda$. Since the second integral in (4.7) vanishes if horizontal barriers are considered, Theorem 4.1 for this geometry is valid for the problem consisting of the conditions (2.5), (2.7), (2.8) and (4.8).

5 Application of the scheme for geometries with non-horizontal cusps

In the present section we shall suggest applications of the scheme of the previous section for the case of obstructions having non-horizontal cusps. For this purpose we use $H = -1/2$ and the vector field $\boldsymbol{V}_\alpha(x, y) = (-x, -\alpha y)$. With the vector field we should assume that $k = 0$ in (2.6), so that the equation of fluid motion is Laplace's one, which appears to describe normal incidence of waves to a cylindrical obstruction. The assumption $k = 0$ is needed to cancel the second term in the integrals over W_R in the identity (3.7).

The vector field $\boldsymbol{V}_\alpha$ is tangential to the field lines described (except the line $x = 0$) by the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \alpha\frac{y}{x},$$

which has the following family of solutions (where c is a parameter)

$$y(x) = c\,|x|^\alpha. \tag{5.1}$$

Author's typeset of the paper published in Q. Jl Mech. appl. Math. ***55(1)****: 127–140, 2002.*

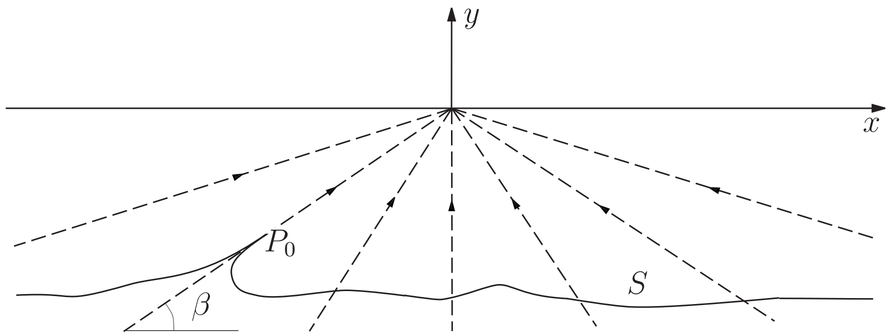

Figure 4: Bottom topography S with one inclined cusp point P_0.

The parameter α must belong to the interval $[0, 2]$, which follows from the condition of the positivity of the matrix $\boldsymbol{Q}$ defined by (3.2).

Consider first the case shown in fig. 4, when the bottom topography S has one cusp point $P_0 = (x_0, y_0)$ and the one-side tangent to S at the point forms an angle β with the x-axis. Without a loss of generality we shall assume the cusp to be turned to the right. We suppose that $\boldsymbol{V}_\alpha \cdot \boldsymbol{n} \geqslant 0$ at all regular points of S (we remind that it means that the vector field nowhere enters the wetted surface of the obstruction). The latter implies that one-side tangent to S at the cusp point should be tangential to one of the field lines as shown in fig. 4. This is easily achieved by a shift of the origin in x, then $x_0 < 0$ and by (5.1) we have $x_0 \tan\beta = \alpha y_0$.

Since by assumption $\boldsymbol{V}_\alpha \cdot \boldsymbol{n} \geqslant 0$, we can follow the scheme of the previous section and write (cf. (4.3))

$$\begin{aligned}(2-\alpha)\int_{W(\varepsilon)} |\partial_x u|^2 \,\mathrm{d}x\,\mathrm{d}y + \alpha \int_{W(\varepsilon)} |\partial_y u|^2 \,\mathrm{d}x\,\mathrm{d}y \\ + \int_{S(\varepsilon)} \boldsymbol{V}_\alpha \cdot \boldsymbol{n}\, |\nabla u|^2 \,\mathrm{d}s + \int_{\partial B(\varepsilon)} \partial_n u\big(u - 2\,\boldsymbol{V}_\alpha \cdot \nabla u\big)\,\mathrm{d}s = 0,\end{aligned} \tag{5.2}$$

where $B(\varepsilon) = \{(x, y) : |x - x_0 + \mathrm{i}(y - y_0)| \leqslant \varepsilon\}$, $W(\varepsilon) = W \setminus B(\varepsilon)$ and $S(\varepsilon) = \partial W(\varepsilon) \setminus F$.

We introduce the polar coordinates (ρ, θ) with origin at P_0 defined as follows

$$x = x_0 - \rho\cos(\theta + \beta), \quad y = y_0 - \rho\sin(\theta + \beta),$$

so that the values $\theta = 0$ and $\theta = 2\pi$ correspond to the lower and the upper sides of the cusp as $\rho \to 0$.

By (A.1) and (A.5) we have

$$\begin{aligned}u(\rho, \theta) &= u(P_0) + c_0\rho^{1/2}\cos(\theta/2) + O\big(\rho^{3/2}\big), \\ \partial_x u(\rho, \theta) &= -2^{-1}c_0\rho^{-1/2}\cos(\theta/2 + \beta) + O\big(\rho^{1/2}\big), \\ \partial_y u(\rho, \theta) &= -2^{-1}c_0\rho^{-1/2}\sin(\theta/2 + \beta) + O\big(\rho^{1/2}\big).\end{aligned} \tag{5.3}$$

Using the asymptotics (5.3) we can find limit of the integrals over $\partial B(\varepsilon)$ in the formula (5.2) as $\varepsilon \to 0$. We have

$$\int_{\partial B(\varepsilon)} \partial_n u \big(u - 2\,\boldsymbol{V}_\alpha \cdot \nabla u\big)\,\mathrm{d}s \xrightarrow[\varepsilon\to 0]{} -\frac{c_0^2}{2}\Big\{x_0 \int_0^{2\pi} \cos\frac{\theta}{2}\,\cos\Big(\frac{\theta}{2}+\beta\Big)\,\mathrm{d}\theta + \alpha y_0 \int_0^{2\pi} \cos\frac{\theta}{2}\,\sin\Big(\frac{\theta}{2}+\beta\Big)\,\mathrm{d}\theta\Big\} = -\frac{\pi c_0^2}{2}\Big\{x_0\cos\beta + \alpha y_0 \sin\beta\Big\}, \tag{5.4}$$

$$\int_{\partial B(\varepsilon)} \boldsymbol{V}_\alpha \cdot \boldsymbol{n}\,|\nabla u|^2\,\mathrm{d}s \xrightarrow[\varepsilon\to 0]{} -\frac{c_0^2}{4}\Big[x_0 \int_0^{2\pi} \cos(\theta+\beta)\,\mathrm{d}\theta + \alpha y_0 \int_0^{2\pi} \sin(\theta+\beta)\,\mathrm{d}\theta\Big] = 0.$$

Finally, combining (5.2), (5.4) and taking the limit $\varepsilon \to 0$ we arrive at

$$\frac{\pi c_0^2}{2}\Big\{x_0\cos\beta + \alpha y_0\sin\beta\Big\} = (2-\alpha)\int_W |\partial_x u|^2\,\mathrm{d}x\,\mathrm{d}y + \alpha \int_W |\partial_y u|^2\,\mathrm{d}x\,\mathrm{d}y + \int_S \boldsymbol{V}_\alpha\cdot\boldsymbol{n}|\nabla u|^2\,\mathrm{d}s. \tag{5.5}$$

From (5.5) by the same scheme as in Theorem 4.1 we prove the following assertion.

Theorem 5.1. *Let u be a solution to the problem (2.5)–(2.8), where $k = 0$ and the geometry S is a depth profile containing one cusp point P_0 turned upwards. Let the origin of the coordinate system (x, y) be chosen in such a way that the vector $\boldsymbol{V}_\alpha(P_0)$ is colinear to the one-side tangent to S at P_0. If $\boldsymbol{V}_\alpha \cdot \boldsymbol{n} \geqslant 0$ at all regular point of S, then $u \equiv 0$.*

The simplest example is the case $\alpha = 1$, when the field lines (5.1) of $\boldsymbol{V}_\alpha$ are straight lines coming to the origin. Some of the lines are shown in fig. 4, where a geometry of bottom is given, for which Theorem (5.1) guarantees non-existence of trapped modes for all frequencies. An important particular application of the latter assertion with $\boldsymbol{V}_1$ is as follows

Proposition 5.1. *A flat barrier adjoint to a horizontal bottom, not piercing the free surface and inclined at any angle cannot support trapped modes described by the problem (2.5)–(2.8), where $k = 0$.*

The field $\boldsymbol{V}_\alpha$ and the above scheme can also be applied to establish absence of trapped modes for bottom topographies with more than one cusp point turned upwards. As in § 4 we denote by $P_i^+ = (x_i^+, y_i^+)$, $i = 1, \dots, N_+$, and $P_j^- = (x_j^-, y_j^-)$, $j = 1, \dots, N_-$ the cusp points turning to the right and to the left resp. Let $\beta_i^\pm$ be the angle between the x-axis and the one-side tangent to S at $P_i^\pm$. Following the scheme we used to obtain (5.5), we arrive at

$$\frac{\pi}{2}\sum_{\pm}\sum_{i=1}^{N_\pm}\Big\{x_i^\pm\cos\beta_i^\pm + \alpha y_i^\pm \sin\beta_i^\pm\Big\}\big(c_i^\pm\big)^2 = (2-\alpha)\int_W |\partial_x u|^2\,\mathrm{d}x\,\mathrm{d}y + \alpha\int_W |\partial_y u|^2\,\mathrm{d}x\,\mathrm{d}y + \int_S \boldsymbol{V}_\alpha\cdot\boldsymbol{n}\,|\nabla u|^2\,\mathrm{d}s, \tag{5.6}$$

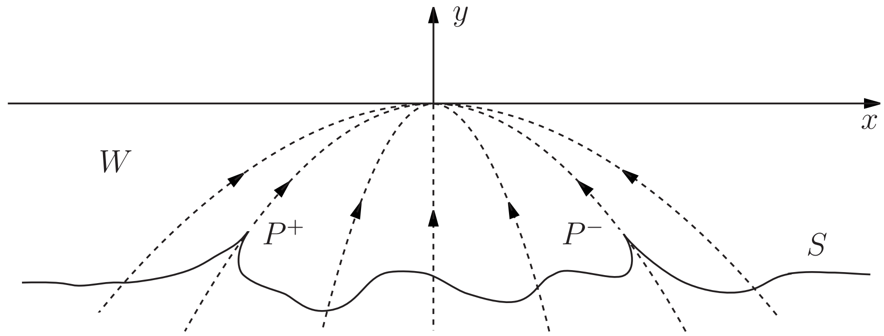

Figure 5: Bottom topography S with two symmetric inclined cusp point P^- and P^+.

where $c_i^\pm$ are coefficients in local asymptotics of the potential at points $P_i^\pm$ (see (4.4)).

It can easily be seen that the condition $\boldsymbol{V}_\alpha \cdot \boldsymbol{n} \geqslant 0$ at all regular point of S implies that the cusps are turned to the y-axis, i.e. $\pm x_i^\pm \leqslant 0$ and $x_i^\pm \cos\beta_i^\pm \leqslant 0$. Thus, by (5.6) we arrive at the following assertion.

Theorem 5.2. *Consider a piecewise smooth depth profile S with a number of cusp points. If $\boldsymbol{V}_\alpha \cdot \boldsymbol{n} \geqslant 0$ at all regular points of S, then the problem (2.5)–(2.8) with $k = 0$ has only the trivial solution.*

The latter assertion is illustrated in fig. 5, where a configuration with two cusp points is shown along with lines of the vector fields $\boldsymbol{V}_2$ guaranteeing non-existence of trapped modes for the geometry.

It is of note that unlike Theorem 4.1 for the case of one inclined cusp point, the latter assertion imposes restriction on the cusps inclination and location for which a value of the parameter α can be found to satisfy the condition $\boldsymbol{V}_\alpha \cdot \boldsymbol{n} \geqslant 0$. The latter is easily seen in the simplest case of two symmetric cusp points. Consider a depth profile S with two symmetric cusps P^+ and P^- such that $P^\pm = (\pm x_0, y_0)$, $x_0 < 0$ and the angle between the x-axis and the one-side tangent to S at P^+ (P^-) is equal to β ($\pi - \beta$) (as shown in fig. 5). The condition $\boldsymbol{V}_\alpha \cdot \boldsymbol{n} \geqslant 0$ at all regular points of S implies that $\alpha = \tan(\beta)x_0/y_0$. Then the condition $\alpha \in [0, 2]$ obviously yields the restriction $\beta \in [0, \arctan(2y_0/x_0)]$.

Conclusion

A boundary-value problem describing localized unforced harmonic motions of an ideal unbounded fluid in presence of submerged obstacles and a bottom topography is considered in the case when the geometry has exterior cusps. A generalization of the Maz'ya integral identity has been obtained including coefficients of singularities of the velocity field. The identity has been used to prove non-existence of trapped modes for some classes of geometries having exterior cusps.

Further work will endeavour to prove analogues of Theorems 4.1, 5.1 and 5.2 for three-dimensional water wave problem. One of particular application of the scheme can be the case of submerged obstructions having angular points with horizontal tangents. It is supposed that non-existence of trapped modes can be established when any cross-section of the fluid domain by the plane $y = h$, $h < 0$, is starlike with regard to the y-axis.

Acknowledgements. The support by Royal Society/NATO/FCO Chevening under a Postdoctoral Fellowship Award grant is gratefully acknowledged. The author is greatly indebted to Dr. N.G. Kuznetsov for sharing his knowledge of Maz'ya's identity. Thanks also due to an anonymous referee for bringing author's attention to the case of non-horizontal cusps.

Appendix A. Asymptotics of potential near corners of obstacles

In this appendix we shall derive asymptotics near a corner of a submerged obstacle for a function u satisfying the Helmholtz equation (4.8) in the fluid domain W, the homogeneous Neumann condition (2.8) on the boundary of the obstacle S and the condition of finiteness of energy $u \in H^1_{\rm loc}(W)$. We introduce a system of polar coordinates (ρ, θ) with origin at the corner point. Let α be the angle between the one-side tangents in the corner point, where $\alpha \in (0, 2\pi]$, measured through the fluid. Let the contour S be smooth at distance ℓ from the corner and let $\chi_{a,b}(s) \in C^\infty(I\!R)$ be a cut-off function, which is equal to one for $s \in [0, a]$ and to zero for $s \geqslant b$. We define the potential

$$\mathcal{U}(\rho, \theta) = \chi_{a,b}(\rho)\, u(\rho, \theta), \quad a < b < \ell. \tag{A.1}$$

Since $\mathcal{U} \equiv 0$ for $\rho > b$, further we shall consider the potential in the subset Ω of the fluid domain, where $\Omega = W \cap \{(\rho, \theta) : \rho < c\}$, $b < c < \ell$.

Taking into account (4.8) we find

$$\begin{aligned} \nabla^2 \mathcal{U} = \rho^2 \partial^2_\rho \mathcal{U} + \rho\, \partial_\rho \mathcal{U} + \partial^2_\theta \mathcal{U} = \rho^2 \left(u\, \partial^2_\rho \chi + 2\, \partial_\rho u\, \partial_\rho \chi + \chi\, \partial^2_\rho u \right) \\ + \rho \left(u\, \partial_\rho \chi + \chi\, \partial_\rho u \right) + \chi\, \partial^2_\theta u = \chi \left(\nabla^2 u + \lambda u \right) + F = F, \end{aligned}$$

where

$$\begin{aligned} F(\rho, \theta; u) = & \rho^2 \left[u(\rho, \theta)\, \partial^2_\rho \chi(\rho) + 2 \partial_\rho u(\rho, \theta)\, \partial_\rho \chi(\rho) \right] \\ & + \rho\, u(\rho, \theta)\, \partial_\rho \chi(\rho) - \lambda \chi(\rho) u(\rho, \theta). \end{aligned} \tag{A.2}$$

Consider also the normal derivative of the potential $\mathcal{U}$ on the boundary of the domain Ω. By (A.1) and (2.8) we find

$$\partial_n \mathcal{U} = u\, \partial_n \chi \equiv G\,. \tag{A.3}$$

Let us now suppose that the potential u is fixed so that $F(x,y;u) = F(x,y)$ and $G(x,y;u) = G(x,y)$, then the potential $\mathcal{U}$ can be considered as a solution to the Neumann problem for the Poisson equation in the domain Ω:

$$\begin{aligned} \nabla^2\mathcal{U} &= F \quad \text{in} \quad \Omega, \\ \partial_n\mathcal{U} &= G \quad \text{on} \quad \partial\Omega. \end{aligned} \tag{A.4}$$

We shall make use of the results by [16, ch. 2] and have to introduce here the functional spaces used in this book. Let the space $V^l_\gamma(\Omega)$ ($l = 0, 1, \ldots; \gamma \in I\!R$) consisting of functions on Ω be defined as closure of $\mathrm{C}^\infty_0(\overline{\Omega} \setminus 0)$ in the norm

$$\left\|u; V^l_\gamma(\Omega)\right\| = \left(\int_\Omega \sum_{i=0}^{l}\sum_{j=0}^{l-i} \rho^{2(\gamma-l+i+j)}|\partial^i_x\partial^j_y u(x,y)|^2 \,\mathrm{d}x\,\mathrm{d}y\right)^{\frac{1}{2}}$$

and the spaces $V^{l-1/2}_\gamma(\partial\Omega)$ ($l = 1, 2, \ldots$) consists of traces on $\partial\Omega$ of functions from $V^l_\gamma(\Omega)$ with the norm defined by

$$\left\|u; V^{l-1/2}_\gamma(\partial\Omega)\right\| = \inf\left\{\left\|v; V^l_\gamma(\Omega)\right\| : v = u \text{ on } \partial\Omega \setminus 0\right\}.$$

From the condition (2.5) and the definition (A.2) it obviously follows that $F \in V^l_\gamma(\Omega)$ for $l = 1$ and $\gamma \geqslant 1$. In its turn the function G is equal to zero in $\{(\rho,\theta) \in \Omega : \rho < a\}$. Under the assumption on smoothness of the boundary $u \in \mathrm{C}^\infty(\overline{\Omega} \setminus 0)$ and, in view of (A.3), G belongs to the space $V^{m+1/2}_\gamma(\partial\Omega)$ for any $m > 0$, in particular, for $m = l = 1$, which is preposed in Theorem 4.2 in [16, ch. 2]. In order to satisfy further conditions of the theorem, where it is assumed that $\gamma - l - 1 \in (0, \pi\alpha^{-1})$, we choose $\gamma = 2 + \varepsilon$, where ε is a small positive value. Then, the theorem guarantees existence of the unique solution to (A.4), $\mathcal{U} \in V^{l+2}_\gamma(\Omega)$. Appealling to the definitions of the functions $\mathcal{U}$ and F we find that $F \in V^3_{2+\varepsilon}(\Omega)$ and, obviously, $F \in V^3_{4+\varepsilon}(\Omega)$, so that we can apply Theorem 4.2 in [16, ch. 2] again. Repeating the procedure we find that $\mathcal{U} \in V^{2n+1}_{2n-1+\varepsilon}(\Omega)$ for all integers $n \geqslant 0$.

In order to satisfy the assumptions of Theorem 4.4 in [16, ch. 2] we shall consider $\mathcal{U}$ as an element of a wider class, so that $\mathcal{U}, F \in V^l_\gamma(\Omega)$, where $l = 2n+1$, $\gamma = 2n+\varepsilon$, $\varepsilon > 0$, and the asymptotics of $\mathcal{U}$ as $\rho \to 0$ is given by Theorem 4.4 in [16, ch. 2] as follows:

$$\mathcal{U}(x,y) = \chi(\rho)\left\{c_0 + c_{01}\log\rho + \sum_{j=1}^{m} c_j\rho^{j\pi/\alpha}\cos\left(j\pi\theta\alpha^{-1}\right)\right\} + w(x,y), \tag{A.5}$$

where c_i are constants, $w \in V^{l+2}_\gamma(\Omega) = V^{2n+3}_{2n+\varepsilon}(\Omega)$, $n \geqslant 0$ and the integer m is defined by the condition of Theorem 4.4 in [16, ch. 2] that the value $\alpha\pi^{-1}(l+1-\gamma)$ should belong to the interval $(m, m+1)$, so that $m = 0, 1, 2, 3$ for $m/2 < \alpha\pi^{-1} \leqslant (m+1)/2$. From the fact that $w \in V^{2n+3}_{2n+\varepsilon}(\Omega)$ it follows, in particular, that $\partial^i_x\partial^j_y w = O(\rho^{2-i-j})$ as $\rho \to 0$. We note that $c_{01} = 0$ in view of the condition (2.5).

References

[1] F. Ursell, *Proc. Camb. Phil. Soc.* **47** (1951) 347–358.

[2] F. Ursell, *Proc. R. Soc. Lond.* A **214** (1952) 79–97.

[3] D.S. Jones, *Proc. Camb. Phil. Soc.* **49** (1953) 668–684.

[4] D.V. Evans and N. Kuznetsov, Trapped modes. *Advances in Fluid Mechanics.* Southampton: Comp. Mech. Publ., **10** (1997) 127–168.

[5] C.M. Linton and D.V. Evans, *Q. Jl Mech. appl. Math.* **44(3)** (1991) 487–506.

[6] N.F. Parsons and P.A. Martin, *J. Fluid Mech.* **284** (1995) 359–375.

[7] N.G. Kuznetsov, Uniqueness in the Water-Wave Problem for a Vertical Shell. *Proc. of 5th Int. Conference on Math. and Numerical aspects of wave propagation.* Santiago de Compostela, Spain (2000) 504–508.

[8] V.G. Maz'ya, *Proc. Sobolev's Semin.* No. 2 (1977) 57–79. Novosibirsk: Inst. of Math., Sibirian Branch, Acad. Sci. USSR. (in Russian)

[9] V.G. Maz'ya, *J. Soviet Math.* **10** (1978) 86–89.

[10] B.R. Vainberg and V.G. Maz'ja, *Trans. Moscow Math. Soc.* **28** (1973) 56–73.

[11] A. Hulme, *Math. Proc. Camb. Phil. Soc.* **95** (1984) 511–519.

[12] N. Weck, *Math. Meth. Appl. Sci.* **12** (1990) 393–404.

[13] N.G. Kuznetsov, *Differential Equations* **27** (1991) 187–194.

[14] M.J. Simon and F. Ursell, *J. Fluid Mech.* **148** (1984) 137–154.

[15] Sir Horace Lamb, *Hydrodynamics.* 6th edition, Dover publications: New York (1932).

[16] S.A. Nazarov and B.A. Plamenevsky, *Elliptic problems in domains with piecewise smooth boundaries.* De Gruyter expositions in math. 13: Berlin, New York (1994).

Author's typeset of the paper published in Q. Jl Mech. appl. Math. ***55(1)****: 127–140, 2002.*